Review *Forma*, **15**, 11–28, 2000

Katachi U Symmetry in the Ornamental Art of the Last Thousands of Years of Eurasia

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(Received December 27, 1999; Accepted January 18, 2000)

Keywords: Eurasian Arts, Katachi and Symmetry, Intuitive Mathematical Discoveries, Technologies, Local Operations in Mathematics of Ornamental Art

Abstract. Artworks of various cultural communities in Northern and Middle Eurasia show that many ornamental patterns are life-style related intuitive (sometimes mathematically interesting) designs. Studies on great part of this rich ornamental art heritage helped to recognize characteristic Katachi and Symmetry principles in construction of patterns by extension, reduction, or coloring of the pre-existing patterns known from natural phenomena and technologies. Katachi and symmetry is also strongly connected in the new model of deduction of the set of ornamental patterns by local operations.

1. Introduction: KATACHI and SYMMETRY in Eurasian Artworks

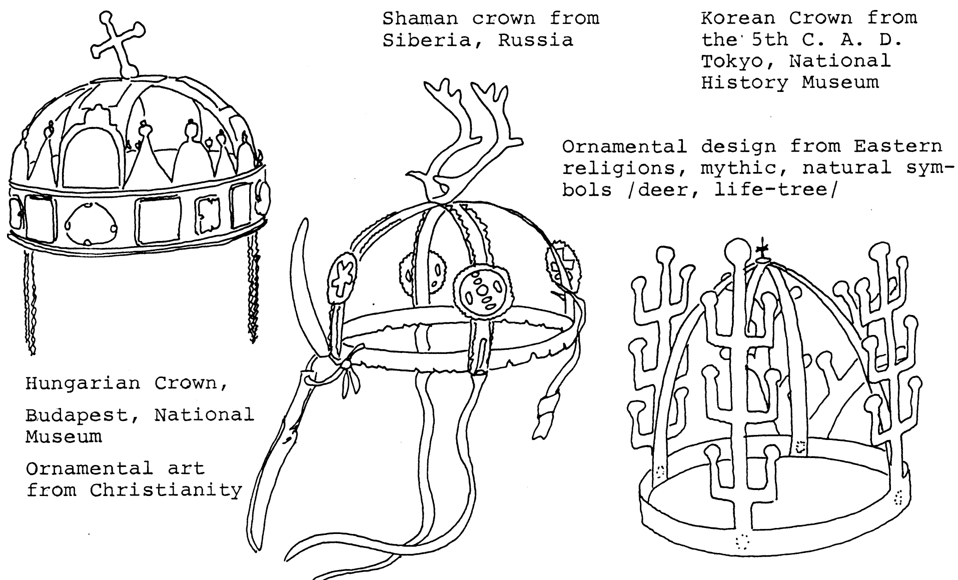
Katachi and symmetry always played essential role in finding the operations which build up patterns, and even today this concept may help in formulating discoveries about structures. Symmetry concept has a more space oriented (western), Katachi concept has a more temporal (eastern) origin about invariances of the structures (YANABU, 1999). Ornaments on archaeologic finds, in architecture and recent artworks reflect the presence of both concepts in the thinking of artists. For example, in weaving the repeated motion (time invariance: KATACHI) produces a woven structure with uniform spatial relations, that is a pattern with SYMMETRY. Studying the art of several Eurasian cultural communities we feel intention to unite KATACHI and SYMMETRY in a concept-machine where they are acting together. Using technology of ornament making we are led to such a concept of pattern construction by local operations. The twin concept of katachi and symmetry and the local operations open new approach to pattern construction and study.

Design of ornaments needs a sequence of recognitions. In principle this sequence of recognitions had common roots for any communities in natural phenomena and technologies. But the sequence of discoveries depends on the environment, on life-style and many other variables. The artproducts tell these differences, ornamental patterns on them may tell something about the sequence of discoveries, too. If we can form constructional series from patterns we may deduce how they were developed by our ancestors. In this geometrical archaeology the structural knowledge of ornamental structures can be used to form the mathematical background for comparisons. Symmetry is one concept from geometry about the order, connections between elements and the whole pattern. This language will be extended in a special way by using operations with technological roots.

