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Pattern Formation on the Vertically Vibrated Granular Layer

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Abstract. An experimental study of the pattern formation on the vertically vibrated thin granular layer is made under atmospheric pressure. The granules are dry glass spheres. The forcing frequencies, their relative amplitudes and phases are controlled. High-speed video imaging is used to study the phase diagram of the patterns and its formation processes. In addition to squares, stripes and hexagons, a pattern similar to quasi-crystal patterns is found.

1. Introduction

Pattern formation of thin granular layer subjected to vertical vibration has been extensively studied since the pioneering work by FARADAY in 1831. In spite of industrial importance, the basic understanding of the physical mechanisms underlying the collective behaviours of this material is still lacking (JAEGER and NAGEL, 1992, 1996; LUBKIN, 1995; JAEGER *et al.*, 1996; etc.). This material shows complicated transition between solid-like and fluid-like states. It is a strongly dissipative system, which is characterized by collision of particles but has some long range order both in spatial and temporal behaviour. Recently many experimental as well as numerical studies have been focused on the vertically oscillated thin granular layer, and important development in understanding a variety of the standing wave patterns has been made (MELO *et al.*, 1994, 1995; UMBANHOWER *et al.*, 1996; METCALF *et al.*, 1997; SANO and SUZUKI, 1998; BIZON *et al.*, 1998; SANO, 1999). All these results reveal a sequence of patterns as the acceleration amplitude is changed, but have different aspects because of a variety of factors that determine the behaviour. As has been noted, the present system has resemblance to parametrically forced water waves (TUFILLARO *et al.*, 1989; MILES and HENDERSON, 1990; MILNER, 1991; CHRISTIANSEN *et al.*, 1992; MULLER, 1993; EDWARDS and FAUVE, 1994). But there still remains a wide gap between the two systems. In this paper, we show some new patterns obtained by two-frequency forcing, and the processes of the formation and transitions among these patterns are discussed.

2. Experimental Apparatus

An experiment of the pattern formation on thin granular materials in a vessel(vessel 1: circular cylinder of a diameter 106 mm and height 31 mm, and vessel 2: rectangular shape of horizontal cross section 8 mm × 91 mm and height 91 mm) which is vibrated vertically under atmospheric pressure, was performed. Glass beads of a diameter $d = 0.13 \pm 0.05$ mm was placed in a container, which is mounted with its principal axis parallel to gravity on an electro-mechanical vibration generator. The block diagram of our experimental apparatus is shown in Fig. 1. The vertical oscillation of the container was given by

$$z = \tilde{a}[(1 - \varepsilon)\sin(2\pi ft) + \varepsilon\sin(2\pi f't + \delta)], \quad (1)$$

where frequencies f, f', their relative magnitude and phase (specified by $\tilde{a}$, ε and δ) were given by function synthesizer. In this paper we report the case $f' = f/2$ and $\delta = 0$. The frequency was prescribed with an accuracy up to 6 decimal places in the range between 10 to 100 Hz. The amplitude of oscillation was measured non-intrusively and instantaneously by means of the magnitude of eddy current induced on the metal plate attached to the vessel, whose accuracy is 0.01 mm. No deformation of the bottom was recognized. The observation was made by means of a high-speed video camera to which a close-up lens is mounted. The images were stored at a rate between 186 and 2048 frame/s. They are later reproduced, and the spatial measurement is made with an accuracy of 0.1 mm.

