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Curves in Traditional Japanese Architecture and Civil Engineering

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Abstract. Curve drawing procedures in traditional architecture and civil engineering in Japan are formulated in terms of western mathematics in order to abstract the fundamental form of the curves and to know what is attempted for its modifications. Confirming the human capability to discriminate such curves, the formations of the images are classified. Based on this framework, a hypothesis on the general characteristics of those curves is proposed: Image of the open ended symmetric curves with axes directed towards the objects of interests is intended and the realization of the impression of elasticity is also attempted.

1. Introduction

This paper is an attempt to propose a framework for studying the curves in traditional Japanese architecture and civil engineering. The first phase consists of abstraction of the fundamental forms of the curves in terms of modern mathematics and analysis of their modification procedures. The second phase is to establish some postulates on the discrimination capability of human being in watching curves. The third is to propose a point of view on the associated physical image with the geometric form of the curves and to infer what was intended by the architects. There might be the fourth phase to discuss the connotation in the sense of philosophy. But such a problem is beyond the scope of this paper.

2. Curves in Terms of Modern Mathematics

Indeed, many curves are observed in traditional Japanese architecture and civil engineering, in Buddhist temples, Shintohist shrines, old castles and even in newly built public bathhouses. Overwhelming Chidori gables constitute the most significant part of the silhouette. Ruled surface of the roof is formed by straight rafters as generatrices with curved ridges as directrices. Because of the concavity of the surface, some part of the tiled roof always twinkles reflecting the sunshine. The curves of the eaves are more delicate. Also, the main entrance is decorated with a so-called Kara gable which is formed from

Fig. 1. Budhist Temple.

concave and convex curves. In case of castles, the whole basements are covered with curved jacket walls corresponding to the curves of the roofs.

Such curves, in most of the cases, are drawn on blue prints according to those of some older architecture with adaptation by the architects and/or carpenters. However, some old manuscripts and textbooks came down in which drawing procedures were described (KOBAYASHI, 1857, also contained in MORINAGA, 1975; SAKUMA, 1975; KITAGAKI; 1987). Some of them are to be presented here in terms of modern mathematics.

2.1. Curves of Chidori gables

Roofs play the most important role in the external appearance of Asian architectures. Among the formers, so called Chidori(plover) gable is the typical in Japanese architecture. It has concave form with a sharp vertex at the top of the building.

Two procedures are known to draw the curve of Chidori gable (MORINAGA, 1975; SAKUMA, 1975; YANAI, 1991). One of them, presented here, forms a parabola, while the remaining procedure forms a sine curve with inclined axis drawn by projecting the equally spaced points on a circular arc.

Set up horizontal abscissa and vertical ordinate with the origin at the lower end of the curve. According to the scale of the gable, we determine length L of the beeline and the

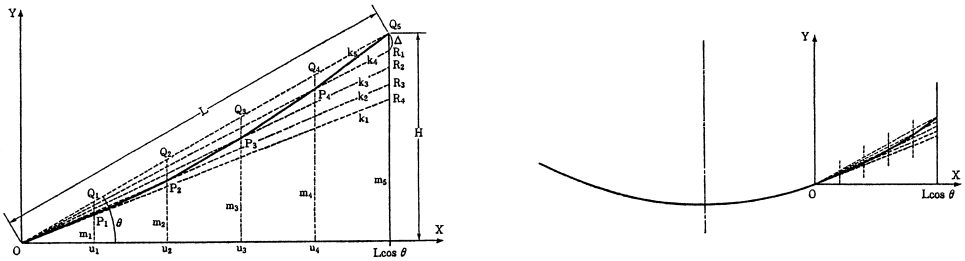


Fig. 2. Chidori Gable.

height H of the summit. Several number of equally spaced vertical lines are prepared; same number of rays including the beeline are drawn from the origin, crossing the vertical lines at an equal vertical distance. The points of intersection among the rays and vertical lines are followed from the origin to the summit stepping one ray up for one vertical line to construct continuous line segments forming the curve of the gable. These line segments tend to a part of a parabola with a vertical axis by increasing the number of lines to infinity. The equation is

$$y(x) = \frac{x}{\cos\theta}\left(\sin\theta - \frac{(L\cos\theta - x)}{NL}\right) \tag{1}$$

where θ is the angle of inclination and N is the number of the rays (Fig. 2).