Eurasian steppe arts includes artworks of both nomadic and settled people who learned from each other. Great Eurasian art centers developed and fluorished in the vicinity of settled and nomadic kingdoms, as: far East in China, Japan and the Altai Mountains cultures, in Central Asia in the kingdoms of Turkestan, Iran, Mesopotamian cultures and in the West in Greece, in Celtic and Viking cultures. In the Carpathian Basin many effects from east and west stratified one above the other and were preserved in Scythian, Celtic, Avar-Onogurian and Old-Hungarian art. The products were always transported by merchants and warriors (PRINCE MIKASA TAKAHITO, 1996).

Historically the knowledge of communities stratifies. Symbolic stratifications of cultural layers can be imagined from two episodes. From great kurgan-excavations it became clear that in Scythian funerals horses were "redressed" as deers: horse was a "new" holy animal, redressing meant the survival of ancient magic deer tradition (Pazyrik kurgan art). Similarly, stratification of art-structural forms can be seen in Romanesque architecture. There old mythology scenes are "Christianized" and appear on church gates as stone-carvings and sculptures, or on the walls as frescoes all over W. Eurasia (LÁSZLÓ, 1943a).

Ornamental pattern knowledge of communities also survives and they can be compared by arranging the artworks in a mathematically defined scheme. On the "background" of a



Crowns from Eurasia: all have similar structure, two half bands crossed and embraced by a circular belt, and all are adorned on nodes of attachement.

mathematically organized set of patterns (i.e. by symmetry groups) discoveries of ancient cultural communities appear as outstanding, unusual structures.

2. Mathematical Background: Elements, Connections, Order

First imagine a regular pattern, an ornament, which is ready, as a whole. Such ornaments are built up from repeating discrete congruent elements in a regular way. There is a strict concept to characterize and classify the various structures we generally call regular. This concept is SYMMETRY. Over the original meaning of proportional, (that is the whole is commensurable to its congruent parts) the modern meaning of symmetry is opeartion (since Weyl and XXth century physics). This operation belongs to the regular pattern. It is a “self-invariance-operation” of its own. Why? Because it can be used (carried out) on this regular pattern without causing any (essential) changes in the pattern. (Symmetry may rearrange some or all of the components, the congruent elements, but keeps the pattern unchanged as a whole.) So symmetry preserves the pattern in its state, leaves the structure invariant. For example: the life-tree scene, which occurs all over the world, but oldest is on the seal cylinders of Sumerians, cuts the mirror symmetric “face” to two parts. There is a lifetree in the centre and deers, goats or lions stand by it left and right. Mirror reflection changes the two sides, but leaves the pattern unchanged.

The symmetry operations of the patterns form a set. There are 7 frieze (along a line) groups and the 17 wallpaper groups on the whole plane. Many cultural communities intuitively used them (CROWE, 1975; NAGY, 1979; WASHBURN and CROWE, 1988; SENECHAL, 1989; URABE, 1999). We shall study them from a new, technological aspect: how they can be built from local operations (Fig. 1.).

Second imagine congruent elements (units) only. They were produced by technology or nature. We must construct a pattern from them. We need steps how to do it. We call these steps: operations. We shall connect congruent elements. We always connect one unit to its neighbour. The connecting operation will be a local operation. We do connections along main directions in the plane. We conncet them in a regular method. Pattern will be constructed by these local, neighbourhood operations. This way of construction we may call pattern making by KATACHI. We may follow both the global and the local way of structure recognition.

Life and nature showed both style of pattern recognition and construction. Such basic patterns were for every community: the structure of feathers on birds (cm type), honeycomb (p6m), patterns reognized from technologies as optimal fitting of furs results in a pmg pattern, some weavings as p4g, p2 etc., some chain connecting resulted in p3, p4 types (Fig. 2). (The technological patterns are present in nature only indirectly; their discovery is forced by complex conditions, the needs for simple repeated operations—katachi—the needs for optimal solutions in spatial arrangements of elements—symmetry.) Over natural patterns, weaving, chain-connecting, tiling and other materials technologies gave idea to new constructions for craftsmen: in ornamental structures intuitive mathematical discoveries by doubling or coloring of an originally existing ornamental thread and also by fractal-like zooming, or complex weaving by knots. Rare or local natural phenomena (i.e. whirls) also could give the idea of a repeating unit with exclusive structure in a new pattern (Celtic, Viking and Japanese artistic whirls from the sea, ornamental knots of seaside cultures).