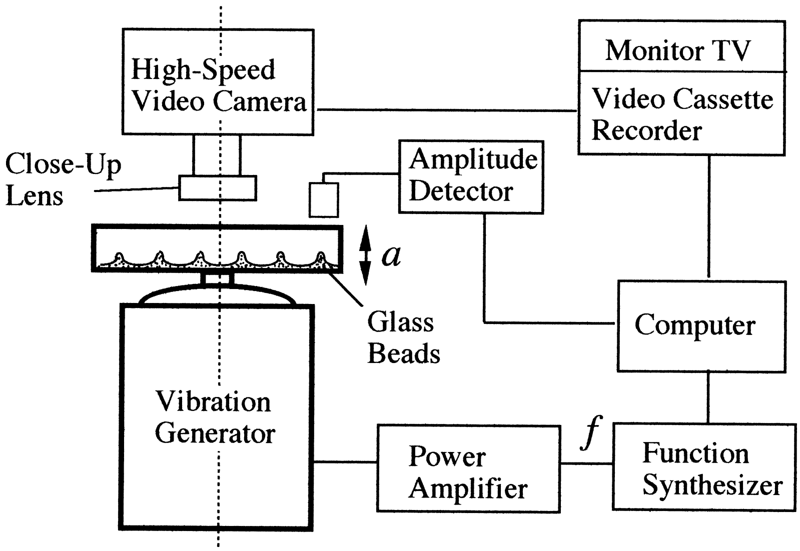

Fig. 1. Block diagram of the experimental apparatus.

3. Experimental Results

3.1. Convection to wave formation

Firstly we filled the glass beads in vessel 1 to the depth H = 15 mm (i.e. about 100 layers of beads), and observed the motion of the material in the vertical cross section under single-frequency ($\varepsilon = 0$). When the dimensionless acceleration amplitude $\Gamma = 4\pi^2 f^2 a/g$ (g is the acceleration of gravity, and a is the maximum amplitude of Eq. (1)) is larger than 1 but is less than about 2, steady convection with several heaps is observed. The position of the heaps, at which glass beads ascend, is not necessarily spaced regularly, nor is influenced by the presence of the side walls. With the increase of Γ up to 2.5, we observe small undulations of the upper free surface, which slide down along the slope of the heap. They merge at the valley region, which are then taken into both sides of the bulk regions along the bottom boundary. Similar patterns are observed for layers of H = 9 mm and 3 mm. When the layer depth H decreases down to 1 mm (i.e. less than about 7 layers of beads), the almost flat lower boundary of the granular layer detaches the bottom boundary at the same frequency as the oscillation of the container. The large scale slope of the upper surface is no longer recognized, and standing-wave-like small scale undulations are observed. The last stage is similar to former experimental and numerical findings (DOUADYS *et al.*, 1989; AOKI and AKIYAMA, 1996; CLEMENT *et al.*, 1996; LAN and ROSATO, 1997). We show the time dependence of the contour of the surfaces in Fig. 2. The frame-to-frame examination reveals that the ridges and valleys alternate with the frequency of oscillation of the vessel, so that the same pattern repeats with a period twice of the period of oscillation (i.e. parametric excitation).

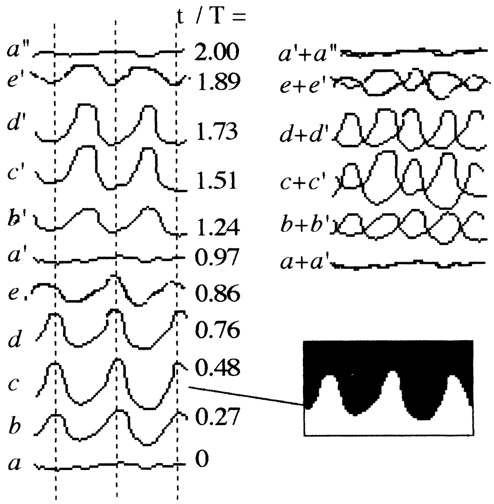

Fig. 2. Contours of the surface at different time t/T (shown in the left column of the figure), where T is the period of vibration of the container. The upper right figures are the superpositions of contours, which are about one period T separated to each other. The lower right figure is an example of original video image, from which contour c is derived (f = 35 Hz, a = 1.10 mm).

