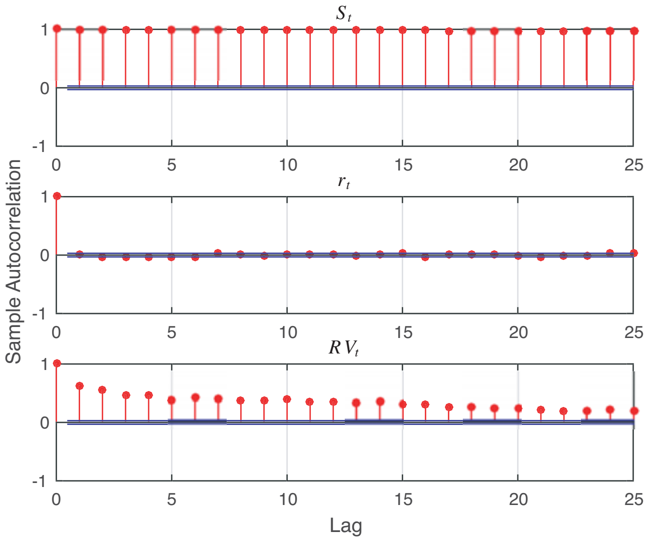

Fig. 8. Sample autocorrelation functions of S_t, r_t and RV_t time series in the TOPIX data (1997–2013).

Table 2. Results of t-ratio $\hat{\tau}_\mu$ estimated by the DF test.

Assumed models	S_t	r_t	RV_t
AR(1)	−0.632	−63.28	−24.68
	(0.421)	(0.001)	(0.001)
AR(1) with drift	−1.929	−63.27	−31.05
	(0.328)	(0.001)	(0.001)
TS	−1.796	−63.28	−31.85
	(0.694)	(0.001)	(0.001)

Note that TS represents trend-stationary and p-values for the null hypothesis are reported in parentheses.

Table 3. Results of the R/S analysis.

Used methods	S_t	r_t	RV_t
Hurst-Mandelbrot			
$\widehat{V}$	1245.19	66.35	561.00
$\widehat{H}$	0.855	0.503	0.759
Lo			
$\widehat{V}$	880.86	65.68	440.13
$\widehat{H}$	0.813	0.502	0.730

r_t at the 0.01 significance level, but the null hypothesis for S_t and RV_t is rejected at this level. Thus, the S_t and RV_t series show apparent serial correlations. As a graphical verification, we present sample autocorrelation functions for S_t, r_t and RV_t in Fig. 8. The correlogram impressively shows that the sample autocorrelation functions of both S_t and RV_t slowly decay, whereas that of S_t is one at all lags.

5.2 Stationarity and long-range dependency

In this subsection, we examine the stationarity and long-range dependency of the series before setting the pricing options under fBm.

First, we perform a unit root test on the series, namely the Dickey-Fuller (DF) test proposed by Dickey and Fuller

Table 4. Estimated results of the ARFIMA model.

Estimates	S_t	r_t	RV_t
$\hat{d}$	0.388	0.119	0.341
$\widehat{H}$	0.888	0.619	0.841

Table 5. Summary of the stationarity and long-range dependency result.

	S_t	r_t	RV_t
Stationarity	No	Yes	Yes
Long-range dependency	Yes	No	Yes

(1979). The unit root problem in a time series arises when either the autoregressive or moving average polynomial of an ARMA model has a root on or near the unit circle (Brockwell and Davis, 2002). We provide a brief theoretical explanation of the unit root test in Appendix D since a unit root in either of these polynomials has important implications for modeling. Table 2 shows the t-ratio $\hat{\tau}_\mu$ estimated by the DF test for each time series analyzed by the two models. While the S_t exhibits no obvious stationarity property, the r_t and RV_t series are probably stationary processes.

Second, to examine the LRD of the data, we conducted an R/S analysis using the Hurst-Mandelbrot and the Lo methods introduced in the previous section. The results of this analysis are presented in Table 3. The Hurst exponent H of the r_t by each method is approximately 0.5, implying that the process follows a standard Brownian motion. In contrast, the S_t and RV_t series are likely to have long-range dependency since their Hurst exponents lie within $(1/2, 1)$.

In addition, we estimated the memory parameter d in the ARIMA model, adopting the Sperio estimator proposed by Reisen (1994). Table 4 shows the estimated d and H for each time series, calculated by Equation (1). All the estimated Hurst exponents in Table 4 are slightly higher than those in Table 3.

5.3 Option pricing under fBm

Finally, we examined option pricing under fBm by the method of Norros *et al.* (1999). We confined this analysis to the RV_t time series since the RV_t data exhibit simultaneous stationarity and long-range dependency properties, as shown in Table 5.

Figure 9 shows how the European call option prices differ between $H = 0.50$ and $H = 0.7592$, estimated by the Hurst-Mandelbrot method. For comparison, the price differences between $H = 0.50$ and $H = 0.7301$ estimated by Lo's method are presented in Fig. 10. In both figures, the differences are enhanced around the at-the-money (here denoting the strike price $k = 100$), and increase as the time to maturity decreases. These figures are plotted identically to Fig. 5 in the simulation study, but they exhibit a distinctly different shape. These shape differences might be explained by the different values of the volatility parameters in the simulation and the empirical study ($\sigma = 0.5$ and $\sigma = 0.01489$, respectively). The volatility is well known as the most sensitive input parameter in pricing options.