Analysis of the Motion of the Pop-up Spinner

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The pop-up spinner is a pop-up card. As the completed card opens and closes, its nested frames spin. We analyze the motion of frames of the pop-up spinner, and represent by a nested epitrochoid mechanism.
Key words: Pop-up spinner, Epitrochoid

1. Introduction

The pop-up spinner is a pop-up card (see Fig. 1). Figure 2 shows a template for the construction of pop-up spinner, having cuts, mountain folds, and valley folds lines. As their names imply, a mountain fold is a crease that bumps outward, while a valley fold is a crease that dents inward. Try cutting template and crease as indicated. As the completed card opens and closes, its nested frames spin. The pop-up spinner has the central frame (like a beak) with one slit and other frames (like wings) with two slits. We can see the animation by A. Nishihara in the Nishihara web page [2]. The name of the pop-up spinner was given by A. Nishihara ([2]). It is not well-known when and who designed this first; Due to [3], the pop-up spinner was invented in Japan by an unknown student at Musashino Art University in 1988. Due to the support web page of "Origami no suuri" [3], it seemed to have already existed in 1970's.

In [3], J. O'Rourke pointed out that the central zig-zag path of mountain and valley creases is a chain with the fixed angle by the construction as in Fig. 3. When the card is closed, this chain is curled up into a spiral configuration. When the card is opened, the chain heads toward the planar staircase configuration which achieves the maximum possible end-to-end distance of the chain, known as the *maxspan*.

In this article, we study the motion of frames of the pop-up spinner when the card is closing. We set x-axis and the origin O in the horizontal line of the opened template as in Fig. 4, y-axis vertical to the opened template. The template closes in the positive direction of y-axis as in Fig. 4. Let n be the number of frames. For $1 \leq k < n$, we denote by O_k and A_k the vertices of the edge of the k-th frame (like a wing) from the outside as in Fig. 4. And, we denote by O_{n-1} and A_n the vertices of the edge of the n-th frame (the central frame like a beak) as in Fig. 4. For convenience, we assume that the length of OA_1 is n, the length of O_kA_k for $1 \leq k < n$ is 1 and the length of $O_{n-1}A_n$ is 1.

We treat the ideal pop-up spinner. That is, we assume that the angle of the crease in the edge of each frame is equal to the fold angle of the card and that the ideal pop-up spinner is symmetric with respect to the xy plane on the way of the folding. Then note that the vertices O_k and A_k are on xy plane.

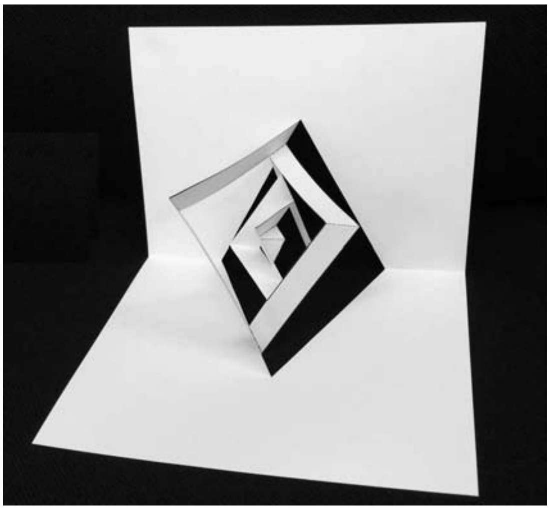

Fig. 1. Pop-up spinner.

We show the following theorem:

THEOREM. *Let n be the number of frames and θ be the fixed angle of the central chain.*

(1) *The angle $A(\theta)$ of the spin of the n-th frame (the central frame like a beak) of the pop-up spinner is given by $A(\theta) = n\theta$.*
(2) *The edge $O_{n-1}A_n$ of the n-th frame of the pop-up spinner has the orbit represented by the following coordinate:*

$$O_{n-1} = \left(\sum_{\ell=1}^{n-1} (-1)^{\ell-1}(n-\ell)\cos(\ell\alpha), \sum_{\ell=1}^{n-1} (-1)^{\ell-1}(n-\ell)\sin(\ell\alpha) \right),$$

$$A_n = O_{n-1} + \left((-1)^{n-1}\cos(n\alpha), (-1)^{n-1}\sin(n\alpha)\right),$$

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