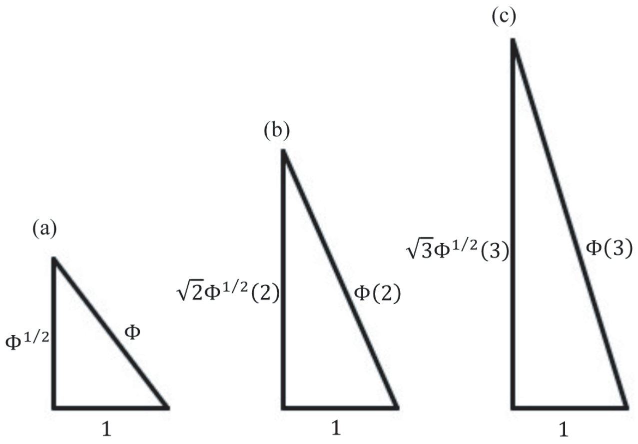

Fig. 2. Examples of the initial three metallic right-triangles: (a) the golden right-triangle (the Kepler triangle); (b) the silver or platinum right-triangle; (c) the bronze right-triangle. The metallic ratios are given in *Definition I* in the text.

the ratio amongst the sides converges such that

$$\begin{aligned}&\left(\sqrt{G_{n-2}(m)},\, \sqrt{m}\Phi^{n/2}(m),\, \sqrt{G_n(m)}\Phi(m)\right)\\ &\to \left(\frac{\Phi^{n/2}(m)}{(m^2+4)^{1/4}}\Phi^{-1}(m),\, \sqrt{m}\Phi^{n/2}(m),\, \frac{\Phi^{n/2}(m)}{(m^2+4)^{1/4}}\Phi(m)\right)\\ &= \left(\Phi^{-1}(m), m^{1/2}(m^2+4)^{1/4},\, \Phi(m)\right)\frac{\Phi^{n/2}(m)}{(m^2+4)^{1/4}}.\end{aligned}$$

Therefore

(the short leg) : (the long leg) : (the hypotenuse)

$$\to \Phi^{-1}(m) : m^{1/2}(m^2+4)^{1/4} : \Phi(m), \tag{8}$$

as n tends to infinity. The ratio satisfies the Pythagorean Theorem.

$$\Phi^2(m) = m\sqrt{m^2+4} + \frac{1}{\Phi^2(m)}, \tag{9}$$

because of (5).

3. Examples

Figure 2 shows the initial three of the metallic right-triangles. Figure 2(a) is the golden right-triangle or the Kepler triangle. Figure 2(b) is *my* silver right-triangle. But this name is used to other triangle, so I may call it the platinum right-triangle. Figure 2(c) is the bronze right-triangle. These metallic ratios are given in *Definition I* above.

How many examples may we find embedded in the nature and artefacts?

4. Conclusion

What we get is the super-set of the Kepler triangle and its kin (Sugimoto, 2020). Those new triangles act as the trivium (three-way crossing) amongst the metallic means, the generalised Fibonacci sequences and the Pythagorean theorem.

Reference

Takeshi Sugimoto (2020) The Kepler Triangle and Its Kin, *FORMA*, **35**(1), 1–2.