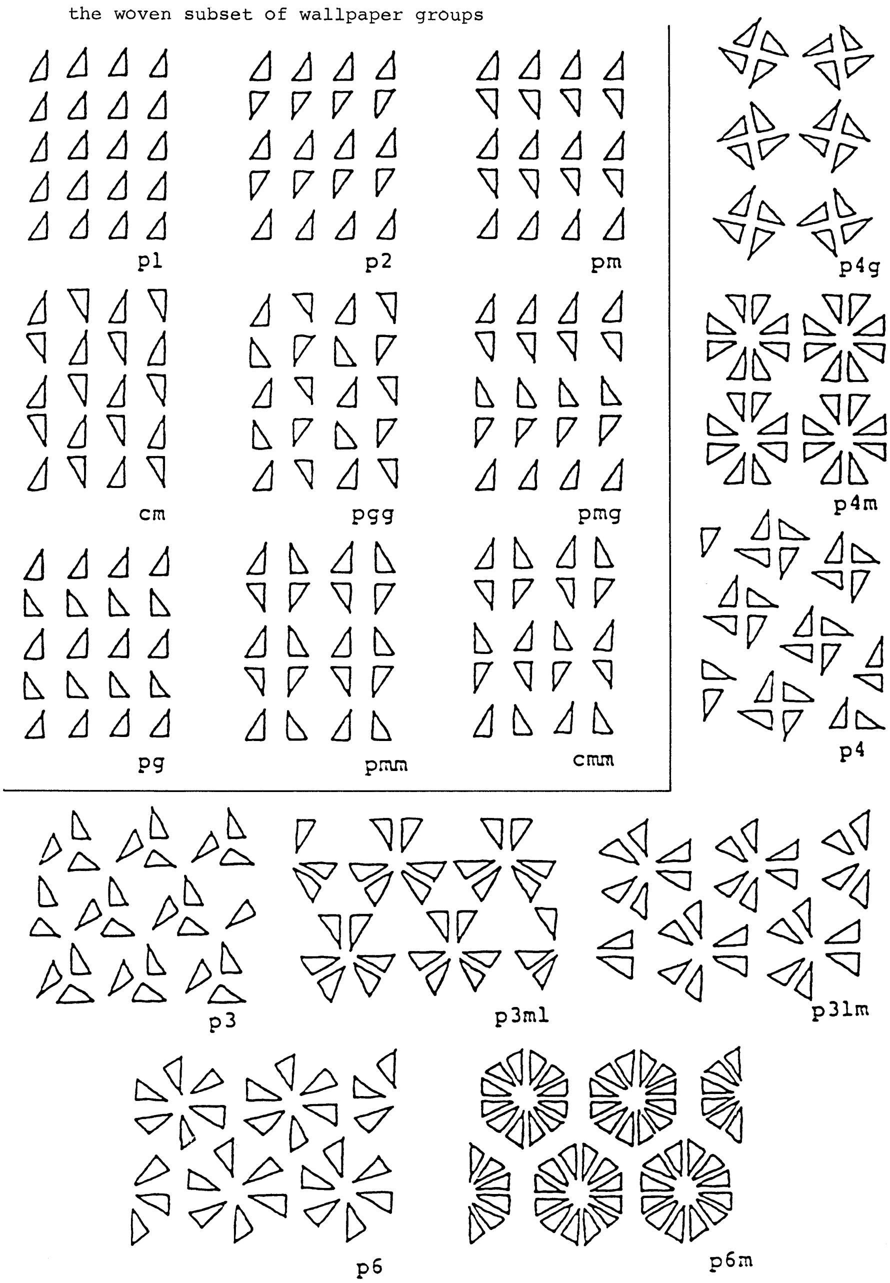


Fig. 1. The 17 wallpaper groups with their simple patterns (all drawings of paper are by the author).