3.2. Planar forms

Secondly, we filled the glass beads in a cylindrical vessel (vessel 2) to the depth of 1.0 mm (about 7 layers) and observed the planar form of the granular layer under vertical vibrations. Examples of the pattern are shown in Fig. 3. We also show the pattern diagram in f–a plane in the case of $\varepsilon = 0$ in Fig. 4. The typical sequence of pattern changes, which we found under single frequency forcing of amplitude a = 1.4 mm, is as follows: initially flat surface disrupts into spot-like pattern (Fig. 3(a)) for frequencies $f \approx 20$ Hz, which diffuses into sheet around spots for $f \approx 25$ Hz, develops to square pattern (Fig. 3(b)) for f = 25 ~ 35 Hz (sometimes pentagons (Fig. 3(c)) or hexagons/triangles (Fig. 3(d)) are observed), and stripe patterns (Fig. 3(e)) with some dislocations for f = 40 ~ 45 Hz (which sometimes evolves into spirals (Fig. 3(f))). Similarly to the two-dimensional waves observed in thin vertical vessel, the ridges and the valley regions in each cellular sÙ,ucture alternate, so that the same pattern repeats with a period twice of the period of oscillation. When we increase the frequency, patterns become irregular for $f \approx 50$ Hz, and cellular patterns of approximately hexagonal shape reappear (Fig. 3(g)) which repeats the pattern with period 4 times of the external vibration T (=1/f) for $f \approx 60$ Hz. Between the last two patterns and the stripe patterns, we observe almost flat surface, although the constituent particles are not at rest. These findings agree with the former works (MELO *et al.*, 1994, 1995; UMBANHOWER *et al.*, 1996; METCALF *et al.*, 1997; SANO and SUZUKI, 1998; BIZON *et al.*, 1998; SANO, 1999), except we have wider region of polygonal cells in the phase diagram. Further increase of acceleration($\Gamma \gtrsim 20$) leads to patterns which are analogous to quasi-crystal patterns (PENROSE, 1974; MACKAY, 1981), and this tendency is enhanced when two frequency oscillation is applied (Fig. 3(h)).

In Fig. 3(b) side wall of the container is also shown. Our container is a circular cylinder, which has no preferred direction of the alignment of the square cells. The same figure also shows that the effect of the boundary is confined to a very narrow region.

3.3. Size of the cell

The size of the cell depends, among all, on f and a. We plot wavelength of the pattern λ^* normalized by particle diameter d (i.e $\lambda^* = \lambda/d$) against normalized frequency f^*(= $f\sqrt{g/d}$) and normalized amplitude a^*(= a/d) in Figs. 5(a) and 5(b), respectively. Data estimated from MELO *et al*. (1994) and METCALF *et al*. (1997) are also plotted. Scaling in this way collapses all these data. From these figures, we anticipate a relation $\lambda \propto f^{\alpha}$, where $\alpha \approx -2$ for lower frequency, while $\alpha \approx -2/3$ for higher frequency, which is analogous to the dispersion relation for capillary/gravity waves. On the other hand, λ is nearly proportional to a, but has apparently a critical wavelength $\lambda^*_c \approx 15$ at a critical amplitude $a^*_c \approx 1$.

3.4. Symmetries

Translational and rotational symmetries of the square cells (as shown in Fig. 3(b) or Figs. 6 (a) and (a′)), and those of triangular/hexagonal cells(as shown in Fig. 3(d) or Figs. 6(c) and (c′)) are apparent. Less obvious is the rotational symmetry of the patterns like Fig. 3(h). In order to clarify the latter, we rotate the pattern by an angle θ around some fixed point, and compare it with the original one. Figure 7 shows the dependence of the overlapping area on θ in the case of image data Fig. 3(h), which suggests the 5- or 10-fold symmetry. We also observed quasi-patterns with 7- to 11-fold symmetry.

