

Fig. 2. A template for the construction of the pop-up spinner.



Fig. 3. A central chain with the fixed angle θ .



Fig. 4. The *x*-axis and the *y*-axis.





Fig. 5. A basic unit of the spin of the frame.



Fig. 6. An outside frame (the gray part) and an inside frame.

where $0 \le \alpha \le \theta$ and when the card is opened (resp. closed), $\alpha = 0$ (resp. $\alpha = \theta$).

The third author and his students conjectured the formula of Theorem (1) by observing the pop-up spinner actually made, and tried designing the template for the construction of a pop-up spinner having bi-directional spin ([1]).

An epitrochoid is defined to be a roulette traced by a point attached to a disc of radius r rolling around the outside of a fixed disc of radius R, where the point has a distance d from the center of the exterior disc.

We consider the following mechanism of epitrochoids with a nested structure: A rolling disc of one epitrochoid mechanism is concentrically attached on a fixed disc of another epitrochoid mechanism as in Figs. 11–14. These two