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The Ishango Artefact: the Missing Base 12 Link

Vladimir PLETSER[1] and Dirk HUYLEBROUCK[2]

[1]*European Space Agency, P.O. BOX 299, NL-2200 Noordwijk, The Netherlands*
E-mail: vpletser@estec.esa.nl
[2]*Sint-Lucas Institute of Architecture, Paleizenstraat 65-67, 1030 Brussels, Belgium*
E-mail: dirk.huylebrouck@pi.be

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Abstract. In 1950, the Belgian Prof. J. de Heinzelin discovered a bone at Ishango, a Congolese village at the sources of the Nile. The artefact has patterned notches, making it the first tool showing logic reasoning. Here, Pletser proposes his new "slide rule" interpretation, rejecting former "arithmetic game" and "calendar" explanations. Counting methods of present day civilisations in Africa provide circumstantial evidence for Pletser's hypothesis. Moreover, it confirms de Heinzelin's archaeological evidence about relationships between Egypt, West Africa and Ishango. It points towards the use of the base 12, which anthropologist Thomas had studied in West Africa some 80 years ago. It appears that the Ishango artefact is the missing link Thomas was looking for. These results where obtained independently; for instance, space scientist Pletser just stumbled over the Ishango artefact as he favoured the project of carrying it into space, as an African equivalent of the Katachi-symmetry relationship.

1. The Ishango bone

The Ishango bone is a 10-cm long curved bone, first described by its discoverer, J. de Heinzelin. He found the tiny bone about fifty years ago, among harpoon heads at a village called Ishango not far from the present border between Congo and Uganda. The fishermen settlement lays on the shores of the Semliki River, one of farthest the sources of the Nile. The bone has a fragment of quartz on the top, most probably for engraving purposes. About 20,000 years old (or even 90,000 years, following other indications), it is the oldest mathematical tool.

The bone carries notches distributed in three columns along the bone length. The central column along the most curved side of the bone, is called the M column (French: Milieu), while G and D indicate the Left (*Gauche*) and Right (*Droite*) columns. The M column shows eight groups of respectively 3, 6, 4, 8, 9 or 10, 5, 5, and 7 notches. The G and D columns each show four groups respectively of 11, 13, 17, 19 and of 11, 21, 19, 9 notches. The groups are labelled with an upper case letter for the column and a lower case letter for the actual group, together with the number of notches between parentheses: Ma(4) to Mh(7), Ga(11) to Gd(19), and Da(11) to Dd(9). The notches are approximately parallel within each group, but sometimes of different lengths and of different orientations. Some uncertainty remains on the number of notches for the Me(10) and Mf(5) groups, as part of the bone was damaged earlier or by infiltrating rainwater.

De Heinzelin thought the bone was an evidence of knowledge of simple arithmetic. Indeed, the first four groups of the M column look like duplication while the G column shows the prime numbers between 10 and 20. The third column indicates 10 ± 1 and 20 ± 1. The most striking is that all numbers in columns G and D add up to $60 = 5 \times 12$, while the sum of the numbers in the M column is $48 = 4 \times 12$.

On the other hand, A. Marshack later suggested that the bone was a lunar calendar. Circumstantial evidence supports this alternate interpretation, since present day African civilisations use bones, strings and other devices as calendars. The lack of such evidence was one of the shortcomings in de Heinzelin's arithmetic game interpretation. For example, no awareness of the notion of prime numbers has been discovered before the classical

