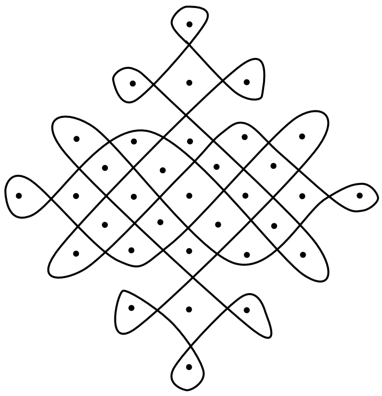

Fig. 1. Example of a *pulli kolam* called Brahma's knot.

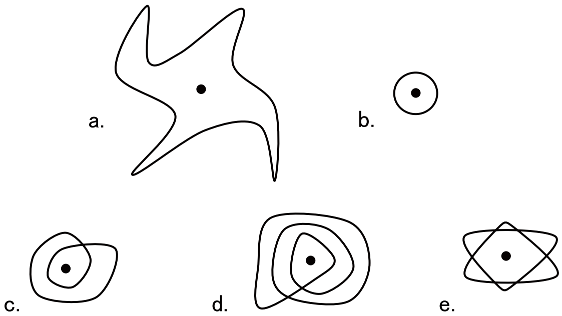

Fig. 2. Examples of *kolam*s around one dot that follow the mandatory rules **M**1–**M**3. An infinite number of *kolam*s are possible. Additional optional (**O**) rules, **O**1–**O**6, can limit the number allowed.

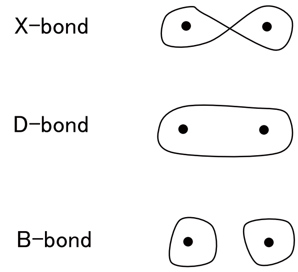

Fig. 3. Infinitely many types of bonds are possible between a pair of dots that follow the rules **M**1, **M**2, **M**3, and optional rules **O**1 and **O**2, three of which are shown here.

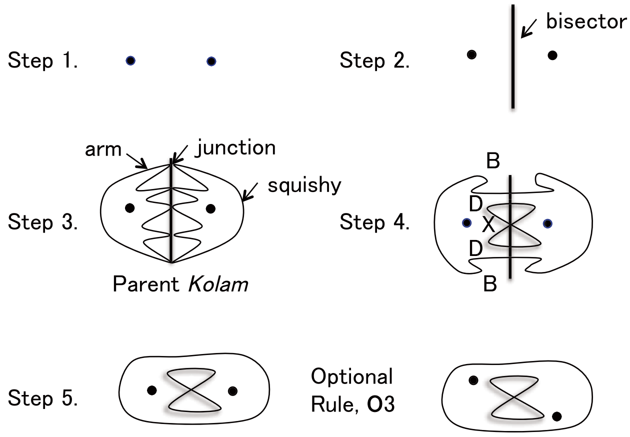

Fig. 4. Illustrating the construction of a *kolam* in 5 steps plus optional rule **O**3: The procedure is shown for $N = 2$ (2 dots) and $J = 5$ (5 junctions). If each junction is restricted have one of 3 types of bonds (X-, D-, or B-), it can lead to $3^5 = 243$ possible *kolam*s. One of these options, namely, B-D-X-D-B, is shown in the figure in Step 4. In the optional rule, the dots have been rearranged as an example of rule **O**3 after the *kolam* is drawn in Step 5.

dots (line crossing, 1, or uncrossing, 0) and convert these tiles into binary number arrays. Nagata [18] also addressed the construction of a primitive *kolam* for an arbitrary dot array with a similar approach. In contrast, the work presented here has a purely topological approach: it defines only 3 mandatory rules for defining a *kolam*, has no standard tiles, generalizes the ideas to any arbitrary arrangement of dots arranged in any shape (not necessarily square arrays), generalizes to interactions between any two dots (instead of only the nearest or next nearest neighbors), and to three or more number of bonds between an interacting pair of dots. The work suggests that for a given number of dots, N, there are a limited number of parent *kolam* types from which all other *kolam*s originate. All parent *kolam*s within a parent *kolam* type are homotopic (or topologically equivalent).

2. How Many *kolam*s for One Dot ($N = 1$)?

Figure 2 depicts a single dot, and a variety of lines circumscribing it that follow the three mandatory rules mentioned above. The *kolam* in general could be amorphous in shape, as in Fig. 2a, and in the special case of Fig. 2b is a circle. Multiple circumscriptions around the dot are possible, as in Figs. 2c, d, and e.

It becomes immediately clear from Fig. 2 that the number of possible *kolam*s thus defined, with only the mandatory rules, is *infinite*. One may arbitrarily impose additional optional (**O**) rules to limit the number of *kolam*s. Here are some:

O1: Only one circumscription of the line is allowed around each dot.

O2: A line circumscribing a dot should be as resourceful (simple) as possible, without additional unnecessary wiggles or flourishes (e.g. Fig. 2b is resourceful vs. Fig. 2a is not).

O3: While a *kolam* may be created by a minimum number of dots N needed for the 5-step method proposed below, one can then eliminate dots from, or add dots to, or move dots in a *kolam* after it has been drawn, provided the process does not violate the mandatory rules. The final *kolam* may thus appear to have N_{final} dots, where N_{final} may or may not be equal to N.

With **O**1 restriction, only 2a and 2b survive. With **O**1 and **O**2, only 2b will survive. Figure 2e, depicting a Star of David is a common *kolam*, which apparently is eliminated by **O**1. However, this *kolam* can also be generated by placing six dots ($N = 6$), one inside each ray of the star, and following the 5-step method proposed below. The 6 dots may later be erased, and one dot placed in the middle ($N_{final} = 1$) according to **O**3 to generate Fig. 2e. Another example is the Brahma's knot in Fig. 1, which can be generated by only $N = 25$ dots. However, Fig. 1 has $N_{final} = 33$ dots; the additional two horizontal rows of 4 dots each (total of 8 dots) in that *kolam* would be placed (according to