2.2. *Curves of Mukuri gables*

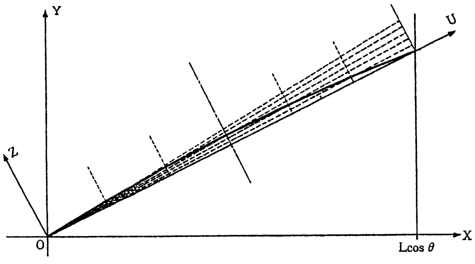
Mukuri (convex) gable is often used above entrances for private use. According to the procedure in textbooks (MORINAGA, 1975; SAKUMA, 1975), the curve is a part of parabola directed downwards and with inclined axis; the procedure is almost the same as the other parabola drawing procedure for the curve of eaves, for example (Fig. 3).

2.3. *Curves of Kara gables*

Kara gable is often used above the main, formal entrance of building. The lower part is drawn as a half period of a sine curve with a larger amplitude and a shorter period in the upper part, as shown in Fig. 3. Although “Kara” means China, it is recognized to be genuine Japanese (O’OKA, 1971). Looked as a curve, however, it is exceptional in form compared to other curves in Japanese architecture (Fig. 3).

2.4. *Curves of Eaves*

Large amount of rainfalls, makes eaves in Japan much deeper than in other dry countries. Accordingly, they are the most significant parts of which people coming in are aware from different distances and different angles with or without consciousness. The architects devoted their attention to the formation of the curve of their eaves. The chronological change in the curves of the eaves is one of the main interests in historical



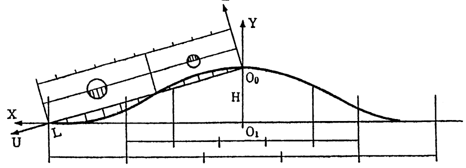


Fig. 3. Mukuri and Kara Gable.

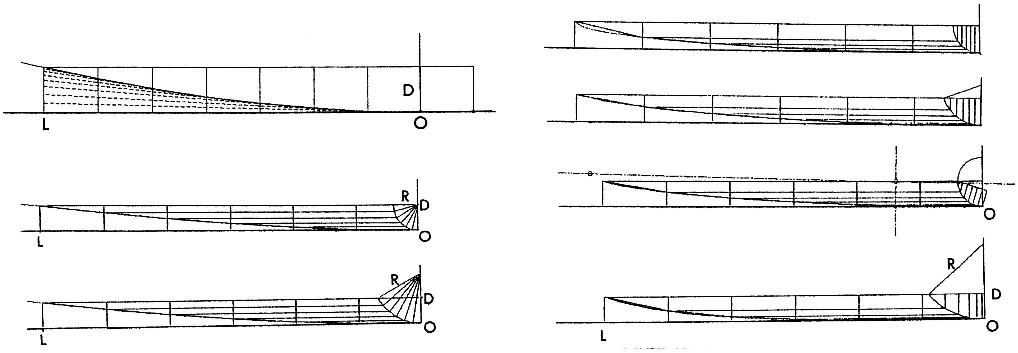


Fig. 4. Various Drawing Procedures for Curves of Eaves.

study in Japanese architecture (O'OKA, 1971). Some are slightly broken at the midpoint. Some have curvatures only at the ends. Some of them are 3-dimensional space curves. The curves of the eaves, however, are determined by those of the lintels supporting them. Many procedures are known to draw the curves.

One of such procedures forms a parabola with a vertical axis by increasing the inclination by a constant amount of tangent for every interval between equally spaced vertical rafters. Sine curve is formed by projecting the points equally spaced along the circular arc; ellipse is formed by affine transformation of a circle. In some procedure, the axis of the ellipse is vertical and inclined in another (Fig. 4). A square root transformation of an ellipse is also proposed; as the result the end of the ellipse is slightly compressed and rounded. All the procedures other than that of parabola seem to intend more round curves than parabola at the ends of the eaves (MORINAGA, 1975; SAKUMA, 1975; YANAI, 1991).

2.5. Stone walls of castles

Most of the stone jacket walls of Japanese castles are not separately standing wall, but steepened slopes of hills covered with stones. Hence the curves must be drawn at the construction sites. One of such procedures is to be presented here according to manuscripts from Goto family, a samurai family belonged to Kaga clan, now Kanazawa (contained in KITAGAKI (1987) and analysed in YANAI (1988)).

