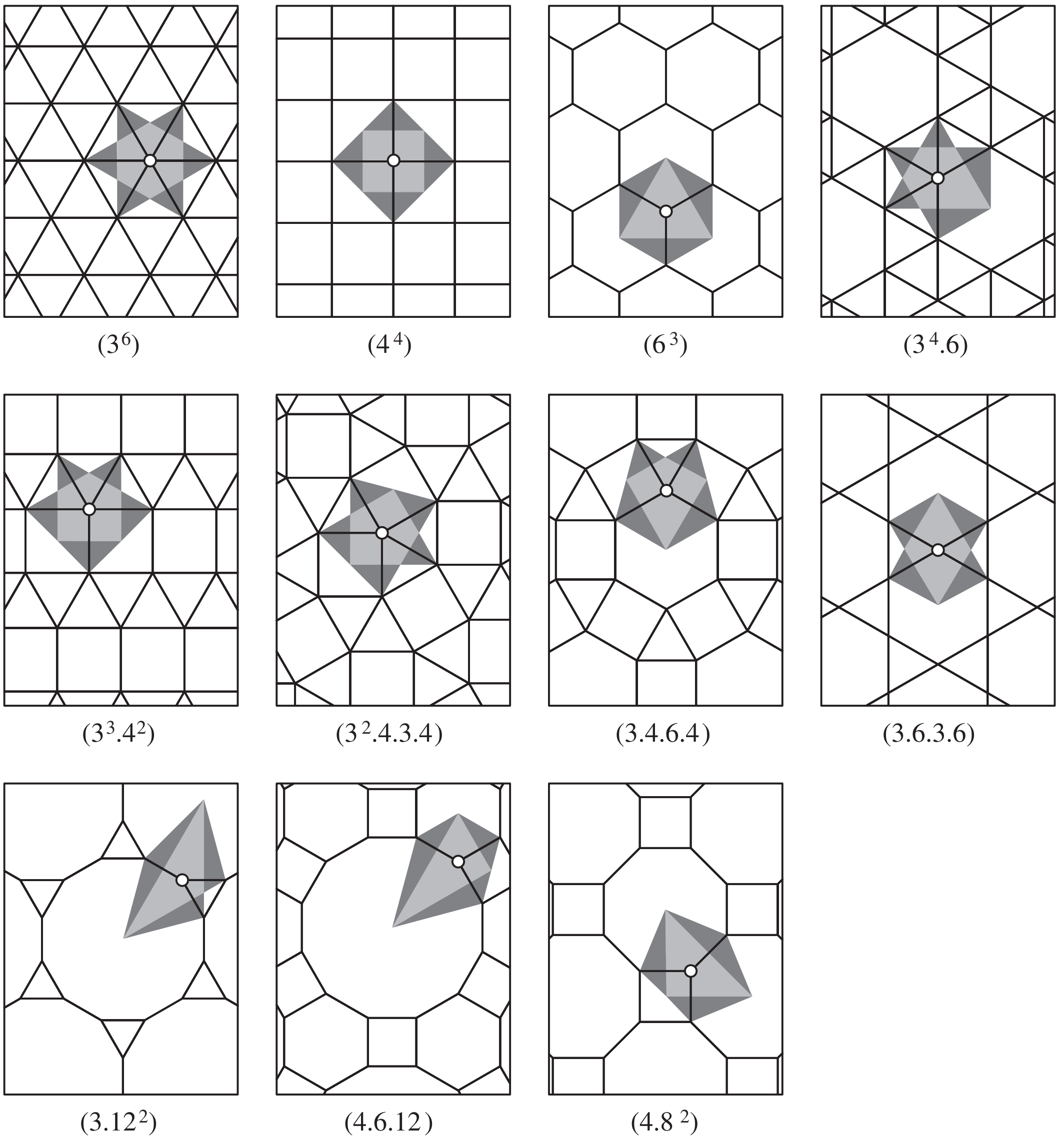

Fig. 3. First and second nearest regions.

given by

$$T(a,b) = \int_0^b \int_0^{\arccos b/a} r^2 \, d\theta \, dr + \int_b^a \int_{\arccos b/r}^{\arccos b/a} r^2 \, d\theta \, dr$$
$$= \frac{ab}{6}\sqrt{a^2-b^2} + \frac{b^3}{6} \ln \frac{a+\sqrt{a^2-b^2}}{b}. \quad (1)$$

Dividing $T(a,b)$ by the area of the triangle $S = b\sqrt{a^2-b^2}/2$ yields the average distance $E(R)$ as

$$E(R) = \frac{T(a,b)}{S} = \frac{a}{3} + \frac{b^2}{3\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{b}. \quad (2)$$

The average distances $E(R_1)$, $E(R_2)$ in Archimedean tilings can be calculated by considering only one vertex, because all vertices are of the same type. Figure 3 shows the regions where the white point is the first and second nearest. We call these regions the first and second nearest regions, respectively. $E(R_1)$ and $E(R_2)$ are then the average distances from a random point in the first and second nearest regions to the white point. $E(R_1)$ and $E(R_2)$ are obtained by partitioning the regions into right triangles. For example, the first nearest region for (3^6) is the hexagon centered at the white point with side length $a/\sqrt{3}$, where a is the side length of a tile. The region is partitioned into 12 right triangles with side lengths $a/\sqrt{3}$ and $a/2$. Using Eq. (1), we have

$$E(R_1) = \frac{12}{S} T\left(\frac{a}{\sqrt{3}}, \frac{a}{2}\right) = \frac{4+3\ln 3}{6 \cdot 3^{3/4}\sqrt{2\rho}} \approx \frac{0.377}{\sqrt{\rho}}, \quad (3)$$

where $S = \sqrt{3}a^2/2$ is the area of the first nearest region and $\rho = 1/S$ is the density of vertices. Partitioning the second nearest region into right triangles, we have

$$E(R_2) = \frac{-4+6\sqrt{3}-3\ln\left(6-3\sqrt{3}\right)}{3 \cdot 3^{3/4}\sqrt{2\rho}} \approx \frac{0.729}{\sqrt{\rho}}. \quad (4)$$

$E(R_1)$ and $E(R_2)$ for the other tilings are similarly obtained as follows: