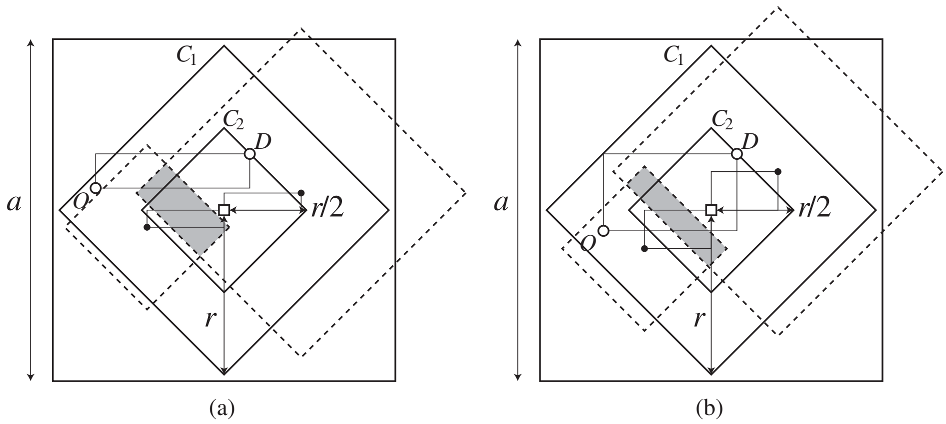

Fig. 6. Calculation of the probability for the grid pattern: (a) $t_y < r/2 \leq t_x - t_y$; (b) otherwise.

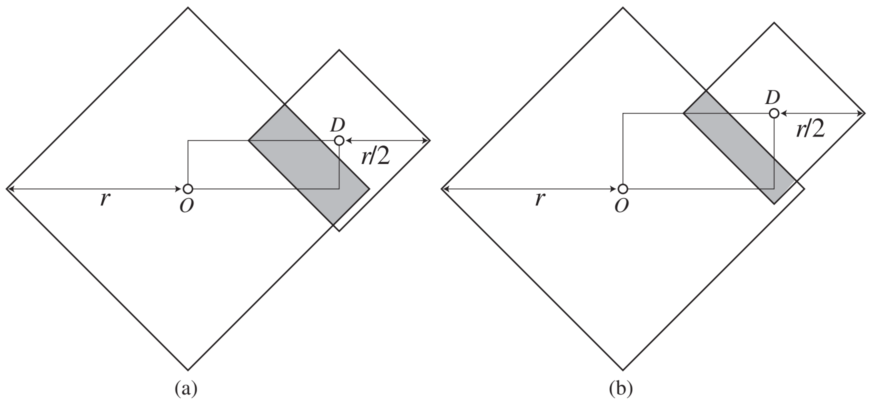

Fig. 7. Calculation of the probability for the random pattern: (a) $t_y < r/2 \leq t_x - t_y$; (b) otherwise.

3.1 Grid pattern

Suppose that stations are regularly distributed on a square grid with spacing a. The density of stations ρ, that is, the number of stations per unit area is then expressed as $\rho = 1/a^2$. The probability $P(t)$ can be calculated by considering only one station. The study region is then confined to the region where a station is the nearest, which is the square centered at the station with side length a, as depicted in Fig. 3.

To make the round trip, both O and D must be in the diamond C—a square rotated at angle 45°, centered at the station with radius r. Recall that a diamond gives the set of points within a given rectilinear distance from its center (Krause, 1987). If both O and D are in the diamond C, the center of the shortest path rectangle is in the shaded region in Fig. 3. This region is the intersection of the two diamonds which are obtained by moving the diamond C by $t_x/2$ to the right and $t_y/2$ to the upward and by $t_x/2$ to the left and $t_y/2$ to the downward. The intersection is assumed to be entirely within the square, i.e., $2r - t_x \leq a$, and the other case is left for future work. $P(t)$ is then the probability that the center of the shortest path rectangle lies inside the intersection of the two diamonds.

Since origins and destinations are selected at random, the center of the shortest path rectangle is uniformly distributed over the square. $P(t)$ is then given by the ratio of the area of the intersection to that of the square. The area of the intersection of the two diamonds is

$$S = \frac{1}{2}(2r - t_x - t_y)(2r - t_x + t_y). \tag{1}$$

$P(t)$ is then

$$P(t) = \frac{S}{a^2} \tag{2}$$

$$= \frac{\rho}{2}(2r - t_x - t_y)(2r - t_x + t_y). \tag{3}$$

$P(t)$ is shown in Figs. 5a and b. $P(t)$ decreases with the trip length t, and increases with the density of stations ρ and the vehicle range r. Note that $P(t)$ for $t_x = t_y = t/2$ is greater than that for $t_x = t, t_y = 0$, even though the trip length is the same. That is, $P(t)$ varies according to the relative position of origin and destination as well as the trip length.

3.2 Random pattern

Suppose that stations are uniformly and randomly distributed. To make the round trip, both O and D must be within the distance r of a station. This means that the station must be in the intersection of the two diamonds centered at O and D with radius r, as depicted in Fig. 4. $P(t)$ is then the probability that the intersection of the two diamonds contains at least one station. The probability that a region of area S contains exactly x stations, denoted by $P(x, S)$, is given by the Poisson distribution as

$$P(x, S) = \frac{(\rho S)^x}{x!} \exp(-\rho S), \tag{4}$$