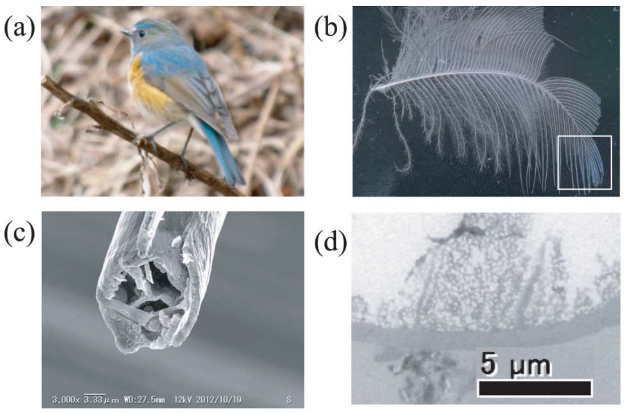

Fig. 1. *Tarsiger cyanurus* and their feather barbs. (a) Adult male of *Tarsiger cyanurus*. (b) A feather of *Tarsiger cyanurus*. (c) SEM image of the cross section of a barb. (d) TEM image of the porous structure of a barb composed of keratin (dark shadow and black) and air pore (light shadow).

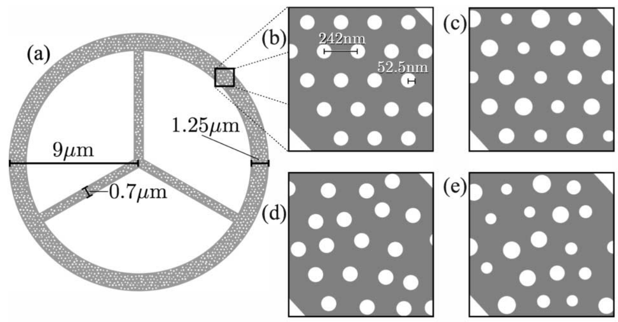

Fig. 2. Numerical models. (a) Cross-sectional structure of a barb composed of three radial partitions and the circular outer wall. (b) Periodic arrangement of the air pores of homogeneous radius. (c) Periodic arrangement of the air pores of random radius. (d) Random arrangement of the air pores of homogeneous radius. (e) Random arrangement of the air pores of random radius.

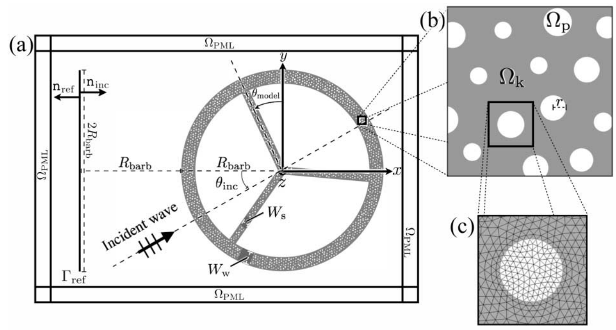

Fig. 3. Numerical computation scheme. (a) Whole scheme for reflectance computing. (b) Porous structure of the outer-wall in close-up. (c) Finite elements around a pore with radius 52.5 nm.

Ueta *et al.*, 1998; Amemiya and Ohtaka, 2003; Urquia, *et al.*, 2020).

In the present study, we investigate optical properties of feather barbs of *Tarsiger cyanurus*, so called red-flanked bluetail, with more precise models taking account of the whole structure of a barb cross-section. The finite element method (FEM) (Ram-Mohan, 2002; Andonegui and Garcia-Adeva, 2013), which is applicable to problems of various fields throughout applied mathematics, engineering and physics, is employed.

The cross-sections of the barbs are observed by SEM and TEM, and the structural dimensions of the barbs are measured. The cross section of a barb is modeled by a two-dimensional (2D) ring with three radial partitions of equiangular interval. Both the ring and the partitions have a large number of pores. In order to reveal the origin of optical properties of the barbs, we propose four types of models of which arrangement and size of pores are different.

2. Modeling of a Feather Barb

2.1 Observation of barb-crosssection

Figures 1(a) and (b) show structural color of *Tarsiger cyanurus* and their barbs, respectively. Only adult males of *Tarsiger cyanurus* have blue colored feathers, whereas young males of less than about 2 years old and females have brown feathers. Barbs only at the tip of feather have blue color as shown in Fig. 1 (b) and a cross-sectional SEM image of the tip part of the blue barb is shown in Fig. 1(c). The cross-section of a barb has a structure which looks like three pointed star composed of a circular outer wall and three radial partitions. Figure 1(d) shows a TEM image of the outer wall in close up. The barb has a porous structure of keratin which includes spheroidal air-pores along the axis direction of the barb.

2.2 Modelling of barb-crosssection

By assuming continuous translational symmetry along the barb axis, we tackle the problem by means of 2D models as in our previous work (Ueta *et al.*, 2014a). In the previous work, the barb was modeled by a infinitely wide dielectrics slab with randomly arranged air rods in which the curvature of the outer wall of a barb was neglected. In the present study, we define a precise 2D model beyond the previous model.

Numerical models of the blue-barb cross-section are shown in Fig. 2. The external radius of the outer-wall is $R_{\text{barb}} = 9.0\ \mu$m, and thickness of the outer-wall and partitions are 1.25 μm and 0.7 μm, respectively. Air-pores are distributed within both the outer-walls and partitions, and we propose four types of model with different arrangements and different sizes of air pores. Figure 2(b) shows homogeneous pores arranged in a triangular lattice. The randomness in the size of air-pores in Figs. 2(c), (e) and that in the arrangement in Figs. 2(d), (e) are determined by TEM observation of a number of barb samples.

The random radius a and the random position of $\mathbf{r}$ are defined by

$$a = \bar{a} + \sqrt{3}\sigma_a\,(2\xi_1 - 1)\,, \tag{1}$$

$$\mathbf{r}_{m,n} = \mathbf{R}_{m,n} + \sqrt{\frac{3}{2}}\sigma_d(2\xi_2 - 1, 2\xi_3 - 1), \tag{2}$$

where $\bar{a}$ and $\mathbf{R}_{m,n}$ are the average radius of an air rod and the lattice vector of the triangular lattice given by

$$\mathbf{R}_{m,n} \equiv d(1, 0)m + d\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)n \tag{3}$$