



Fig. 4. The most fundamental concentric tiling of regular pentagons is generated by arranging five “golden trapeziums” ($108^\circ - 108^\circ - 72^\circ - 72^\circ$) dextrally around a vertex in the central area and the pattern is concentrically inflated by adding more tiles around it as shown.



Fig. 5. A concentric tiling of regular pentagon is generated by arranging five “golden trapeziums” differently than in Fig. 4.

4. Tiling and Twinned Crystal

Senechal (1995) commented: “... while from a crystallographic perspective, the structure of Fig. 4 is just a twinned crystal for which the twin boundaries can be filled nicely with the same trapezoidal shape as is used to make the crystals”. Closer examination of the areas between adjoining vertices of pentagon and laying a lattice reveals period parallelograms containing identical pieces of the tile matching fundamental domains having twice the area of golden trapezium and displaying translational symmetry, characteristic of periodic tiling. The central region displaying pentagonal-chiral symmetry could be interpreted as a 5-fold intergrowth structure of a twinned crystal. If subjected to laser beam experiment, such a tiling would reveal pseudo-fivefold rotational symmetry with periodic bright spots. The diffraction pattern would thus be a sum of discrete periodic diffraction patterns.

However, Bagley (1965) citing experimental evidence, suggested that it is unlikely that twinning could produce such a structure in small sized nanoparticles, instead, formation of pentagonal dipyrmaid nucleus and its subsequent growth is a simpler and more probable mechanism.

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