Let the hypotenuse of right angled triangle ABC be the slope. The upper 2/3, the interval AE, is to be curved. Divide it into n equal subintervals. The horizontal straight line passing through the i th node Q_i from E is represented by l_i in Fig. 5. On the other hand, we put a horizontal plank at the summit of the slope, on which $n + 1$ points R_0, R_1, $\cdots$; R_n are marked. The distances between succeeding points constitute a decreasing arithmetic progression. We stretch a string from R_0 to $Q_0 = E$. We represent the point of intersection between this string and the level line l_1 by P_1. Stones are laid along this string one another until the point P_1 is attained. Then we stretch a new string from R_1 to P_1. Again stones are laid along this new string until the point P_2, where P_2 is the point of intersection between this string and the level line l_2. Following such procedures until the highest level is attained, the whole curve is completed. If the number n is increased to infinity, these line segments

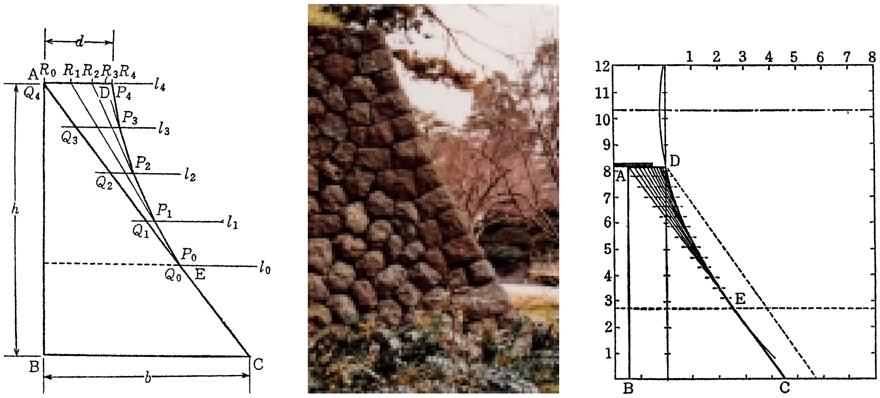


Fig. 5. Stone Wall of Castle.

are regarded as the envelop of the strings in terms of modern western mathematics, which is represented by

$$y = \frac{bu}{h} + d\left(1 - \frac{3u}{2h}\right)^2 \tag{2}$$

where y is the abscissa, u is the ordinate which goes downwards; h, b and d are the parameters representing the height, width of the fundamental triangle and length of the plank. This is a part of a parabola with a horizontal axis as shown in Fig. 5. The axis lies, however, far above the summit of the wall according to the parameters in Goto's manuscripts.

2.6. Fundamental form of the curves

So far as we have observed, most of the curves used in traditional Japanese architecture and civil engineering are conic sections, sine curves and their variations. However, parabolas are the most frequently observed among them, although they are partly modified in some cases. In this sense, parabolas are regarded to be the fundamental form of the Japanese curves.

However, if the people watching them could not recognize the curves as parabolas or the like (not in terms of mathematics, but in the sense of the form) nor discriminate them from others, it would be useless to prepare such delicate curves. In this regard, we have to confirm the human capability of discrimination by watching curves.

3. Human Capability of Discrimination

So far as the architectures are of real existence and presentations of human concept, there must be some intention in designing the curve in addition to engineering rationality. Then, what kind of information can be delivered through the curves as media? Although this is partly a philosophical problem, it is also deeply related to the function and its limit of the human eyes and information processing capability. Not in details, but we know that the capability of human eyes are very high; they can recognize even very slight displacements by means of densely distributed minute cells of optic nerves. Such optical information is analysed to obtain interpolation and extrapolation of the curve.

SANUKI (1980) made a suggestive conjecture about the high capability for human being to inter- and extrapolate curves: Observing a curve, people imagine their own movement along the curve; they move in their imagination along the curve with a constant speed on a train or skis for example. They can feel the centrifugal force associated with the movement, which means the curvature or the second derivative of the curve.

On the other hand, an experimental analysis was tried, in which a part of conic section enclosed in a rectangle is shown to people and they are asked to extrapolate by hands being just informed that it is a conic section. The resulting curves are superposed with the remaining part of the conic section in Fig. 6. A large majority of the extrapolated curves are closely around the conic section. According to these experiments human eyes could be recognized to have the capability to distinguish the difference among conic curves and to find the directions of the axes just by watching a part of them (KABURAKI *et al.*, 1992).

By all these observations, human intrinsic capability is regarded to be high enough to set up following postulates:

1. Curves can be recognized with first and second derivatives. Hence, their curvatures are also recognized.
2. Curves can be extrapolated.
3. Symmetry in curves can be found if any, with their axes and/or centres.

4. Formation of the Image of the Curve

Given a curve to be watched, people would construct their own image of the curve as a whole by means of the capabilities described above. Restricting ourselves to physical aspects, we classify the types of formation of the image from two points of view (YANAI, 1993).

