

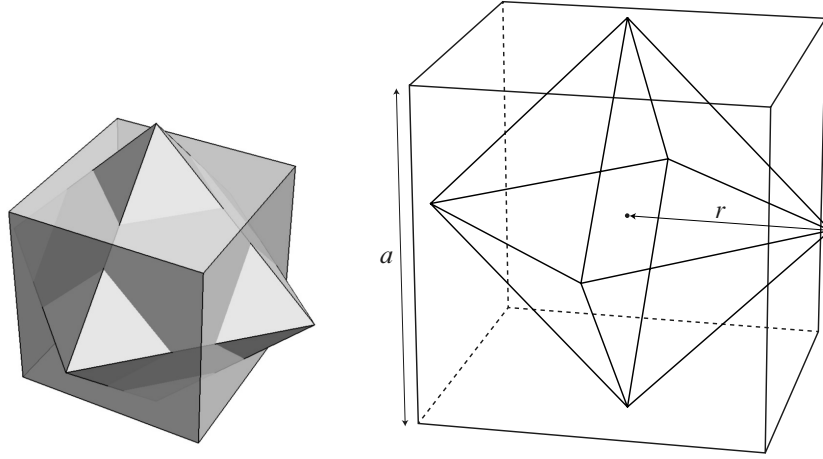
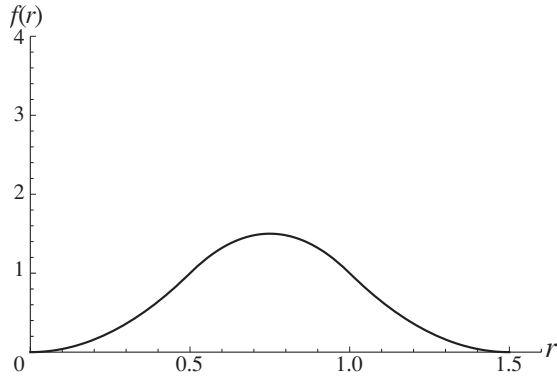
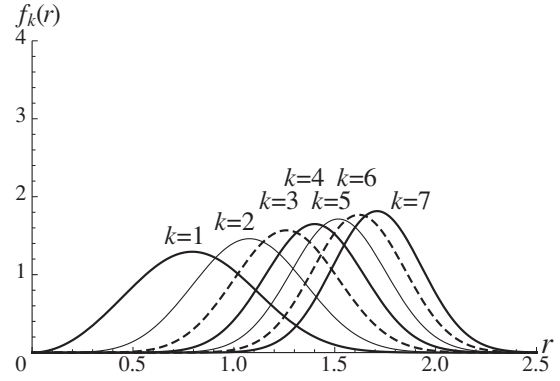
Fig. 5. Region such that $R \leq r$.

Fig. 6. Distribution of the rectilinear nearest neighbor distance for the grid pattern.

Fig. 7. Distribution of the rectilinear k th nearest neighbor distance for the random pattern.

in Fig. 5. The volume of the octahedron in the cube is

$$V(r) = \begin{cases} \frac{4}{3}r^3, & 0 < r \leq \frac{a}{2}, \\ \frac{4}{3}r^3 - 4\left(r - \frac{a}{2}\right)^3, & \frac{a}{2} < r \leq a, \\ a^3 - \frac{4}{3}\left(\frac{3}{2}a - r\right)^3, & a < r \leq \frac{3}{2}a. \end{cases} \quad (13)$$

Substituting V , $V(r)$, and $a = \rho^{1/3}$ into Eq. (1) and differentiating with respect to r yield the distribution of the nearest neighbor distance

$$f(r) = \begin{cases} 4\rho r^2, & 0 < r \leq \frac{1}{2\rho^{1/3}}, \\ -8\rho r^2 + 12\rho^{2/3}r - 3\rho^{1/3}, & \frac{1}{2\rho^{1/3}} < r \leq \frac{1}{\rho^{1/3}}, \\ 4\rho r^2 - 12\rho^{2/3}r + 9\rho^{1/3}, & \frac{1}{\rho^{1/3}} < r \leq \frac{3}{2\rho^{1/3}}. \end{cases}$$

$f(r)$ for the grid pattern is shown in Fig. 6. The average nearest neighbor distance is

$$E(R) = \frac{3}{4\rho^{1/3}} \approx \frac{0.750}{\rho^{1/3}}. \quad (14)$$

The average rectilinear distance for the grid pattern is greater than the average Euclidean distance for the random pattern as well as that for the grid pattern.

3.2 Random pattern

The cumulative distribution function $F(r)$ is the probability that the octahedron with radius r contains at least one point. The volume of the octahedron is $4r^3/3$. Using the Poisson distribution (6), we have

$$\begin{aligned} F(r) &= 1 - P\left(0, \frac{4}{3}r^3\right) \\ &= 1 - \exp\left(-\frac{4}{3}\rho r^3\right). \end{aligned} \quad (15)$$

Differentiating $F(r)$ with respect to r yields the distribution of the nearest neighbor distance

$$f(r) = 4\rho r^2 \exp\left(-\frac{4}{3}\rho r^3\right). \quad (16)$$

The average nearest neighbor distance is

$$E(R) = \left(\frac{3}{4\rho}\right)^{1/3} \Gamma\left(\frac{4}{3}\right) \approx \frac{0.811}{\rho^{1/3}}. \quad (17)$$

The cumulative distribution function of the k th nearest neighbor distance is the probability that the octahedron with