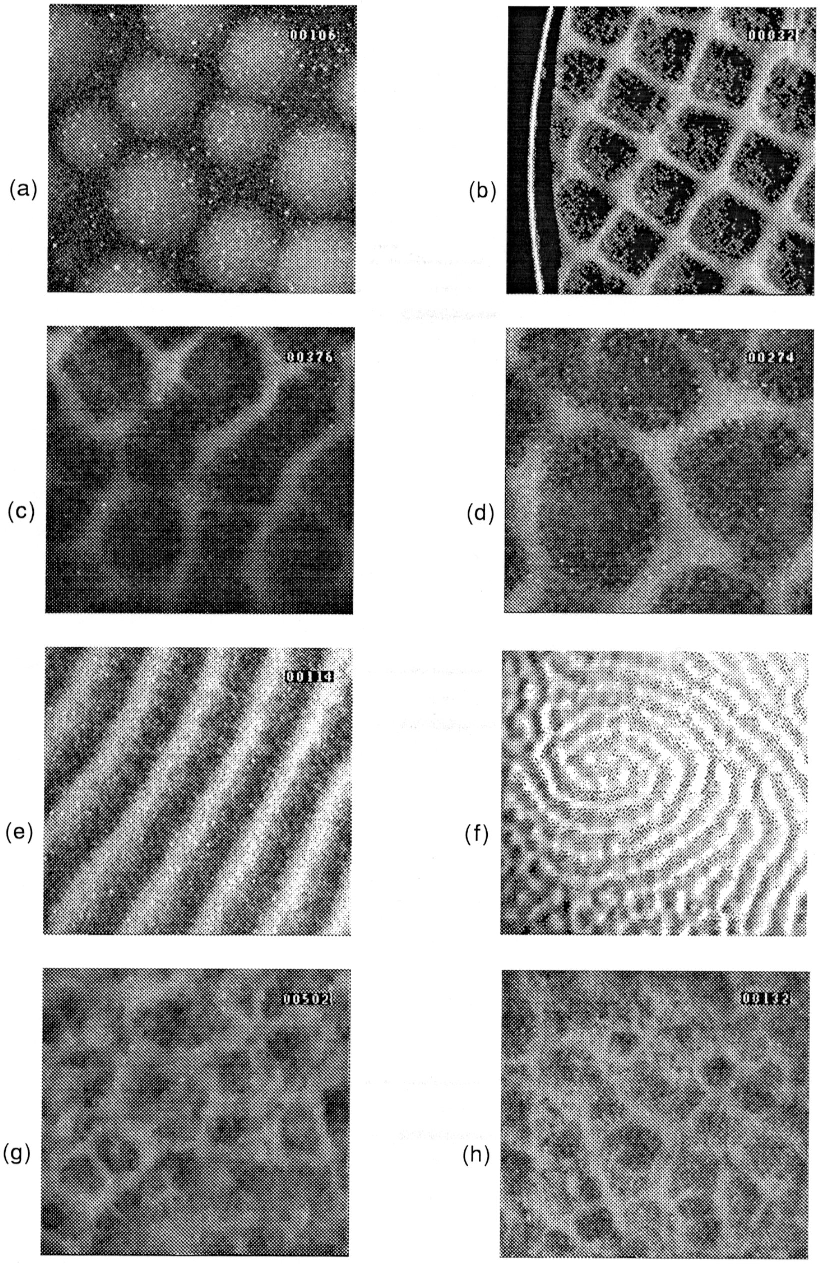

Fig. 3. Typical patterns observed on vertically vibrated thin granular layer; (a) spots (f = 20 Hz, a = 1.08 mm, ε = 0.3), (b) squares(f = 27 Hz, a = 1.59 mm, ε = 0.1), (c) pentagons (f = 30 Hz, a = 1.81 mm, ε = 0.1), (d) hexagons (f = 30 Hz, a = 2.18 mm, ε = 0.1), (e) stripes (f = 45 Hz, a = 1.00 mm, ε = 0.3), (f) spiral(f = 45 Hz, a = 1.20 mm, ε = 0), (g) polygons with period $4T$ (f = 65 Hz, a = 1.26 mm, ε = 0.1), and (h) quasi-patterns (f = 60 Hz, a = 1.61 mm, ε = 0), where the vertical oscillation of the vessel is given by $z = \tilde{a}[(1-\varepsilon)\sin(2\pi ft) + \varepsilon \sin(\pi ft)]$, and a is the maximum amplitude.