(a) Open and closed curves; Some curves would be imaged as a part of open ended

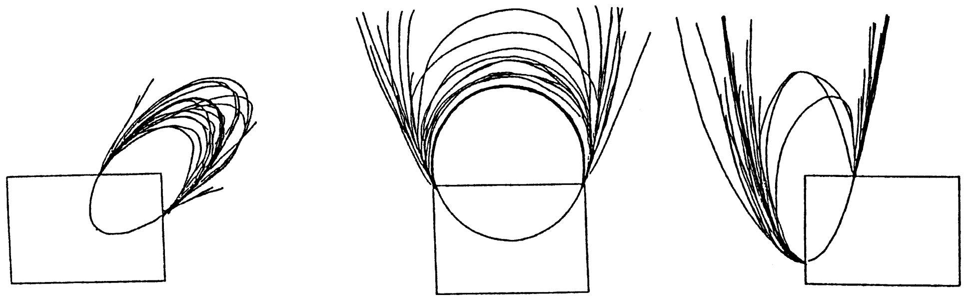


Fig. 6. Intrinsic Extrapolations.

curve like catenarys or parabolas; some would be imaged to form closed curves like circles or ellipses. The axes of the curves extrapolated in image are also remarked.

(b) Image of elasticity: People imagine existence of elasticity in some curves and not in others.

By the capability described above and by daily experience people can discriminate the curves of a suspended chain and a bow spanned with a string. Some catenarys and curves of bent wires are compared in Fig. 7. The former curve is of complete slackness, while the latter is that of a bent elastic body in which the curvature is widely distributed along the whole length. The former presents a cosine hyperbolic curve which is closely approximated by parabolas, while the latter is well approximated by polynomials of the 4-th order, although the exact mathematical expression depends on the boundary conditions. The former is open ended, while a closed circle is formed by joining the both ends of an elastic wire. It must be also remarked that, the axes of symmetry are always vertical in case of suspended chains, while the directions of the axes are not definite in the case of bent wire. Reflecting all these observations, we classify the types of the images into the two above.

5. General Characteristics of the Curves in Japanese Architecture

As described above, the fundamental form of the curves is parabola. From this fact, we can infer that parts of open ended, symmetric curves are intended in traditional Japanese architecture. In addition, it should be remarked that the axes of the extrapolated curves are directed upwards to heaven in case the building is of religious and/or public use, downwards above entrances for private use and toward the enemy in the case of jacket walls of castles. Even in a Kara gable above the main entrance, the direction is downward at the centre and upwards at the both ends.

Although parabola is very close to catenary, the architects are not contented themselves with catenary. In fact, catenary is used to be applied as a mean of approximation at construction sites, however, it was always necessary to modify by suspended weights along the cable to fit it to the original curve in the blue print (YANAI and OKAMURA, 1985).

On the other hand, in significant parts like the eaves, not only other curves like sine curves or ellipses are applied, but also many procedures are invented for further modification

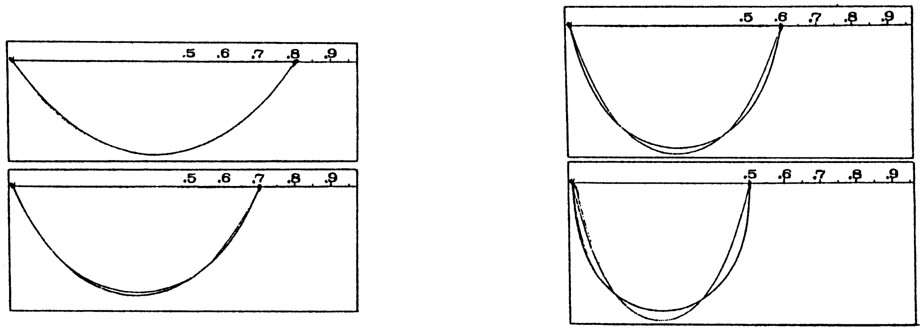


Fig. 7. Catenary and Elastic Bending.

of these curves. Moreover, according to these procedures, the curvatures of the eaves are strengthened at their ends, which could be interpreted to be the attempt to represent the image of the bent elastic body.

From these observations, the general characteristics of the curves in the traditional Japanese architecture and civil engineering could be described as

Image of parts of open ended, symmetric curves
with the axes directed toward the objects of interests is intended.
Realization of the impression of elasticity is also attempted.

This is of course a temporary hypothesis. Further examination is necessary in order to approach the connotation hidden in the curves, probably from the point of view of psycology, cultural anthropology and so on, which is beyond the scope of this paper.

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