Review — *Forma*, **14**, 309–313, 1999

Virtual Construction of Human Lung

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(Received November 19, 1999; Accepted December 27, 1999)

Keywords: Airway, Branching System, Lung, Space Filling, Acinus

Abstract. An algorithm to produce a branching system is proposed in order to simulate the human lung. This system has constraints that the end points of the airway should fill the 3D space uniformly, that the energy to supply air to the end points should be small and that the quantity of material to construct this system should be small. Several rules are set up to satisfy these constraints and the system is constructed virtually by the use of microcomputer. The optimum one among resulting airways proves to be quite similar to the real lung.

1. Introduction

Problems how to fill 3D space are not confined to those in crystallography, particle dynamics, or cell dynamics. Fine structures in organs in living systems also provide interesting problems as space filling ones. It would be quite natural to assume that these structures are constructed almost deterministically so that functions needed in organs are created effectively and that biological systems possess an abilities to produce such structures via morphogenesis.

Since general approach to this problem is not expected at the present stage, it would be a meaningful way to make case studies for various organs or parts of biological systems, in which their forms and functions are described mathemathically and certain relations are discussed between them. The lung treated here is an organ to supply air to all elements filling the space in itself through a branching ductal system, to make gas exchange there and to transport blood both through arteries and veins. It is an organ with the largest volume in human body, but 90% of its volume is occupied by the air.

WEIBEL (1963) and HORSFIELD *et al.* (1971) proposed structural airway models including airway dimensions and connectivity, which do not include information of the spatial structure and are looked upon as 1D models. 2D geometric airway models were proposed by NELSON and MANCHESTER (1988) and by MARTINEN *et al.* (1994). Recently, GLENNY and ROBERTSON (1995) and PARKER *et al.* (1997) proposed 3D models of the pulmonary arterial system for simulating blood flow distributions in the lung. They proved to be useful for prediction of lung functions, while branching structures in these models are symmetric unlike in the real lung. KITAOKA *et al.* (1999) proposed a new algorithm to create the branching system of the human lung airway. The purpose of this paper is to explain the basic ideas and result of this work.

2. Algorithm for Generating Airway

2.1. Geometrical principles

At any branching generation the airway has two kinds of geometrical characteristics. One is concerned to geometrical properties of itself, i.e. its location, length, thickness and direction. Another is concerned to those of a region in the lung governed by a branch, i.e. its volume and contour shape. Now, it is required that the air should be supplied equally to any region, hence the branching should me made so that terminal branches fill the space uniformly and the amount of fluid delivery through a branch should be proportional to the volume of the region governed by the branch. These requirements lead to rules to determine branching angles, diameters and lengths of branches, directions of planes on which lie the mother and daughters.

2.2. Dynamical principles

The dynamical characteristics of the airway are the flow rate and the momentum conservations during branching. In addition the viscous dissipation of energy at the branching is an important factor. Since the air compressibility is neglected in real situations, the volume flow rate should be conserved at the branch point. The momentum of air is also conserved so that the duct does not receive excess stress. This requirement means that the mother and daughter branches lie on the same plane.

Some past studies treated the energy dissipation in ductal systems (MURRAY, 1926; GROAT, 1948; SUWA *et al.*, 1963; KAMIYA *et al.*, 1974). They derived the following formula for diameters of the parent d_0 and daughters d_1, d_2 and also that for branching angles θ_1, θ_2 (see Fig. 1):

$$d_0^n = d_1^n + d_2^n, \tag{1}$$

$$\cos\theta_1 = \frac{1 + r^{4/n} - (1-r)^{4/n}}{2r^{2/n}}, \quad \cos\theta_2 = \frac{1 + (1-r)^{4/n} - r^{4/n}}{2(1-r)^{2/n}}, \tag{2}$$

where n is a constant called a diameter exponent, and r is a flow dividing ratio ($0 \le r \le 0.5$). The minimum energy loss requires $n = 3$ (MURRAY, 1926; KAMIYA *et al.*, 1974), while morphometric data vary from 2.6 to 2.8.