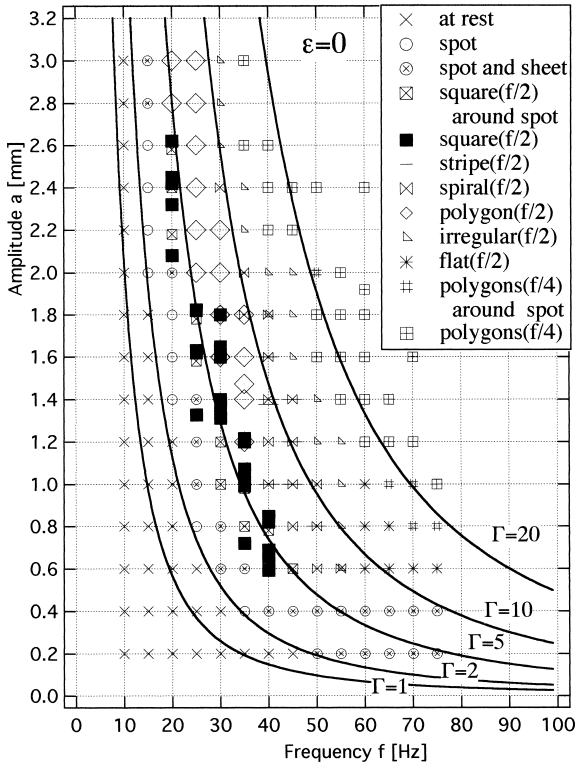

Fig. 4. Phase diagram of the patterns obtained in the experiment.

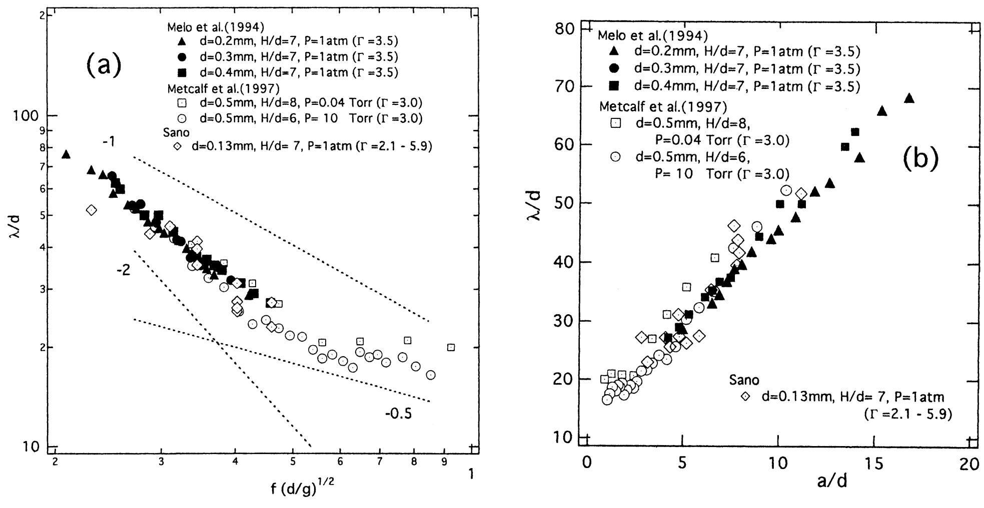

Fig. 5. Dependence of the (a) frequency and (b) amplitude on the cell size.

3.5. Pattern formation process

We show the typical pattern formation process in Fig. 6. Figures 6(a) and (a′) are the square cells with $f = 35$ Hz, $a = 0.99$ mm, $\varepsilon = 0$, in which two mutually perpendicular waves similar to Fig. 2 develop to form square array of standing-wave patterns. Cross point of the ridges in Fig. 6(a) turns into valley region of Fig. 6(b) after a time T, and vice versa. The last statement is valid to other types of patterns like pentagonal cells ((b)(b′) $f = 30$ Hz, $a = 1.81$ mm, $\varepsilon = 0.1$), or triangular/hexagonal cells ((c)(c′) $f = 33$ Hz, $a = 1.56$ mm, $\varepsilon = 0.1$), where (b′) and (c′) are corresponding patterns after a period T. We also checked time variation of the pattern for Figs. 3(g) and 3(h). By counting the overlapping region among successive image data, we found that they alternate with period $4T$.