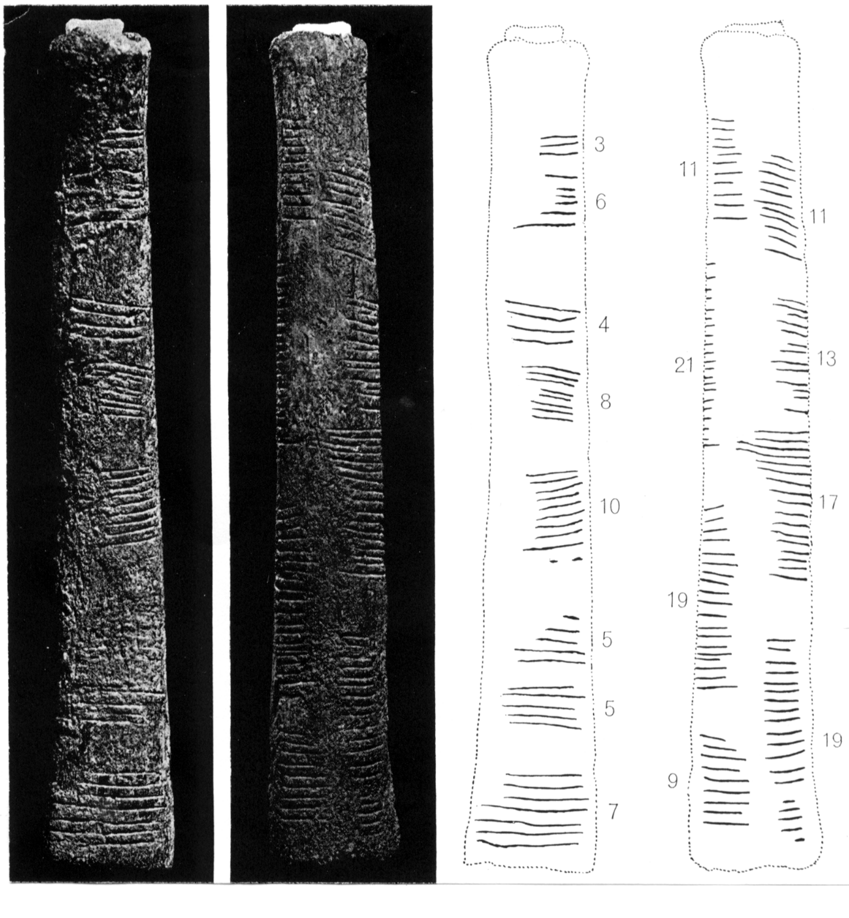

Fig. 1. De Heinzelin's detailed drawing of the Ishango bone.

Greek period.

The new interpretation proposed here combines solid circumstantial evidence with a straightforward mathematical insight.

2. The Slide Rule Hypothesis

The middle M column is central to the understanding of the number system displayed on the Ishango bone. The approximate length in mm of each notch and the vertical distance between the eight groups, as well as the orientations of the different groups imply interesting deductions, when read from top to bottom. The first four groups suggest duplication, but the process looks more like rearranging a new group in a particular way. Mb(6) appears as a set of three notches of the same length, like in Ma(3), together with two longer notches, one on each side, and an even longer notch on only one side to obtain six. Similarly, Md(8) seems a subgroup of two notches in the middle, with on each side a subgroup of three notches longer than the first two but of similar length. This regrouping concept is not essential (and probably did not exist in the mind of the notch maker), but it puts in evidence the fundamental role of the two bases 3 and 4. If s, m and L stand for small, medium and long, it can be schematised by

Ma(3): 3s; Mb(6): 1m + 3s + 1m + 1L; Mc(4): 4L; Md(8): 3L + 2s + 3m;
Me(9′): 4m + 4m + 1L or 3m + 2s + 3m + 1L or 8m + 1L; Mf(5): 3m (?) + 2L;
Mg(5): 1m + 1L + 3m or 2L (?) + 3m or 1m + 4L (?); Mh(7): 3m + 4L.

The notation 9′ was used to express the uncertainty about this number. The corresponding number of notches could well be 10, a case that was studied separately by Pletser, but that was omitted here for reasons of conciseness.

Surprisingly, the D column bears the numbers 10 ± 1 and 20 ± 1 in a disordered manner. The usual explanation of linking these numbers to a base 10 stresses more the influence of the actual modern use of the base 10 than intrinsic relations to the simple arithmetic displayed in the M column. However, these seem more significant in view of the use of the bases 3 and 4. If these operations of addition and subtraction of 1 from a base are correct, it would make more sense to see the ordered series of numbers in the G column as displaying the numbers 12 ± 1 and 18 ± 1 from the derived base 12 and the number 18, one and half times the derived base 12.