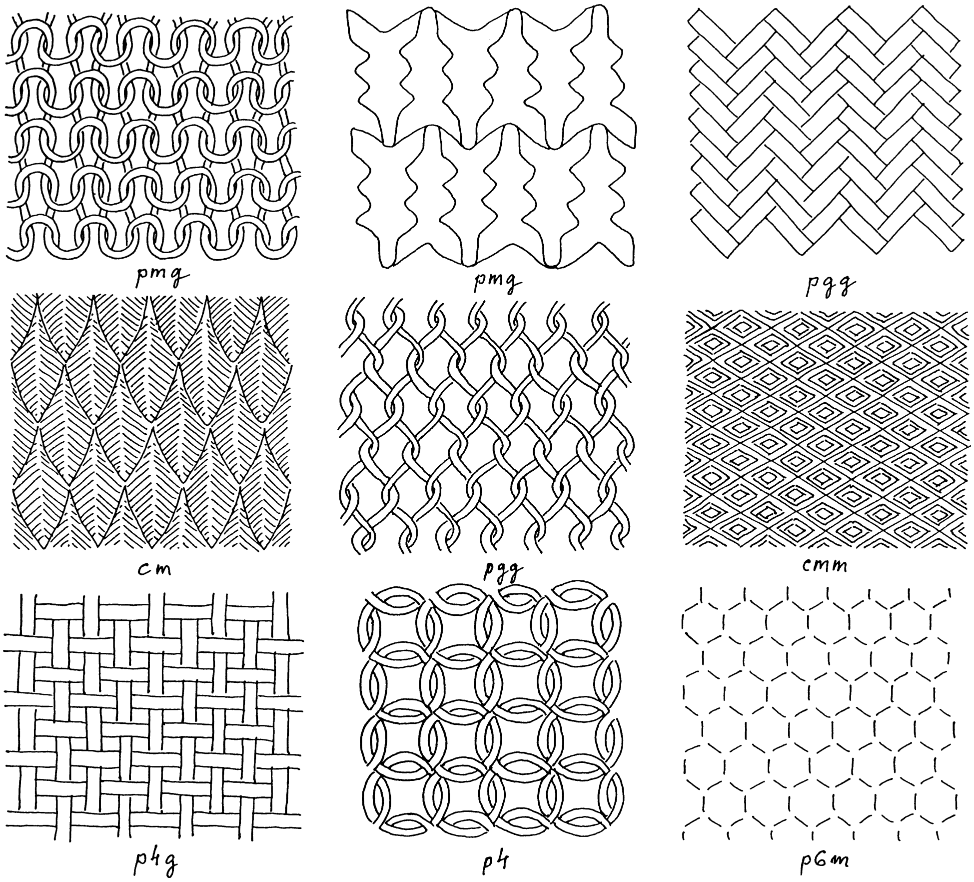


Fig. 2. Ornamental patterns from the nature and from technologies.

3. Patterns by Local Operations: Steps toward the Construction of Cellular Automata Mosaics

In the rich set of Avar-Onogurian ornamental structures doubled ornamental threads frequently occur. This suggested to the author as early as in 1984 to develop another way how patterns can be generated. (At that time I did not know about katachi, which would be useful to formulate the principle what follows.) The new principle is connected to the local operations of the pattern generation. This new concept regards the main aspect of technology, when we construct material systems. In material producing technologies we always attach and fix elements (congruent units) to each other. The elements are directly connected to their neighbours. Such a way the classification of the double-friezes triggered a new meaning for the classical symmetry concept: the doubling operation suggested that a principle from the technology should be involved into the new system of patterns (BÉRCZI, 1989). This is the neighbourhood relation.

The classical symmetry concept used global type order for local connections. Symmetry was the order of the WHOLE ruling on its repeating, congruent ELEMENTS. (This ruling was represented by the symmetry operations.) Our new symmetry concept uses local (neighbour) operations instead of global (symmetry) operations. Double friezes required a one-step (local) operation concept in order to generate the other one from the pre-existing original thread. (I related this local operation to the cellular automatic operation concept because cellular automata "concept-machine" works with neighbourhood operations, BÉRCZI, 1990.)

The concept of LOCAL OPERATION (neighbour-operation) opens new strategy in forming patterns. With the eyes of a craftsman it may mean: if you have recognized the global order, then you formulate the local operation. (This is a translation of the global order for the needs of constructional sequence: "what to do in the next step" when I fit and fix the repeating units, the elements, the—so called—cells.) In this strategy of thinking the sum of local operations (neighbourhood relations) add up the final "state" of the global pattern. Global order is result of local steps. If local operations are regular (they form a sequential KATACHI), in the main (neighbour) directions, the produced pattern exhibits global order (a kind of pattern with SYMMETRY).

