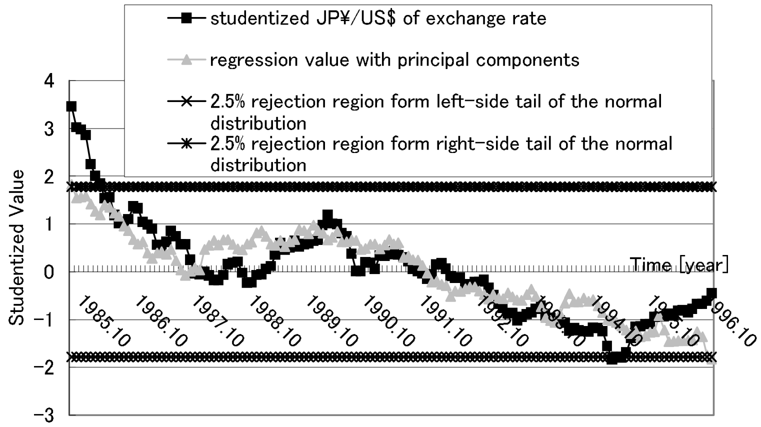

Fig. 2. This figure compares the central rate time series data and the series produced by the multiple regression model. The model uses two principle components of the Japanese Wholesale Price Index, the Japanese Gold and Foreign Exchange Reserves, the Japanese Short-term Interest Rate, the Japanese Current Balance, the U.S. Money Supply, and the U.S. Index of Industrial Production.

$$v_j = \varsigma_{j+1} - \varsigma_j \quad (j = 1985.11.1 - 2001.1.30).$$

We assume the set of these days to be K at the following, when the central rate is sampled.

We analyzed the time series after the Agreement in this paper. We calculated the correlation dimension of each expected attractor on which a dynamical system generates time series of the central rate. We have concluded that it is necessary for two essential variables, principal components z_1, z_2,

$$\hat{\varsigma}_t = 1.58 z_1(t) + 0.37 z_2(t) \tag{1}$$

to regress the time series $\{\varsigma_j\}_{t \in K}$ with the multiple regression analysis where ς_t is a regression value at time t (Matsugi *et al.*, 2001). The determination coefficient on the multiple regression formula was more than 0.80. However, we consider squares of error sum at both sides of the interval remarkable (see Fig. 2); therefore the linear model is not enough to express the central rates. Our purpose of this study is what kind of mathematical models describe the variations of central rates and their difference.

The differenced time series seem to be complex in comparing with time series data of the central rates (Fig. 1). The variations on the difference of them are more furious than the time series data and seem to be random. The differential coefficients do not necessarily converge to bound values. We mathematically show the fact with calculation on their correlation dimension and run tests in next section. Based on the property of the differenced time series, we apply the method proposed to construct a SDE for a description of the JP¥/US$ exchange rate (Takada *et al.*, 2001). We also verify assumptions of the reducing method and evaluate the SDE with the numerical simulation here.

3. Empirical Study for Differenced Time Series and Their Randomness

We mathematically compare the time series data of the central rate with the differenced time series. We show that the later is more complex than the former with calculation of the correlation dimensions and statistics.

3.1 A method of fractal analysis

Using the embedology, we calculated the correlation dimension of the reconstructed attractor on which a dynamical system generates time series of JP¥/US$ exchange rates and their difference. We compose the following delay coordinates that are m dimension vector system:

$$\boldsymbol{x}_j = \begin{pmatrix} x_j & x_{j+\tau} & \cdots & x_{j+(m-1)\tau} \end{pmatrix}, \tag{2}$$

where τ is the sampling time and negative sign of τ, 2τ, ..., $(m-1)\tau$ mean delay time. An orbit with delay coordinates (2) is embedded in m dimensional phase space (see Appendix A). Assuming that a dynamical system on n dimensional compact manifold generates time series, it is possible to think that an attractor on the dynamical system can be reconstructed as the orbit if the time series is observed for a long while. According to Takens' theorem (Takens, 1981), a transform from the time series to the attractor is embedding on the condition of $m \geq 2n + 1$ (Appendix A). The attractor and the manifold are isomorphism.

It is important on the theorem mentioned above that the transformation Φ onto the delay coordinate is embedding. If the transformation Φ is embedding, it is immersion; therefore the fractal dimension of the attractor is preserved (Ikeguchi and Matozaki, 1996). Calculating the fractal dimension of the attractor with delay coordinates; we can analyze the geometrical structure of the manifold M.

The phase space in which we have embedded coordinates with delay time can be delimited with hyperspheres whose radius is assumed to be ε. The number of these hyperspheres is assumed to be $n(\varepsilon)$. The probability that the embedded points are contained in that sphere whose center is x_j is assumed to be P_j. The correlation dimension of the attractor with delay coordinates is defined as follows: