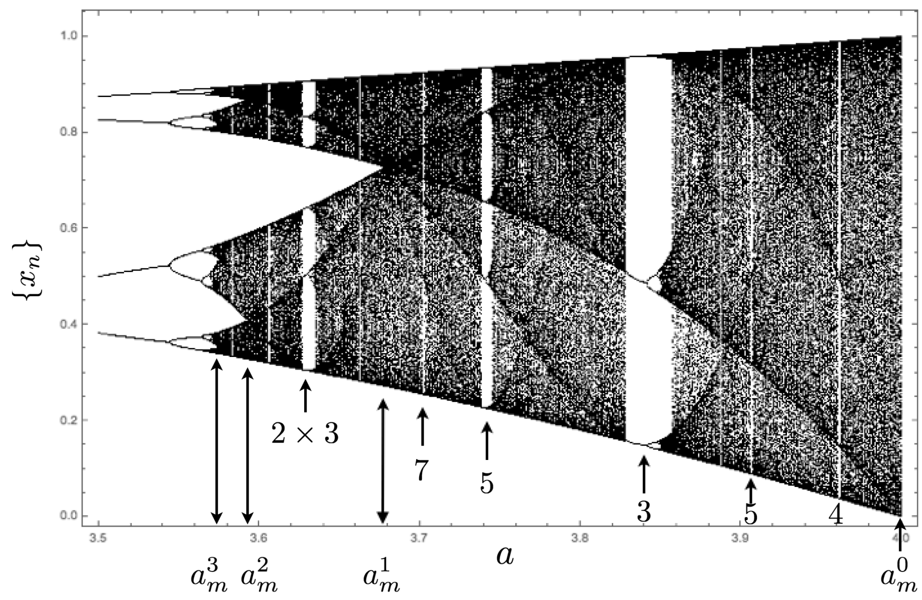

Fig. 1. Bifurcation diagram of the logistic map where $a_m^0 = 4$, $a_m^1 = 3.678573$, $a_m^2 = 3.592572$ and $a_m^3 = 3.574804$. A natural number 3 represents the window of period-3 orbit.

Here, a fraction 1/2 represents an initial point and the arrow ($\rightarrow$) means the orbital order. The code is determined as $RLRR$. The number of symbols in code does not accord with the length of period. Thus, in the kneading theory, a symbol C is added in front of this code and new code $CRLRR$ is defined. Symbol C means the center of interval.

Next, using $CRLRR$, we explain the minimum representation for code. The position represented by C is $x = 1/2$ and the mapping function has the maximum point at this point. This fact implies that next position represented by R (next symbol of C) is the maximum orbital point. In the unimodal map, the maximum orbital point is mapped to the minimum one. Thus, the position represented by L (next symbol of R) is the minimum one. We name the representation $LRRCR$ the minimum representation for code. In the following, we use the minimum representation for codes.

We use two symbols 0 and 1 in consideration of correspondence with the binary representation where a symbol 0 (1) means L (R). Thus, the code for P is 0 and that of Q is 1. Next, we give a meaning of symbol C. Two periodic orbits which appear through the tangent bifurcation constitute (0-1)-pair which means a pair of the stable and the unstable periodic orbits (Hall, 1994). In the two dimensional map, (0-1)-pair means the saddle-node pair. There exist periodic orbit with code where C is replaced by 0 and that with code where C is replaced by 1. The code $LRRCR$ means two periodic orbits 01101 and 01111. The set of these codes is an example of (0-1)-pair.

We comment on the codes for periodic orbits which appear through the period-doubling bifurcation. For example, let us consider the code 0111, which is the code for the daughter periodic orbit which appears through the period-doubling bifurcation of the period-2 orbit. We exchange a symbol 1 at the second-to-last to 0 and have new code 0101 which is the repetition of word 01. Thus, 0101 is meaningless as a code. The code for the periodic orbit which appears through the period-doubling bifurcation does not have a partner code of (0-1)-pair.

The code obtained here is the same as the code determined by the tent map T defined on [0, 1]. In the following, we explain this fact. The logistic map f at $a = 4$ is converted into the tent map T.

$$T \ : \ X_{n+1} = 1 - |2X_n - 1| \tag{3}$$

by the translation formula

$$x_n = \sin^2((\pi/2)X_n). \tag{4}$$

Thus, the logistic map at $a = 4$ and the tent map T are conjugate. Here, we take the orbit of logistic map at $a = 4$. If the orbit enters the interval [0, 1/2], the symbol is defined as 0. If the orbit enters the interval [1/2, 1], the symbol is defined as 1. For the point $x = 1/2$, we can use 0 or 1. This is originated from the fact that there are two representations to an irreducible fraction, for example, $1/2 =_{\bullet} 10^{\infty}$ and $1/2 =_{\bullet} 01^{\infty}$. Here, a symbol $_{\bullet}$ is a decimal point and the right hand sides are the binary representation.

For example, suppose that the code 011 is obtained. In the tent map, there exists the periodic orbit 011. Conversely, the periodic orbit in the tent map exists in the logistic map. From these facts, we can study the periodic orbit with a given code in the tent map. Translating the orbital points in the tent map by Eq. (4), we have the orbital points in the logistic map at $a = 4$. The orbital order of periodic points in the tent map is the same as that in the logistic map.

2.2 Block representation

First, we introduce two block symbols $E(2) = 01$ and $F(2) = 11$ (Yamaguchi and Tanikawa, 2009, 2016). Block symbol $E(2) = 01$ represents the code for the daughter periodic orbit which appears through the period-doubling bifurcation of Q. Block symbol $F(2) = 11$ is introduced for convenience sake and there is no periodic orbit represented by $F(2)$. Suppose that the periodic orbit of period