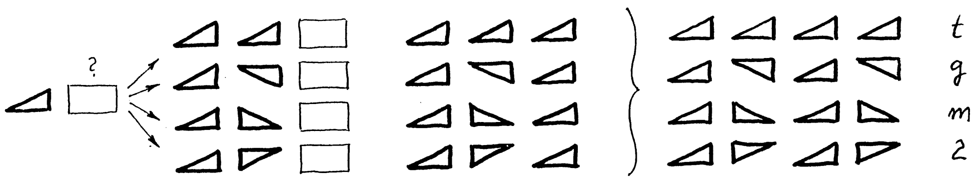
4. Four Local (Neighbourhood) Operations of the Plane which Generate Cell Mosaic Threads (the Equivalents of Friezes)

Let us reduce the great variety of forms (planar structures) to a skeletal unit of a triangle with right angle. This triangle is placed into a rectangle. Let us consider that the rectangle is the tiling unit and triangle on it is the repeating element of the pattern. Using this method *first we construct the frieze patterns along the line*, then we *extend* the produced frieze structure *by doubling* it into a double-thread pattern and finally we extend the "frieze-regular" multiplication of the initial thread onto the whole plane. The system we get forms a rich background for patterns of the Eurasian ornamental art.

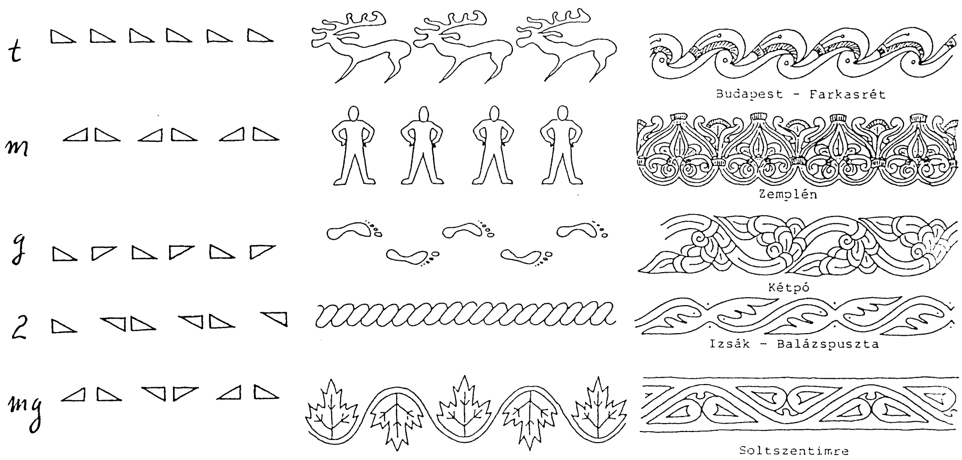
So: we have a 2D unit element: the rectangle with triangle in it. *There are four simple congruency operations which can multiple this unit along a thread in one direction*. They are the following ones: translation, mirror reflection, glide reflection and half turn (Fig. 3a). The basic frieze patterns in our system are those which were generated by these four congruency operations. We add plus one more frieze-thread to this set: that one which was their combination. This fifth type, a composite one, can be given as an alternation of half turn and mirror reflection: mg. This 5 types will form the basic set of elementary thread patterns. (Later we construct more complex patterns using this set.) We give the set with their skeletal frieze pattern, with natural-phenomena patterns and with patterns from Old-Hungarian (Carpathian Basin) ornamental art (Fig. 3b).

The first step for a thread to have connection to another direction is the doubling the thread. This operation builds another thread on one side of the initial thread. (Taking a metaphorical association from the world of molecules, this operation is a kind of "DNA-doubling" of the thread: doubling along one side, —in the plane, —with different kinds of doubling operations.) The doubling of the five threads also may use the earlier given four local neighbour generator. If we generate (like in technology we really do it) neighbour elements along one side of the basic frieze-thread we double it. Doubling means step by step (local) generation of the second frieze-thread. (With Celtic and Avar-Onogurian double frizese see Figs. 4a and 4b) We may form this neighbourhood thread by any of the four operations, so we have 20 double-frieze patterns. We may arrange them into a matrix according to the thread-generating (perpendicular) and the side-generating (or doubling: horizontal) local operations (Table. 1).