2.3. Some anatomical conditions

The mode of branching is assumed to be dichotomous. The outer shape of the lung should be given to define the 3D space in which an airway is constructed. The ratio of length to diameter of a branch is assumed to be 3 according to morphological data. The initial

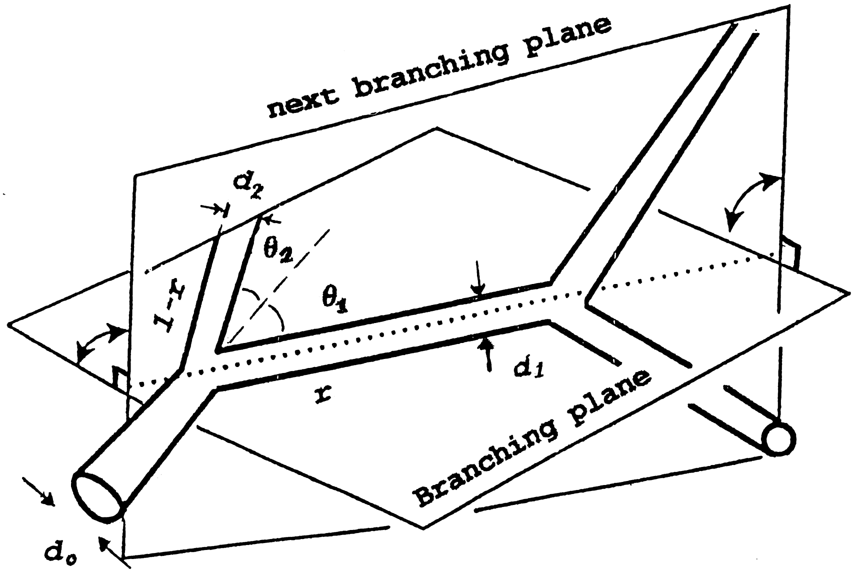

Fig. 1. Geometrical definition sketch.

condition for branching, i.e. the locations and the diameters of the main trunk and a few generations from it are given based on anatomical data.

The branches with flow rates less than a threshold is assumed to be a terminal. Beyond it the gas diffusion dominates the process of air transport.

2.4. *The algorithm with morphogenetic rules*

Based on the considerations mentioned above we propose the following 9 rules for constructing branching system:

Rule 1: Branching is dichotomous.
Rule 2: The parent and its two daughters lie in the same plane (branching plane).
Rule 3: The flow rate is conserved after branching.
Rule 4: The region supplied by a parent is divided into two daughter regions by a plane (space dividing plane), which is perpendicular to the branching plane.
Rule 5: The flow dividing ratio *r* is equal to the volume dividing ratio of daughter regions.
Rule 6: Diameters and branching angles of the two daughters are determined by Eqs. (1) and (2).
Rule 7: The length of each daughter branch is three times its diameter.
Rule 8: The branching plane is perpendicular to the preceding branching plane.
Rule 9: The branching process terminates whenever the flow rate becomes less than a specified threshold or the branch extends out of its own region.

Some of these rules are modified if resulting airway is too unfavorable (for precise refer KITAOKA *et al.* (1999)).

Resulting airway tree is evaluated by the use of the following five quantities. They are (i) failure flow rate, i.e. sum of flow rates in branches which terminated because of extending out of their own region, (ii) variation of the density of terminals defined by the ratio of the mean to the standard deviation, (iii) variation of acinar volumes, (iv) volume ratio of the total airway volume to the total lung volume, and (v) energy loss per terminal. Evaluation of airway tree is made by a simple sum of these five quantities.

