on the left belongs to the right(left) branch of the quadruplet, the triplet, and the doublet, respectively. Note that the odd number belongs to the right branches only and that this sequence contains many right branches. The number of the dichotomous branching node of the quadruplet, triplet, and doublet nodes are equal to 16, 28, and 5, respectively. The estimation from the transition matrix 0.4 : 0.5 : 0.1 was a good approximation of the ratio 16 : 28 : 5 = 0.326530612 : 0.571428571 : 0.102040816.

We clarified how many odd numbers appear from an initial positive integer to 1. In a long Collatz sequence, the ratio of visiting $Q_r(n) = 24n - 20$ and $Q_l = 72n - 56$ is 24 : 72 = 1/4 : 3/4. In a similar manner, the ratio of visiting $T_r(n) = 12n - 2$ and $T_l = 72n - 8$ is 12 : 72 = 1/7 : 6/7. Taking into account the visiting frequency of the quadruplet and triplet, we estimated the ratio of the right branch to the whole of the quadruplet and triplet as $4/10 \times 1/4 + 5/10 \times 1/7 = 6/35$. Next, we considered the relative ratio of the initial numbers on the Sharkovskii branch $6(2n-1) \times 2^p$ $(p = 0, 1, \cdots)$. The ratio of the odd number $3(2n-1) = 6n - 3$ of the doublet to the initial numbers between 1 and N was given by $(N/6)/N = 1/6$ for $n \ll N$. The largest number on the Sharkovskii branch was estimated as $6(2n-1) \times 2^{p_{max}} = N$, leading to $p_{max} \propto \log N$ for $n \ll N$, which is much less than $N/6$. Ignoring the case of the initial number located at the Sharkovskii branch aside from the single odd number, the ratio of the initial number coinciding with the odd number $6n - 3$ was equal to 1/6, and 5/6 otherwise. In the latter case, the ratio of choosing the right branches including odd numbers was 6/35, as shown above. Thus, the ratio of odd numbers to the stopping time from the initial number to 1 was equal to $5/6 \times 6/35 + 1/6 = 13/42 \approx 0.31$. Starting from power-of-two numbers $n_p = 2^p$, we had $n_p(1/2)^T = 1$ for the stopping time T, i. e., $T = \log n_p / \log 2$. Starting from an arbitrary initial number, we had $n_p \lambda^T = 1$ for the roughly estimated stopping time T, where λ was estimated as $\lambda = \frac{42-13}{42}(-\log 2) + \frac{13}{42}\log 3 \approx -0.139$. The former and latter corresponded to the contribution from the even and the odd number, respectively. This averaged behavior was underestimated because we ignored the additional part

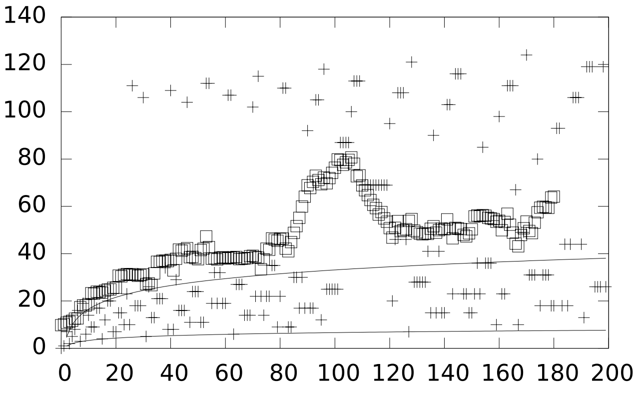

Fig. 1. The stopping time is plotted against the initial number with symbol + and that of the moving average over the succeeding 20 plots with symbol □. The shortest stopping time for power-of-two numbers (dashed line) and the rough averaged stopping time (solid line) are shown.

1 of $3x + 1$ of the iteration of an odd number x. The stopping time (symbol +), that of the moving average over the preceding 20 points (symbol □), the shortest stopping time for power-of-two numbers (dashed line), and the rough averaged stopping time (solid line) are shown in Fig. 1.

The quadruplets, triplets, and doublets seem randomly arranged on the Collatz tree. However, for a specific dichotomous branching node $54M - 38$ (positive integer 16 mod 54), there always exists a periodic backward sequence in order of the quadruplet, triplet, and doublet tracking the left-most branches Q_l, T_l, and D_l. The periodic sequence located at the nearest neighbor of the final loop $4 \to 2 \to 1 \to 4$ is given by $\cdots \to 262144 = D(14564) \to 65536 = T(3641) \to 16384 = Q(911) \to 4096 = D(228) \to$

Table 1. Collatz sequence from 27 to 4.

27	DR	445	QR	1154	QR
82	D(5)	1336	Q(75)	577	QR
41	TR	668	DL	1732	Q(97)
124	T(7)	334	D(19)	866	QR
62	QR	167	TR	433	QR
31	QR	502	T(28)	1300	Q(73)
94	Q(6)	251	TR	650	QR
47	TR	754	T(42)	325	QR
142	T(8)	377	TR	976	Q(55)
71	TR	1132	T(63)	488	DL
214	T(12)	566	QR	244	D(14)
107	TR	283	QR	122	QR
322	T(18)	850	Q(48)	61	QR
161	TR	425	TR	184	Q(11)
484	T(27)	1276	T(71)	92	DL
242	QR	638	QR	46	D(3)
121	QR	319	QR	23	TR
364	Q(21)	958	Q(54)	70	T(4)
182	QR	479	TR	35	TR
91	QR	1438	T(80)	106	T(6)
274	Q(16)	719	TR	53	TR
137	TR	2158	T(120)	160	T(9)
412	T(23)	1079	TR	80	QL
206	QR	3238	T(180)	40	Q(3)
103	QR	1619	TR	20	DL
310	Q(18)	4858	T(270)	10	D(1)
155	TR	2429	TR	5	TR
466	T(26)	7288	T(405)	16	T(1)
233	TR	3644	QL	8	QL
700	T(39)	1822	Q(102)	4	Q(1)
350	QR	911	TR		
175	QR	2734	T(152)		
526	Q(30)	1367	TR		
263	TR	4102	T(228)		
790	T(44)	2051	TR		
395	TR	6154	T(342)		
1186	T(66)	3077	TR		
593	TR	9232	T(513)		
1780	T(99)	4616	QL		
890	QR	2308	Q(129)		