Let us now compare the consecutive sums of two or three consecutive numbers in the M column to those displayed in the same approximate vertical range in the middle of the G and D columns. For the groups on the extremities of the bone, we note that they can be represented as:

- Ga(11) : (2m + 1L + 2m + 1L) + 4L + 1L, indicating that the values of Mb(6) and Mc(4) should be added (6 + 4), together with an additional 1;
- Gd(19): 2m + ((7L) + (3L + 2m) + (1s + 3m + 1s)), showing that the number 2 is added to the numbers 7, 5 and 5 in the reversed order of the M column's last three groups.
- Dd(9): 2s + (1L + 1m + 3L + 1m + 1L), showing an addition of 2 and 7.

Considerations of this kind led Pletser to a diagram of which a slightly different version is presented here, again for reasons of conciseness:

M									G	D
3 +	6		~~+2~~							11
~~1+~~	6 +	4						→	11	
		4 +	6 +	3				→	13	
		4 +	8 +	9′				→		21
			8 +	9′				→	17	
				9′ +	5 +	5		→		19
~~2+~~					7 +	5 +	5	→	19	
						~~2+~~	7	→		9
6	12	12	24	30′	12	12	12		60	60

This ordering as presented by the markings on the bone in the last group of the G column yields solely occurrences of the base 12 and directly derived numbers. For the additional 2 added in the first line, Pletser had no straightforward explanation, but his study of the relative positions of the notches of the Ma(3), Ma(6) and Da(11) groups proved that this was not done thoughtlessly.

Summarising, the hypothesis becomes likely that ancient people of Ishango made use of the bases 3 and 4 for counting and building further small numbers up to 10. The derived base 12 could have been used for counting larger numbers. It is not impossible that these bases coexisted with other natural counting bases like 10 or 20. Although addition of simple small numbers is apparently well mastered, knowledge of multiplication by other factors than 2 is not apparent. Furthermore, the hypothesis of mastering the knowledge of prime numbers appears too audacious in view of the basic arithmetic displayed.

On the other hand, the proposed hypothesis of considering the bone as an ancient 'slide rule' to display simple addition arithmetic fits well with the various notch geometrical patterns. It shows a systematic occurrence of the derived base 12 and the central role that this base played in the proto-mathematics of the ancient Ishango people.

3. Circumstantial Evidence: Counting Methods in Africa

One can wonder if such a number system as proposed in the slide interpretation has ever existed. A survey of the various counting methods in Africa turns this question into a rhetorical one. Zaslavsky wrote a complete book about related subjects, with the transparent title *Africa counts*. Many other papers on the subject added more information, showing that the base ten system is certainly not the only one that was ever preferred. For example, the Congolese Baali system is a mixed base 4 and 6 number system (Question marks indicate forms that are difficult to explain):

1 = *imoti* 2 = *ibale* …	6 = *madia*; 7 = *madea neka* (6 + 1?)
8 = *bapibale* (6? + 2)	9 = *bapibale nemoti* (6 + 2 + 1)
10 = *bapibale nibale* (8 + 2)	11 = *akomoboko na imoti* (10? + 1)
12 = *komba* 13 = *komba nimot*	14 = *komba nibale* …
24 = *idingo*	25 = *idingo nemoti* …
36 = *idingo na komba*	37 = *idingo na komba nemoti*

48 = *modingo mabale*	49 = *modingo mabale nemoti*
576 = *modingo idingo* (= 24^2)	577 = *modingo idingo nemoti* (= $24^2 + 1$) …

Combining number bases, like with 3 and 4 as proposed in the previous sections, has been frequently noticed too. The Nyali from Central-Africa use a mixed system forming numbers through combinations of 4, 6 and $24 = 4 \times 6$. Four corresponds to *gena*, 8 to *bagena* (= plural form of four), 24 is *bwa* and 576 *mabwabwa* (= 24^2). The Ndaaka express 10 as *bokuboku*, and 12 as *bokuboku no bepi*, but for 32 there is a special word, *edi*. Now 64 becomes *edibepi* (= 32×2) while 1024 is *edidi* (or 32^2). A number like 1025 is expressed as *edidi negana* or $32^2 + 1$.