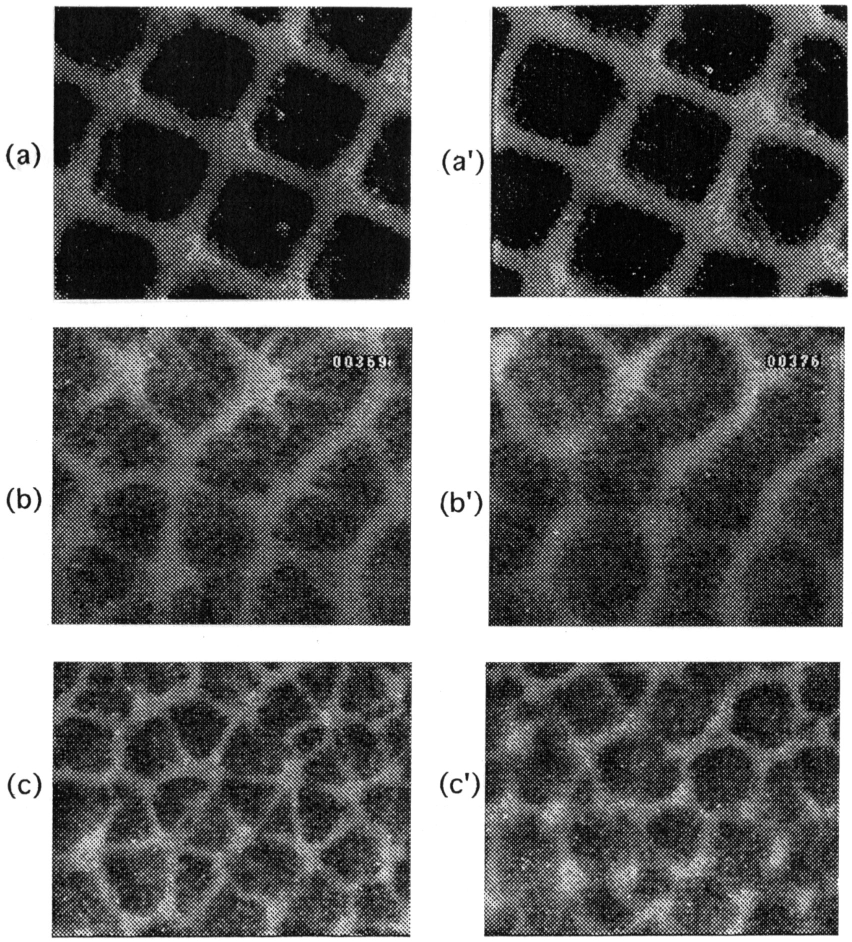

Fig. 6. Pattern formation process: (a)(a′) $f = 35$ Hz, $a = 0.99$ mm, $\varepsilon = 0$, (b)(b′) $f = 30$ Hz, $a = 1.81$ mm, $\varepsilon = 0.1$, (c)(c′) $f = 33$ Hz, $a = 1.56$ mm, $\varepsilon = 0.1$. Figures (a′), (b′) and (c′) are the patterns after one period T of (a), (b) and (c), respectively.

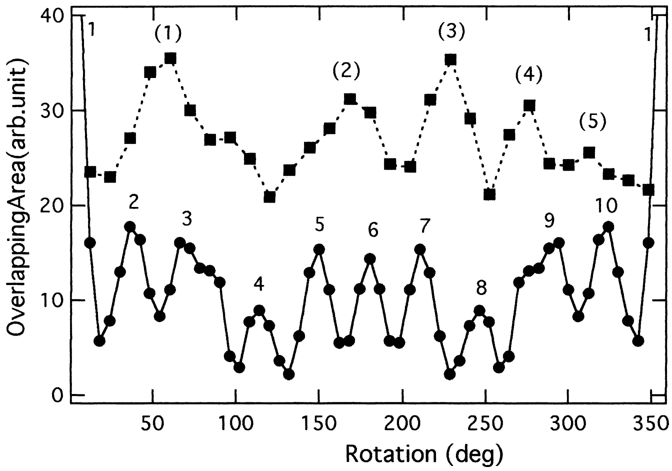

Fig. 7. Dependence of the overlapping regions between the original image and the one rotated around a fixed point. Image data is taken from Fig. 3(h), and different marks correspond to different point of centre.

4. Discussion

We have shown variety of patterns with special attention to their spatial symmetries and temporal periodicity. Our granular layer seems to have strong similarity to water surface waves, which is understood on the basis of the Navier-Stokes equation. On the other hand, we have no basic equations for the former. The ballistic character, and hence local ordering, seems to explain the negligible influence of the boundary shape of the container. How this short range interaction leads to long range order (or patterns) is an open question to be answered in the future investigation.

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