3. Results of Simulation

Figure 2 shows an anterior and a right lateral view of this 3D model with the mostly optimum conditions. The rules have several parameters for modification, as noted above, and there is a room to look for the best result. It should be noted that this model resembles the real airway remarkably. It has 27,306 terminal branches, and the mean generation numbers down to terminals is 17.6 ± 3.4 varying from the minimum 8 to the maximum 32.

We also tried cases with other geometric shapes for outer boundary, such as a sphere, a cylinder and a cube. The results were similar to those obtained with the lung shape. This suggests that the algorithm is quite robust and capable of generating efficient tree structures in many different organ shapes.

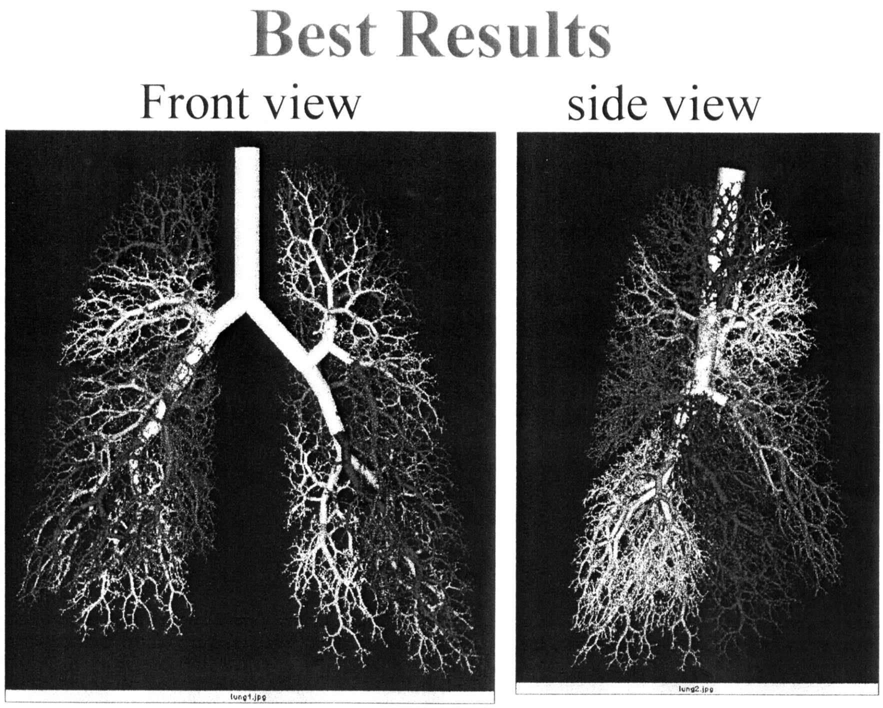

Fig. 2. Resulting 3D airway tree with the best choices of parameters. Left: anterior view, Right: right lateral view. The parameters to produce this model are slightly different from those used in the previous study (KITAOKA *et al.*, 1999).

4. Conclusions and Discussion

The success of the present lung model suggests that the algorithm proposed here is quite reasonable. It should be noted also that resulting trees exhibit self-similar properties, which is caused by the independence of the algorithm on the size of region except for branches close to outer boundaries.

Some discussions are made on the present algorithm. First, the deterministic algorithm can not produce an inter-individual variations or an inter-species differences. However, introduction of fluctuations into parameters such as length-diameter ratio will account for these variations.

When an outer boundary of the lung was given very faithfully to the real one, the model provided a large failure flow rate (*Qf*). It is caused by the existence of concave surfaces due to the existences of the heart and the aorta, where branching stopped too early (Rule 9). In the present algorithm constraction of branching system is associated with a space division. In other words, a hierarchical space division, associated with material transport, can be produced by constructing a branching system. This nature is considered to be common in all biological branching systems.

The present authors would like to express their cordial thanks to Prof. B. Suki of Boston University and to Prof. H. Itoh for their helps during this work. Mr. M. Kashiwabara helped us in preparing computed images, for which the present authors are thankful.

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