One of the reasons for the great creativity found in the vocabulary for number words can be the various counting gestures. Zaslavsky pointed out that sometimes stretched fingers designate numbers (like in Western Europe), while sometimes the folded fingers do. The Tete cross the fingers they want to show with the left thumb. The Soga tribe shows 6 by holding the left forefinger to the closed right hand, while 7 corresponds to adding the middle finger to that left forefinger. The Chagga take the fingers of the right hand, beginning with the little finger, with the whole left hand, to indicate numbers from 6 to 9. In Rwanda and Western Tanzania, four corresponds to snapping the forefinger from the ring finger of one hand onto the middle finger (and so on: this was just a small indicative sample).

A remarkable fact in the denominations and gestures for the numbers from six to nine is that these can be formed by different principles. Sometimes 6 and 8 are expressed as 3 + 3 and 4 + 4, while in some regions compositions based on 5 are preferred: 6 = 5 + 1, 7 = 5 + 2. Other descriptions are frequent for the cases of 7 and 9, like 10 – 3 and 10 – 1.

Number words can vary according to what has to be counted. For example, if in Burundi one has to do with large quantities of cattle, groups of five have to be used. In this case, the usual word *itandatu* or 3 + 3 changes in *itano n'umwe,* meaning 5 + 1. Similarly, *indwi* or 7 changes into *itano n'iwiri* or 5 + 2, etc.

Pletser's hypothesis affirms furthermore that the Ishango number system would have involved the number 12 in particular. Now one of the many African counting methods, similar to the ones given above, uses the base 12: the thumb of a hand counts the bones in the fingers of the same hand. Four fingers, with each three little bones, evidently yield 12 as a counting unit. Also, each dozen is counted by the fingers of the other hand, now including the thumb, and the multiple $5 \times 12 = 60$ provides an additional indication of the often simultaneous occurrence of the duodecimal and sexagesimal base.

Thomas reported on such number words in his study of the West-African tribes in the region of the actual Nigeria. The Yasgua, Thomas asserted, count as follows:

1 = *unyi*	2 = *mva*	3 = *ntad*	4 = *nna*
5 = *nto*	6 = *ndshi*	7 = *tomva*	8 = *tondad*
9 = *tola*	10 = *nko*	11 = *umvi*	12 = *nsog*
13 = *nsoi* (=12+1)	14 = *nsoava* (=12+2)	15 = *nsoatad*	16 = *nsoana*
17 = *nsoata*	18 = *nsodso*	19 = *nsotomva*	20 = *nsotondad* …

Thomas reported that 4-6 combinations like 10 = 6 + 4 was in use with the Bulanda