It is interesting imagine phenomena which may show double friezes. On remarkable sculptures of Parthian kings their dress exhibit a pair of wide ornamental frieze, running from top to bottom on their coat and trousers, one on the right and one on the left side (BAKAY, 1997, 1998) (Fig. 5).

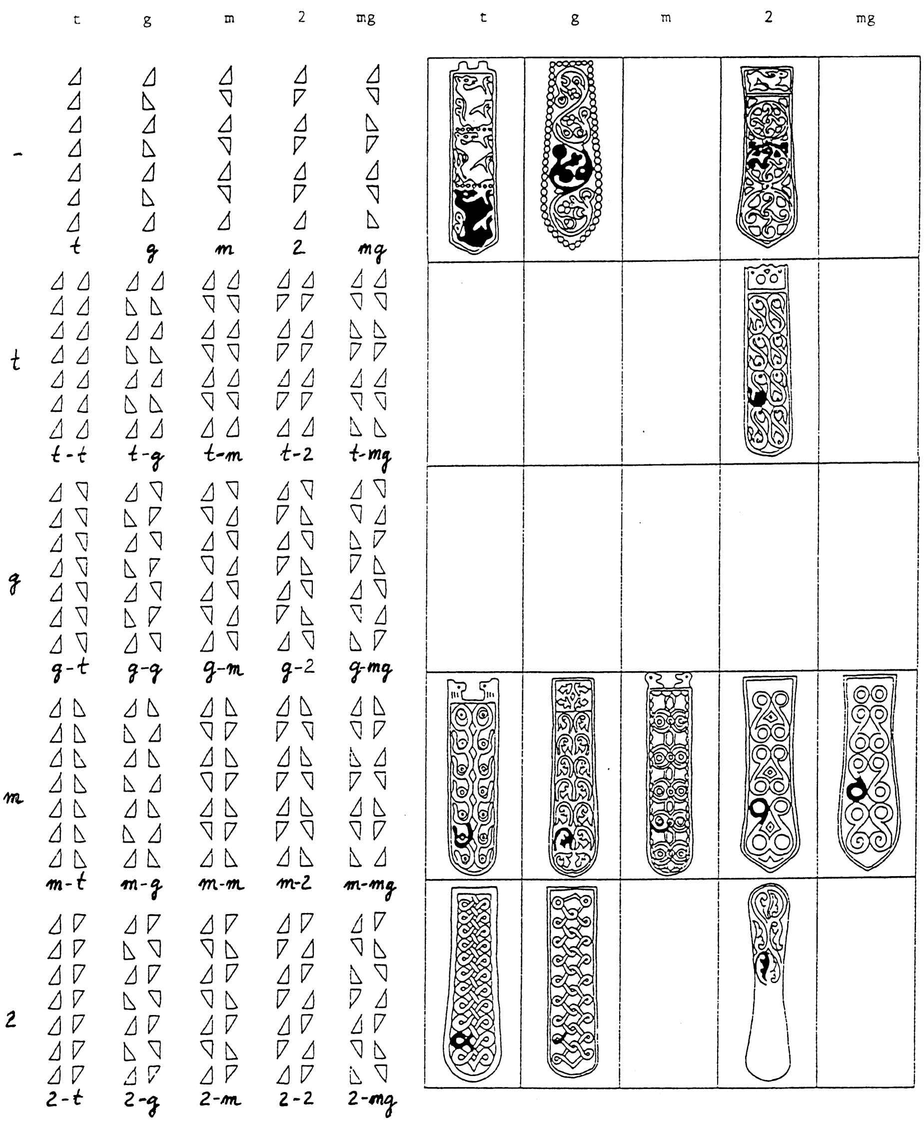


(a)



