Table 2. Density of stations required to achieve $p(u;t) \geq \alpha$.

t		0.7			0.9			1.1		
α	u	0.8	0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.4
0.2		1.28	0.83	0.63	1.20	0.79	0.63	1.41	1.03	0.90
0.4		2.93	1.90	1.44	2.75	1.81	1.44	3.22	2.37	2.07
0.6		5.26	3.41	2.59	4.93	3.25	2.58	5.77	4.25	3.71
0.8		9.24	5.99	4.54	8.66	5.70	4.53	10.14	7.46	6.52

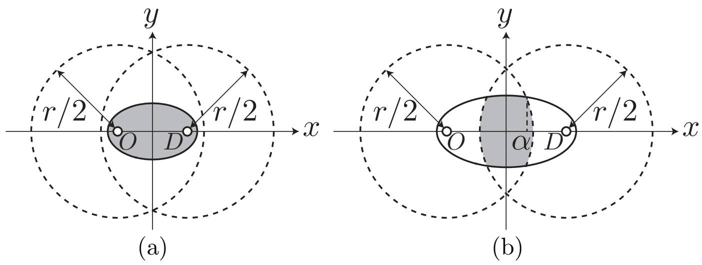

Fig. 6. Calculation of the probability: (a) $t \leq u \leq r - t$; (b) $r - t < u \leq r$.

determined according to the traffic condition in the study region. If long distance trips are dominant, we should use a large value for both t and α. If drivers are reluctant to make a deviation to refuel their vehicles, the value for u should not be much greater than that for t.

4. Fuel is Available at Either Origin or Destination

Next, we assume that fuel is available at either origin O or destination D. Without loss of generality, we assume that fuel is available at only O. Since the round trip is considered, the vehicle is required to reach D with at least half a tank remaining. If $t \leq r/2$, the vehicle can make the round trip without refueling. If $t > 3r/2$, the vehicle cannot make the round trip without refueling more than once. Hence, we focus on the case where $r/2 < t \leq 3r/2$. If $r/2 < t \leq 3r/2$, the vehicle can make the round trip if O is within the distance r of a station and D is within the distance $r/2$ of the station (Miyagawa, 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D, turn round, fill up again at that same station, and return to O.

To refuel at a station and complete the round trip, the station must be in the intersection of the circle centered at O with radius r and the circle centered at D with radius $r/2$. To visit the station within a deviation distance u, the station must also be in the ellipse (1). Thus, $p(u;t)$ is the probability that the intersection of the two circles and the ellipse contains at least one station, as shown in Fig. 4. The area of the intersection is, if $t \leq u \leq 2r - t$,

$$S = \frac{\sqrt{u^2 - t^2}}{u} \int_{\alpha}^{u/2} \sqrt{u^2 - 4x^2}\,\mathrm{d}x + 2 \cdot \int_{t/2-r/2}^{\alpha} \sqrt{\left(\frac{r}{2}\right)^2 - \left(x - \frac{t}{2}\right)^2}\,\mathrm{d}x, \tag{6}$$

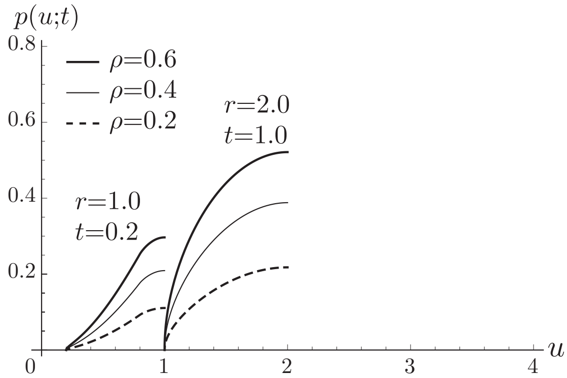

Fig. 7. Probability of making the round trip within a deviation distance u.

and if $2r - t < u \leq 3r/2$,

$$S = \frac{\sqrt{u^2 - t^2}}{u} \int_{\alpha}^{\beta} \sqrt{u^2 - 4x^2}\,\mathrm{d}x + 2\int_{t/2-r/2}^{\alpha} \sqrt{\left(\frac{r}{2}\right)^2 - \left(x - \frac{t}{2}\right)^2}\,\mathrm{d}x + 2\int_{\beta}^{r-t/2} \sqrt{r^2 - \left(x + \frac{t}{2}\right)^2}\,\mathrm{d}x, \tag{7}$$

where

$$\alpha = \frac{u^2 - ru}{2t},\ \beta = \frac{2ru - u^2}{2t}. \tag{8}$$

The probability $p(u;t)$ is obtained from Eq. (5) and shown in Fig. 5. Note that $p(3r/2;t)$ is identical with the result obtained by Miyagawa (2013a).

Table 2 shows the density of stations required to achieve $p(u;t) \geq \alpha$ for $r = 1$. Observe that more stations are required than the previous case to achieve an even lower level of service.