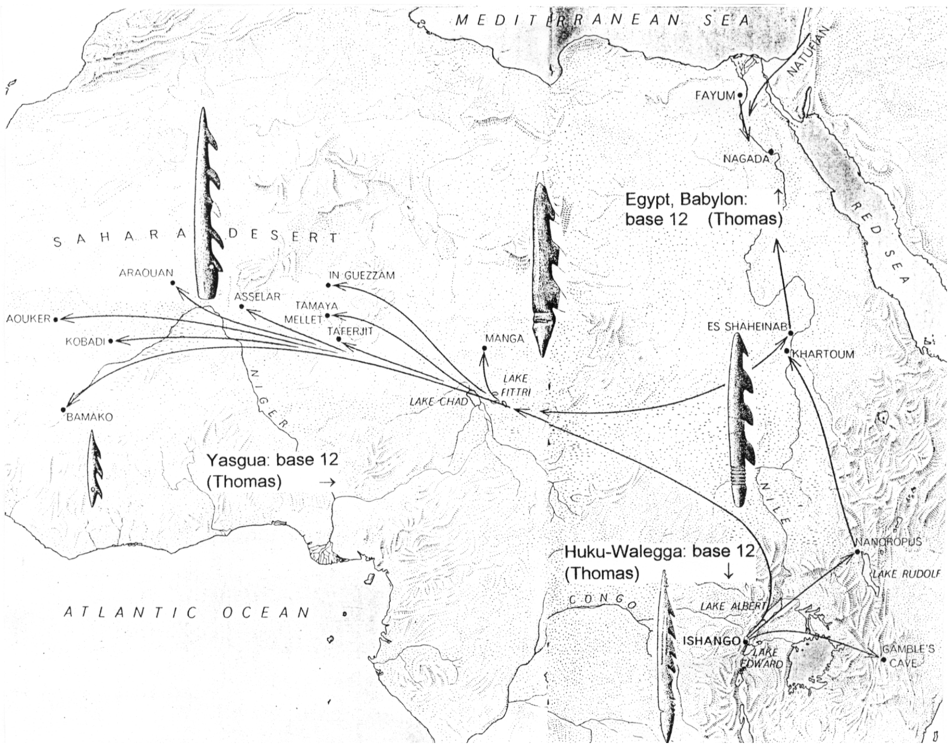

Fig. 2. Based on harpoon head findings, archaeologist de Heinzelin proposed an itinerary for the Ishango knowledge from Central Africa in western and northern direction. The liberty was taken to add some harpoons heads on the original drawings from Scientific American, as well as some information related to Thomas' paper mentioned in the text.

tribe, while the Bola would have expressed 12 as 6×2 and 24 as 6×4.

This apparently well known use of the base 12 (or base 6?) in some areas, surprising to many Western readers, makes Pletser's base 12 hypothesis plausible.

4. The Ishango Bone as Missing Link in the Base 12 Dissemination

De Heinzelin concluded his Ishango paper in Scientific American by a hypothesis about the impact of the bone, based on findings of harpoon heads:

> '*The first example of a well-worked-out mathematical table*', says de Heinzelin, '*dates from the dynastic period in Egypt. There are some clues, however, that suggest the existence of cruder systems in predynastic times. Because the Egyptian number system was a basis and a prerequisite for the scientific achievements of classical Greece, and thus for many of the developments*

in science that followed, it is even possible that the modern world owes a great one of its greatest debts to the people who lived at Ishango. Whether or not this is the case, it is remarkable that the oldest clue to the use of a number system by man dates back to the Central Africa of the Mesolithic period. No excavations in Europe have turned up such a hint'.

A part of itinerary proposed by de Heinzelin points to West Africa. Now Thomas, in his paper about West African tribes written in 1920, asserted that the base 12 systems he had discovered, had to be rather old and that they should go back two thousand years at least. He knew about Babylon as *"the best known reference to such a base"*, but wondered if there was a link between both:

It remains to add that if we find no duodecimal system among any people likely to have been in contact with Nigerian tribes, we must assume an independent origin for the system. If it had been transmitted from Babylonia via Egypt, it must surely have left some traces on its road. For those who believe the duodecimal notation can have been invented once only, it is an interesting problem to bring the Nigerian duodecimal area into relation with Babylonia.

The Ishango bone could be the missing link Thomas was looking for 80 years ago. Thomas already guessed where to look, since he mentioned that there was one other tribe in Africa with computational practices similar to those he studied in West Africa. It are the Huku-Walegga, who live in an area Northwest of the Lower Semliki, the same river on which shores the Ishango bone was found. The Ishango bone was found about 30 years later, and de Heinzelin made his map about influence from Ishango to Egypt or to West-Africa only in 1962.

In 1999, Pletser interpreted the bone as a base 12 pattern and this again without having read Thomas' work. He actually is a microgravity space scientist, who was involved in trying to get the Ishango bone into space. He stumbled over his slide-rule interpretation when he was informed about that Ishango bone space project. There seems to be too much coincidence involved in these three research results for them to be sheer accident.

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Notes

1. Acknowledgement: I. Jadin and A. Hauzeur of the Royal Museum of Natural Sciences of Belgium in Brussels helped the authors at many occasions. In particular, the museum provided the voluminous original report of Prof. de Heinzelin to both authors.
2. Appeal: Like the mathematical knowledge was maybe transferred from Central Africa to ancient Egypt and then to Europe, the bone followed a somewhat similar odyssey. It was excavated after thousands of years and brought to Europe by its discoverer. This odyssey is not completed since it may fly one day in space on board the International Space Station. It would be a symbol for the link between mathematical knowledge and art, even at their very beginnings. Carrying the artefact into space, as a gesture for the small step it took to reach modern times, would mimic the ellipse in Stanley Kubrick's acclaimed movie *"2001: A Space Odyssey"*.