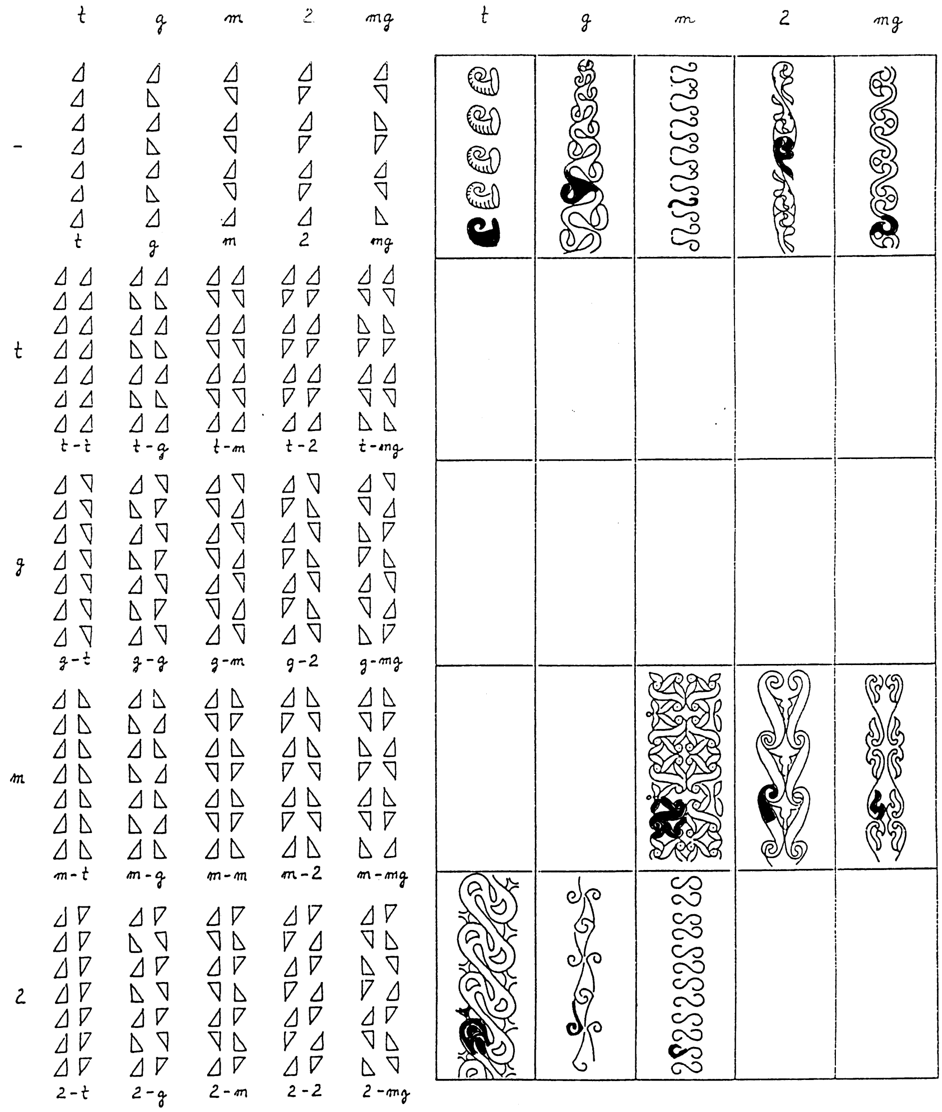
(b)

Fig. 3. The basic frieze patterns: a. friezes generated from a skeletal unit of a triangle with right angle: this tiling unit may be continued by translation, glide reflection, mirror reflection and half turn, b. the basic friezes and the only one complex mg type occurs in ornamental art of Old-Hungarians. We give them with skeleton, with natural phenomena pattern and one ornamental pattern.



(a)

Fig. 4. Double frieze sets: a. Avar-Onogurian, b. Celtic.



(b)

Fig. 4. (continued).

Table 1. Double friezes: columnar friezes are doubled by the horizontal operation.

	Doubling operation	t	g	m	2	mg
Translation	t	t-t	t-g	t-m	t-2	t-mg
Glide reflect	g	g-t	g-g	g-m	g-2	g-mg
Mirror reflect	m	m-t	m-g	m-m	m-2	m-mg
Half turn	2	2-t	2-g	2-m	2-2	2-mg

Example: 2-t means the order of the double frieze: 2 (= half turn) horizontally, t (= translation) vertically.

