Table 1. Density of stations required to achieve $p(u;t) \geq \alpha$.

	t	1.2			1.4			1.6		
α	u	1.3	1.4	1.5	1.5	1.6	1.7	1.7	1.8	1.9
0.2		0.61	0.43	0.35	0.73	0.53	0.45	1.05	0.79	0.71
0.4		1.39	0.97	0.80	1.68	1.21	1.02	2.40	1.82	1.62
0.6		2.49	1.75	1.43	3.02	2.18	1.84	4.31	3.26	2.90
0.8		4.38	3.07	2.51	5.30	3.83	3.23	7.57	5.72	5.09

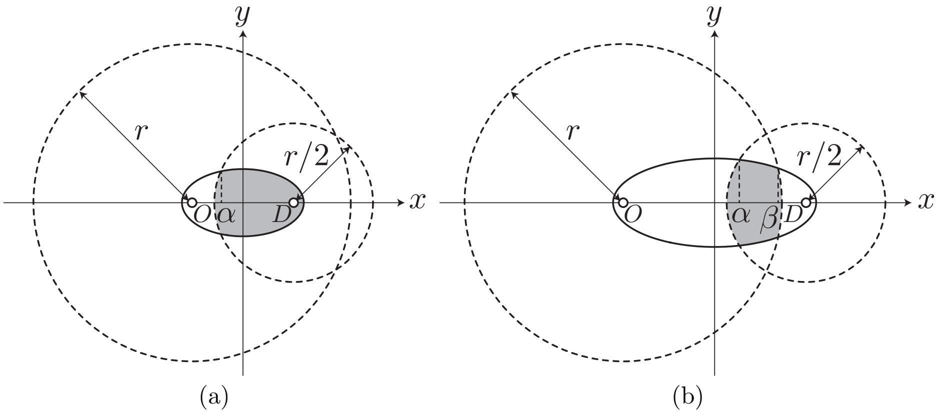

Fig. 4. Calculation of the probability: (a) $t \leq u \leq 2r - t$; (b) $2r - t < u \leq 3r/2$.

full tank of fuel. If $t \leq r$, the vehicle can reach D without refueling and return to O. If $t > 2r$, the vehicle cannot reach D because more than one refueling is needed. Hence, we focus on the case where $r < t \leq 2r$. If $r < t \leq 2r$, the vehicle can make the round trip if both O and D are within the distance r of a station (Miyagawa, 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D, fill up again at D, turn round, fill up again at that same station, and return to O.

To refuel at a station and complete the round trip, the station must be in the intersection of the two circles centered at O and D with radius r. To visit the station within a deviation distance u, the station must also be in the ellipse (1). Thus, $p(u;t)$ is the probability that the intersection of the two circles and the ellipse contains at least one station, as shown in Fig. 2. The probability that a region of area S contains exactly x stations, denoted by $P(x, S)$, is given by the Poisson distribution as

$$P(x, S) = \frac{(\rho S)^x}{x!} \exp(-\rho S), \tag{2}$$

where ρ is the density of stations (Clark and Evans, 1954). The area of the intersection is, if $t \leq u \leq 2r$,

$$S = \frac{2\sqrt{u^2 - t^2}}{u} \int_0^{\alpha} \sqrt{u^2 - 4x^2}\,\mathrm{d}x + 4 \cdot \int_{\alpha}^{r-t/2} \sqrt{r^2 - \left(x + \frac{t}{2}\right)^2}\,\mathrm{d}x, \tag{3}$$

where

$$\alpha = \frac{2ru - u^2}{2t}. \tag{4}$$

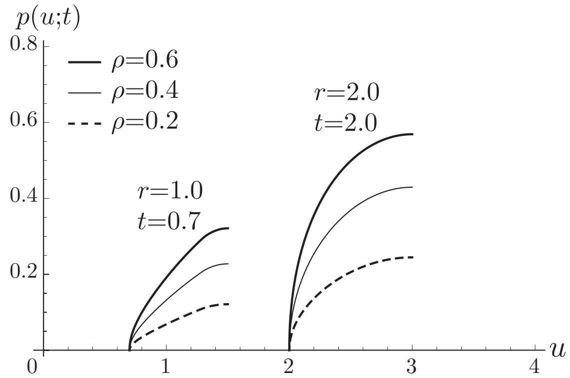

Fig. 5. Probability of making the round trip within a deviation distance u.

The probability $p(u;t)$ is obtained as

$$p(u;t) = 1 - P(0, S) = 1 - \exp(-\rho S). \tag{5}$$

Although the final form is not provided due to the limited space, the probability can be expressed in a closed form. The probability $p(u;t)$ is shown in Fig. 3. It can be seen that $p(u;t)$ increases with the deviation distance u and the density of stations ρ. Note that $p(t;t) = 0$ because the vehicle cannot make a deviation and that $p(2r;t)$ is identical with the result obtained by Miyagawa (2013a).

Using the probability $p(u;t)$, we can calculate the density of stations required to achieve a specified level of service. Table 1 shows the density of stations required to achieve $p(u;t) \geq \alpha$ for the vehicle range $r = 1$. The required density increases with the trip length t and the target probability α, and decreases with the deviation distance u that drivers can tolerate. The target service level should be