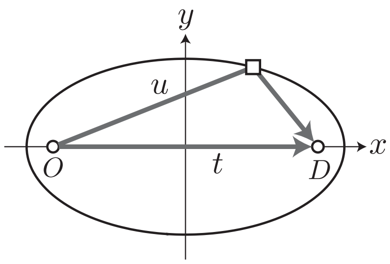

Fig. 1. Deviation distance to visit a station.

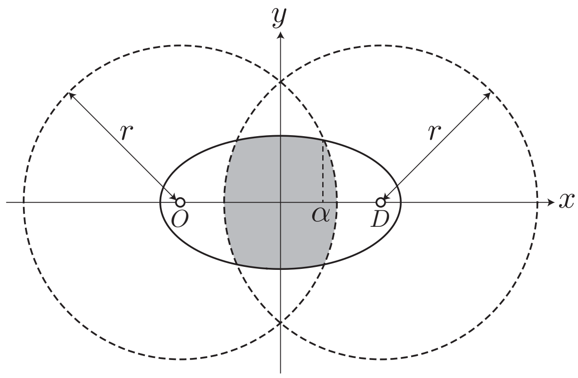

Fig. 2. Calculation of the probability.

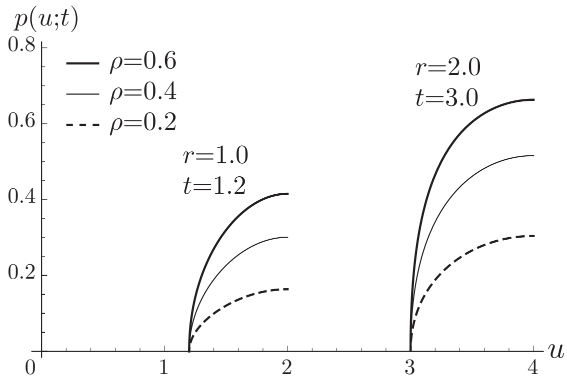

Fig. 3. Probability of making the round trip within a deviation distance u.

plied the FRLM to the location of hydrogen stations in Florida. Lim and Kuby (2010) presented three heuristic algorithms for the FRLM. Capar and Kuby (2012) and Capar *et al.* (2013) proposed efficient formulations of the FRLM to solve large problems. The FRLM was extended by Kuby and Lim (2007) to add candidate sites along network arcs, Upchurch *et al.* (2009) to include the capacity of refueling facilities, and Kim and Kuby (2012) to allow drivers to deviate from their shortest paths. Wang and Lin (2009) presented a set-covering model to minimize the cost of refueling stations. In these location models, the number of stations to be located is an input. Our model will thus supplement location models of alternative fuel stations.

The rest of this paper is organized as follows. The next section develops a model for determining the sufficient density of alternative fuel stations. The following sections provide the density of stations required to achieve a specified level of service for three cases of the refueling availability at origin and destination. The final section presents concluding remarks.

2. Model

Consider trips using alternative fuel vehicles. Let r be the vehicle range—the maximum distance that the vehicle with full tank of fuel can drive. Origins and destinations are selected at random within a study region. The random travel demand can be used as the first approximation for the actual travel demand and serves as a basis for further analysis with more realistic travel demand. For example, travel demand that depends on the trip length can be considered by incorporating the trip length distribution, as discussed by Miyagawa (2016).

Drivers are assumed to deviate from their shortest paths to refuel their vehicles. Let t be the trip length between origin O and destination D and u be the deviation distance to visit a station. The deviation distance is defined as the sum of the distances from O to the station and from the station to D. Distance is measured as the Euclidean distance. The region that a driver can cover within a deviation distance u forms an ellipse, the foci of which are at O and D. Recall that an ellipse is defined as the locus of points such that the sum of the distances to two fixed points (foci) remains constant. Set the coordinate system as shown in Fig. 1. The ellipse is then expressed as

$$\frac{4x^2}{u^2} + \frac{4y^2}{u^2 - t^2} = 1, \qquad (1)$$

where $u \geq t$ (Miyagawa, 2013b). If the ellipse contains a station, the station is available within the deviation distance u.

Let $p(u;t)$ be the probability that the vehicle can make the repeated round trip between randomly selected origin and destination within a deviation distance u. Refueling is allowed only once for each one-way trip. Thus, we focus on short distance trips which need at most one refueling. Note that if multiple refueling is allowed for such short distance trips, the sufficient density of stations decreases, but the inconvenience of drivers increases. Since $p(u;t)$ depends on the refueling availability at origin and destination, three cases are considered: fuel is available at both origin and destination, fuel is available at either origin or destination, and fuel is available at neither origin nor destination. The first or second case can be applied to plug-in electric vehicles, whereas the third case can be applied to hydrogen and natural gas vehicles.

Refueling stations are assumed to be randomly distributed. This assumption is not entirely unrealistic because when stations are sparse in an early stage of the development, the pattern of stations makes little impact on the basic properties of the probability of making the round trip. By comparing the probabilities for grid and random patterns of stations, Miyagawa (2013a) demonstrated that the difference between them is relatively small.

3. Fuel is Available at Both Origin and Destination

First, we assume that fuel is available at both origin O and destination D. The vehicle can then start at O with