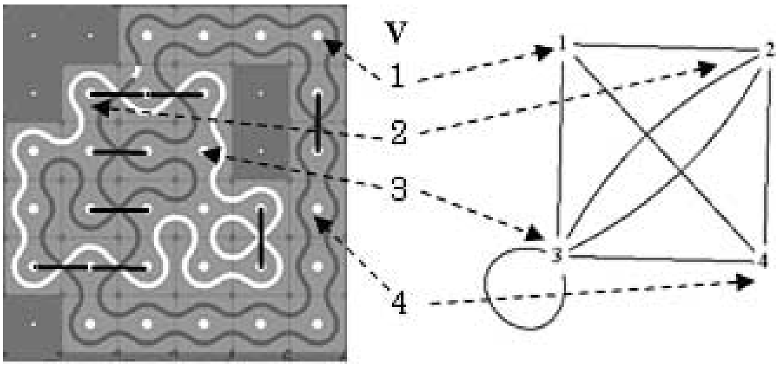

Fig. A.1. In Kolam (left), white dots represent vertices, but some dots surrounded by the lines with two-point-connections are combined to one vertex, the N-lines of black linear lines represent the planar graph (right), and then the lines belonging crossings at the medial positions between two adjacent and combined dots are the medial graph. This Kolam becomes a knot-link with up-down crossings. Components of the medial graph are called left-right cycles sometimes. The numbers in the center are assigned to the vertices representing combined dots of Kolam. Note one vertex represents some combined dots.

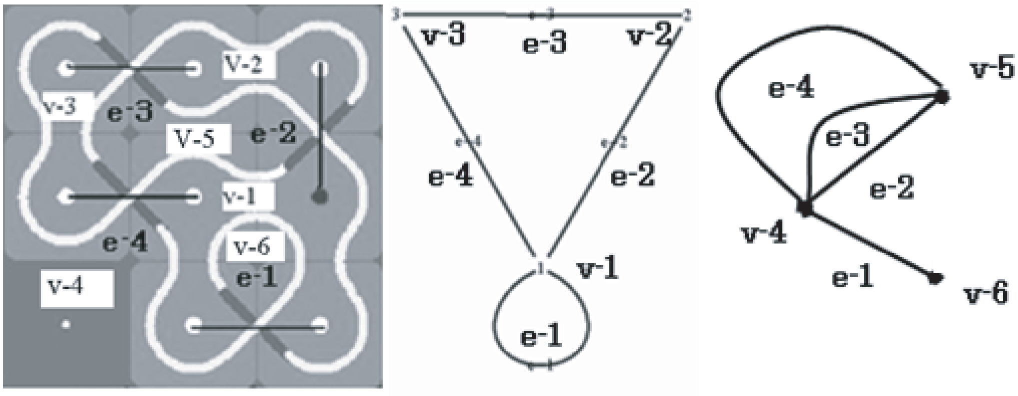

Fig. A.2. A sample Kolam (left. the medial graph of G, or the diagram D) and N-lines (G: black lines), the Tait graph G (center. the planar graph) and the dual graph (right).

and self-contained proof of the following:

THEOREM A.1 (Silver and Williams). Let L be a link arising from a medial graph M(Γ) by resolving vertices. The number μ(L) of components of L is the nullity of the mod-2 Laplacian matrix Q2(Γ). This Theorem was first given in the paper by C. Godsil and G. Royle[8].

The process for getting the component number is the following: Start with the graph G and number the vertices with the maximum vn. Make the adjacency matrix A (entry in ith row and jth column is number of edges from i to j, with loops counted twice). Obtain the Laplacian matrix Q = D − A, where D is a diagonal matrix of degrees of vertices from A. Finally, calculate the nullity of the mod-2 Laplacian matrix Q2(G). and then the result is the component number of the medial graph of G. The following calculation example is of the sample Kolam (Fig. A.1) using Mathematica code by the author;

Each matrix of the process for Fig. A.1 is the following respectively: Adjacent matrix ma with vertex number vn of four, Diagonal matrix md of degrees of vertices, md[i=j] = sum{ma[I,1] + ma[I,2] + $\cdots$ + ma[I,vn]}, md[i!=j] = 0. Laplacian matrix mq = md - ma, modular matix m2 = Mod[mq,2], and Mod-2 row reduced matrix mq2 from m2.

$$ma = \begin{array}{c} \\ v1 \\ v2 \\ v3 \\ v4 \end{array} \begin{array}{c} \begin{array}{cccc} v1 & v2 & v3 & v4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array},$$

$$md = \begin{bmatrix} 3\,0\,0\,0 \\ 0\,4\,0\,0 \\ 0\,0\,6\,0 \\ 0\,0\,0\,3 \end{bmatrix}, \quad mq = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 4 & -2 & -1 \\ -1 & -2 & 4 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix},$$

$$m2 = \begin{bmatrix} 1\,1\,1\,1 \\ 1\,0\,0\,1 \\ 1\,0\,0\,1 \\ 1\,1\,1\,1 \end{bmatrix}, \quad mq2 = \begin{bmatrix} 1\,0\,0\,1 \\ 0\,1\,1\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \end{bmatrix}.$$

The nullity is the number of zero rows in the matrix mq2. From the rank-nullity theorem, Nullity = vn − MatrixRank[mq2] = 2. Finally, we obtain the output 2, which means that the component (loop in Kolam) number is two. Note the rank of [mq2] is calculated using arithmetic modulo 2 as an example of ma = [[0,1,1][1,0,1][1,1,0]], mq2 = [[0,1,1][1,0,1][1,1,0]], the rank of mq2 = 2, and then the