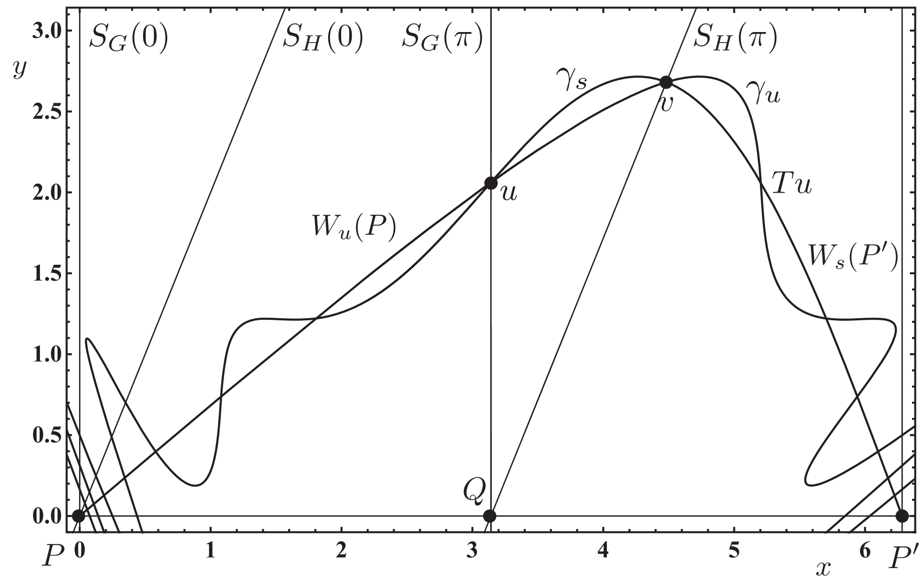

Fig. 1. The unstable manifold $W_u(P)$ of the saddle fixed point $P = (0, 0)$ and the stable manifold $W_s(P')$ of the saddle fixed point $P' = (2\pi, 0)$ are displayed at $a = 1.5$. Here, $S_G(0)$, $S_H(0)$, $S_G(\pi)$ and $S_H(\pi)$ represent the symmetry axes. Two intersection points u and v of $W_u(P)$ and $W_s(P')$ are the primary homoclinic points. Two arcs $\gamma_u = [v, Tu]_{W_u(P)}$ and $\gamma_s = [u, v]_{W_s(P')}$ are also displayed.

rotation number p/q, a pair of saddle and elliptic orbits exist in the standard map. Since these orbits satisfy the order preservation, these are called the monotone periodic orbits or the Birkhoff periodic orbits (Yamaguchi and Tanikawa, 2007). Here, the order preservation means that for any pair of points in an orbit, the order of the x-coordinates of their iterates does not alter on the universal cover. We use the notation p/q-BE for the order preserving symmetric elliptic periodic orbit of rotation number p/q.

Through the linear stability analysis, the eigenvalues $\lambda_\pm$ of the linearized matrix M are determined. If these are complex conjugate, i.e., $\lambda_\pm = \alpha \pm i\beta$ ($|\lambda_\pm| = 1$), we call the periodic orbit the elliptic periodic orbit. If these satisfy the conditions $\lambda_- < -1 < \lambda_+ < 0$, the periodic orbit is a saddle periodic orbit with reflection. If these satisfy the conditions $0 < \lambda_- < 1 < \lambda_+$, the orbit is a saddle periodic orbit.

If a map is represented by the product of involutions, we say that the map has the reversibility in the meaning of Birkhoff (1927). The standard map T is reversible. Using the involutions G and H, we represent T as $T = H \circ G$ where $H \circ H = G \circ G = \mathrm{id}$ and $\det\nabla H = \det\nabla G = -1$. Here, we give the actions of G and H.

$$G\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} y + f(x) \\ -x \ (\mathrm{Mod}\ 2\pi) \end{pmatrix}, \tag{2}$$

$$H\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} y \\ y - x \ (\mathrm{Mod}\ 2\pi) \end{pmatrix}. \tag{3}$$

The set of fixed points of involution is called the symmetry axis. Let S_G be the symmetry axis of G and S_H be the symmetry axis of H. We give the representations for them on cylinder (see Fig. 1).

$$S_G(0) = \{(x, y) \ : \ x = 0\}, \quad S_G(\pi) = \{(x, y) \ : \ x = \pi\}, \tag{4}$$

$$S_H(0) = \{(x, y) \ : \ y = 2x\},$$
$$S_H(\pi) = \{(x, y) \ : \ y = 2(x - \pi)\}. \tag{5}$$

Definition 2.1 An orbit is symmetric if and only if it has a point on the symmetry axis.

Proposition 2.2 A periodic orbit is symmetric if and only if it has two of the points on the symmetry axis or axes.

By proposition 2.2, a $1/(2k+1)$-BE has one point z_0 on $S_H(0)$ and the other point z_k on $S_G(\pi)$.

2.2 Involutions for T^{2k+1}

Using the two involutions G and $T^{2k}H$ ($k \geq 0$), we express T^{2k+1} as

$$T^{2k+1} = T^{2k}H \circ G. \tag{6}$$

Let $S_{T^{2k}H}$ be the symmetry axis of $T^{2k}H$. Thus, T^{2k+1} has two symmetry axes $S_{T^{2k}H}$ and $S_G(\pi)$. Greene (1979) named $S_G(\pi)$ the dominant axis (DA). In this paper, we call $S_{T^{2k}H}$ the subdominant axis (SD). The initial point z_0 is on $S_H(0)$ and z_k on $S_G(\pi)$. We remark that z_k is the intersection point of DA and SD.

The representation of SD is $T^k S_H$. In fact,

$$T^{2k}H(T^k S_H) = T^{2k}T^{-k}HS_H = T^k S_H.$$

Let us operate $T^{-(2k+1)}$ to $S_{T^{2k}H}$.

$$\begin{aligned} T^{-(2k+1)}S_{T^{2k}H} &= G(HT^{-2k}S_{T^{2k}H}) \\ &= G(T^{2k}HS_{T^{2k}H}) = GS_{T^{2k}H}. \end{aligned} \tag{7}$$

The operation G to $S_{T^{2k}H}$ is equivalent with that of $T^{-(2k+1)}$ to $S_{T^{2k}H}$.

We use the following representation of involution G whose symmetry axis is $S_G(\pi)$.

$$G\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} y + f(x) \\ 2\pi - x \end{pmatrix}. \tag{8}$$