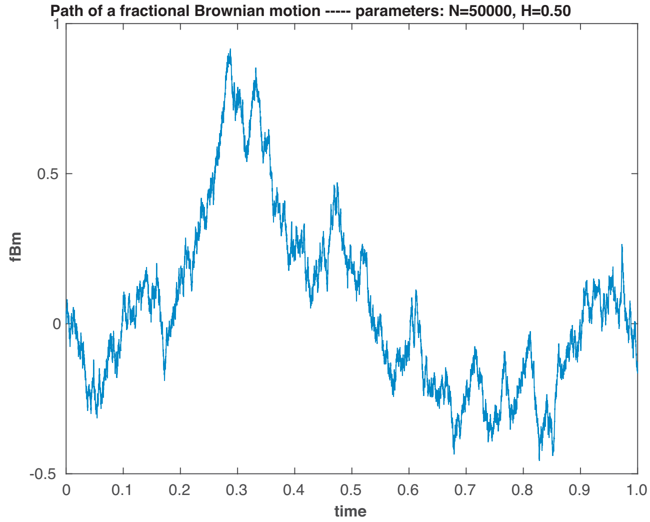


Fig. 1. Simulated fBm with $H = 0.50$.

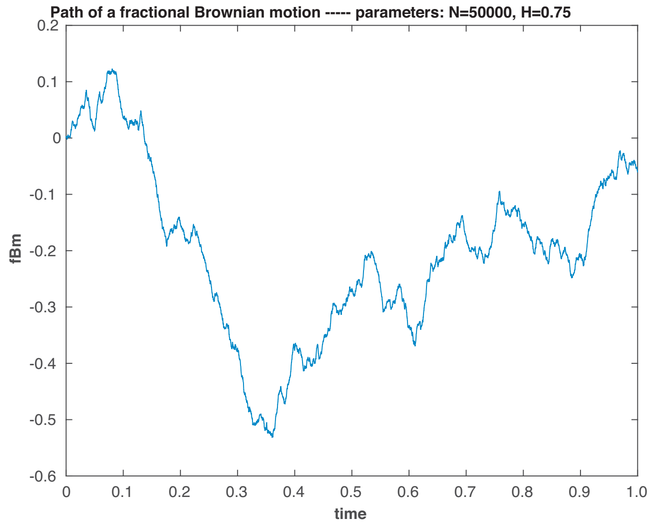


Fig. 2. Simulated fBm with $H = 0.75$.

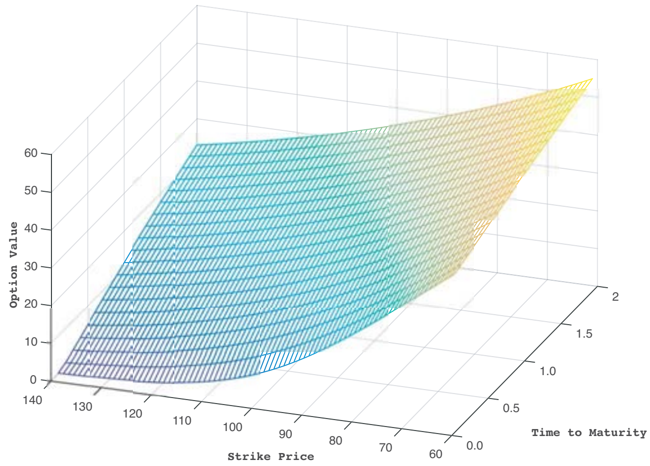


Fig. 3. Call option prices with $H = 0.50$ (current stock price $s = 100$; volatility $\sigma = 0.5$; risk-free interest rate $r = 0.1$).

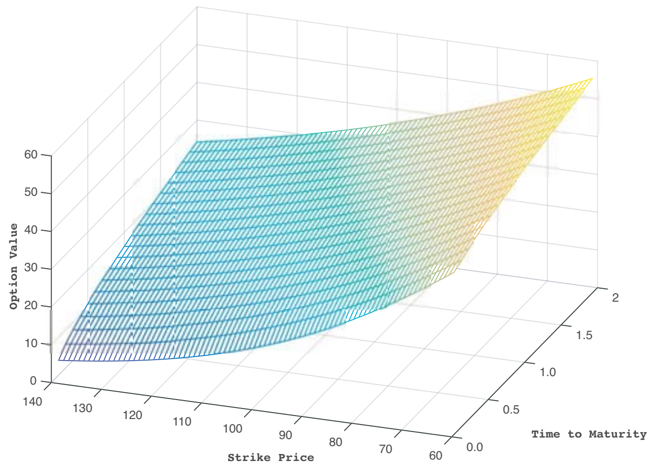


Fig. 4. Call option prices with $H = 0.75$ (other parameters fixed as in Fig. 3).

long memory parameter $H$ (i.e., the Hurst exponent). The fractional European put option price $P_{\text{fbs}}$ is also determined by the standard put-call parity relation

$$P_{\text{fbs}} = C_{\text{fbs}} + ke^{-rt} - s.$$

## 4. Simulation Study

In the section, we examine how the fractional European call option prices are evaluated in simulation studies.

Figures 1 and 2 are examples of fBm paths with $H = 0.50$ and $H = 0.75$, respectively. Simulations were performed using the dvfBm[*1] package function circFBSM() in the R language. As noted above, $H = 0.50$ (Fig. 1) describes standard Brownian motion, while $H = 0.75$ (Fig. 2) describes a process with a long memory property. Generation of the fractional Brownian paths is detailed in Coeurjolly (2000).

[*1] http://cran.r-project.org/web/packages/dvfBm/

Figures 3 and 4 show the European call option prices calculated by the fractional Black-Scholes formula introduced above. The Hurst parameters are set to $H = 0.50$ and $H = 0.75$ respectively, the current stock price $s = 100$, the volatility $\sigma = 0.5$, and the risk-free interest rate $r = 0.1$.

Since Figs. 3 and 4 are visually very similar, we clarify their differences in Figs. 5 and 6.

Figure 5 shows how the European call option prices differ between $H = 0.50$ and $H = 0.75$ with other parameters fixed at $s = 100$, $\sigma = 0.5$, and $r = 0.1$. According to this figure, the price difference is enhanced around the at-the-money (here denoted by the strike price $k = 100$), and widens as the time to maturity reduces. Figure 6 shows how the European call option prices as the Hurst exponent $H$ varies from 0 to 1. Other parameters are fixed at $s = 100$, $\sigma = 0.5$, $r = 0.1$, and $k = 100$. From this figure, we observe that the difference greatly increases as the time to maturity reduces and the Hurst exponent $H$ increases.

## 5. Empirical Study

In this section, we examine the long memory property of real financial data such as time series of stock prices, log-