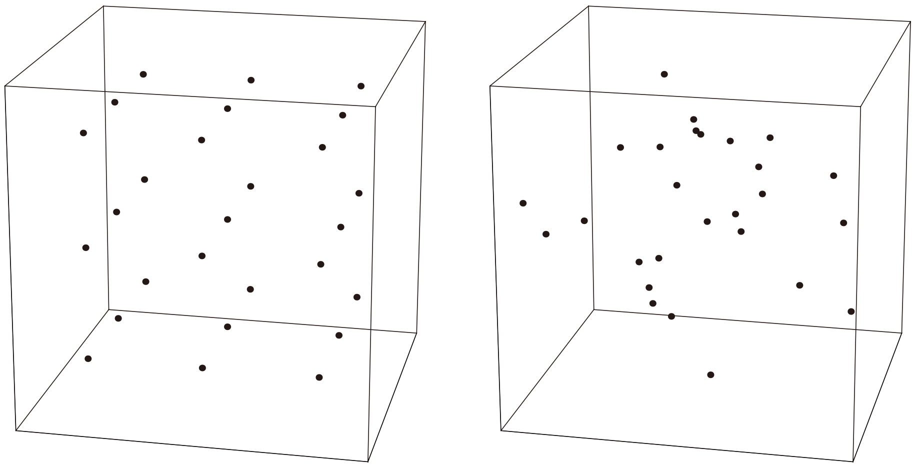

Fig. 1. Point patterns in three-dimensional space: (a) Grid; (b) Random.

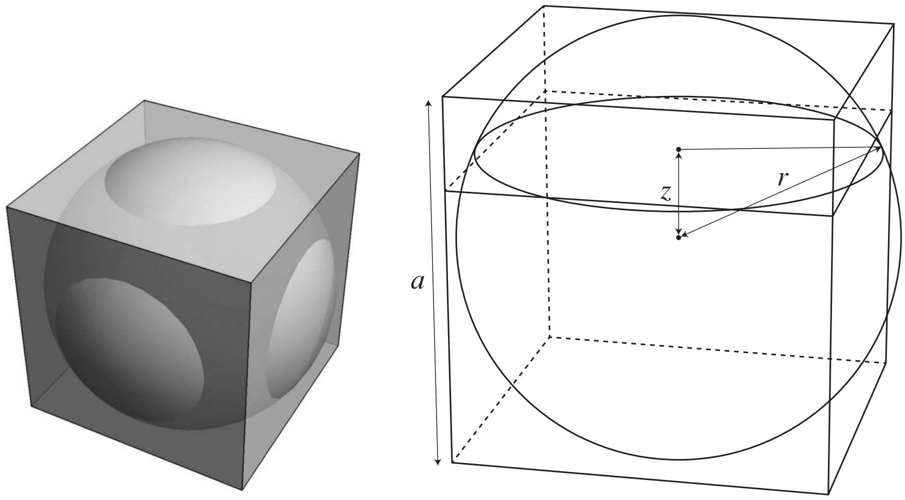

Fig. 2. Region such that $R \le r$.

where V and $V(r)$ are the volume of the study region and the volume of the region such that $R \le r$ in the study region, respectively. Since we assume that the grid pattern continues infinitely, the study region can be confined to the region where a point is the nearest. The study region is then given by the cube centered at the point with side length a. The region such that $R \le r$ is expressed as the ball centered at the point with radius r. $F(r)$ is thus the ratio of the volume of the ball in the cube to the volume of the cube, as shown in Fig. 2. The volume of the cube is $V = a^3$. The volume of the ball in the cube is

$$V(r) = \begin{cases} \frac{4}{3}\pi r^3, & 0 < r \le \frac{a}{2}, \\ 2\int_0^{a/2} S_1(z)\,\mathrm{d}z, & \frac{a}{2} < r \le \frac{a}{\sqrt{2}}, \\ 2\int_0^{a/2} S_2(z)\,\mathrm{d}z, & \frac{a}{\sqrt{2}} < r \le \frac{\sqrt{3}}{2}a, \end{cases} \tag{2}$$

where $S_1(z)$ and $S_2(z)$ are the cross sectional areas of the ball in the cube expressed as

$$S_1(z) = \begin{cases} \pi(r^2 - z^2) + a\sqrt{4(r^2 - z^2) - a^2} \\ \quad -4(r^2 - z^2)\arccos\frac{a}{2\sqrt{r^2 - z^2}}, \\ \qquad 0 < z \le \sqrt{r^2 - \frac{a^2}{4}}, \\ \pi(r^2 - z^2), \\ \qquad \sqrt{r^2 - \frac{a^2}{4}} < z \le \frac{a}{2}, \end{cases} \tag{3}$$

$$S_2(z) = \begin{cases} a^2, \\ \qquad 0 < z \le \sqrt{r^2 - \frac{a^2}{2}}, \\ \pi(r^2 - z^2) + a\sqrt{4(r^2 - z^2) - a^2} \\ \quad -4(r^2 - z^2)\arccos\frac{a}{2\sqrt{r^2 - z^2}}, \\ \qquad \sqrt{r^2 - \frac{a^2}{2}} < z \le \frac{a}{2}, \end{cases} \tag{4}$$