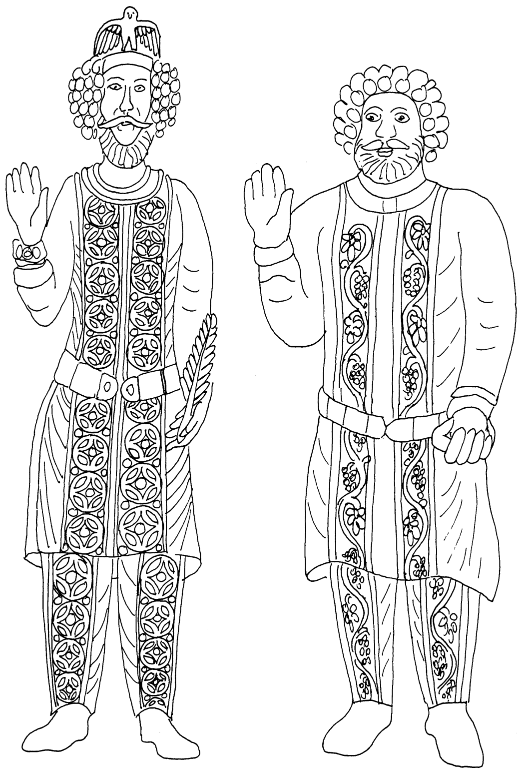


Fig. 5. Parthian kings wear on their dress a pair of ornamental frieze, running from top to bottom on their coat and trousers, one on the right and one on the left side. These cloths (coat edges in a mirror symmetric position) could give the idea to design double frieze patterns (Parthian art from Central Asia).

5. From Ornamental Threads toward the Ornamental Plane Patterns

Let us continue the operation of doubling in a "frieze-regular" way. This means that we multiple the double frieze-thread by adding (attaching) to it new and new copies of the initial thread in a frieze regular way. Frieze regular way means that the units along a line

Table 2. Correspondence of double friezes and woven wallpaper patterns given by their groups signs.

Plane symm. group / Double frieze		Columnar	Frieze types	Weaving	The planar	Pattern
	Weaving friezes	t	g	m	2	mg
Translation	t	p1 t-t	pg t-g	pm t-m	p2 t-2	pmg t-mg
Glide reflection	g	pg g-t	pgg g-g	cm g-m	pgg g-2	g X mg g-mg
Mirror reflection	m	pm m-t	cm m-g	pmm m-m	pmg m-2	cmm m-mg
Half turn	2	p2 2-t	pgg 2-g	pmg 2-m	p2 2-2	2 X mg 2-mg
Half turn + mirror refl.	mg	pmg	g X mg	cmm	2 X mg	mg X mg

Example: 2-t means the order of the double frieze: 2 (=half turn) horizontally, t (=translation) vertically.

perpendicular to the initial thread form also a basic frieze thread. Using this neighbourhood operation the pattern will extend on the plane. Therfore the matrix of double frieze-threads corresponds to a matrix of plane patterns (Table 2). There the plane symmetry patterns appear at the crossboxes of rows and columns. Completing this matrix with the fifth enlarging generator operation—the composite mg alternating operation—a fifth row is added to it and so the whole matrix becomes symmetric. In the matrix in Table 2 the matrix elements are given by the symbols of the double frieze patterns and of the plane symmetry groups, respectively. If we compare Table 2 matrix to Fig. 1 we can see that the first 9 plane symmmetry groups of the whole set of 17 appear in Table 2. The common characteristic of these nine types is that they all have rotational symmetry not higher than second order. The nine types are the following: p1, p2, pm, pg, cm, pmg, pgg, pmm and cmm. This system of construction from threads to plane patterns gives a rich background to study Eurasian ornamental art. We project some remarkable artworks on this background.

6. Scythian Tradition of Wallpaper Patterns on Textiles and Paintings

Applicated patterns were among the first ornaments by technologies, too. Application may result in two sharply different operation: overturned (backside up) forms may give mirror shapes as repeating element, but could give different colors, too. If the elements with similar orientation were repeated in a given background net, like as on curtains or shabrack applications of Pazyrik (Altai Mts.) or Kelermes (Pontus-Kuban steppe) Scythian kurgan excavations, they have the most simple p1 type wallpaper symmetry. Application is similar to coloring in that sense, that both coloring and application may reduce the symmetry of the background pattern. (On the Kelermes plane pattern the higher symmetry—cm—of the net was reduced by translational symmetry of the deers and goats—to p1.) Cm and cmm patterns frequently occur as background nets (Fig. 6). On the frescoes of Afrasiab pg type patterns were preferred (BAKAY, 1997, 1998, Fig. 7).

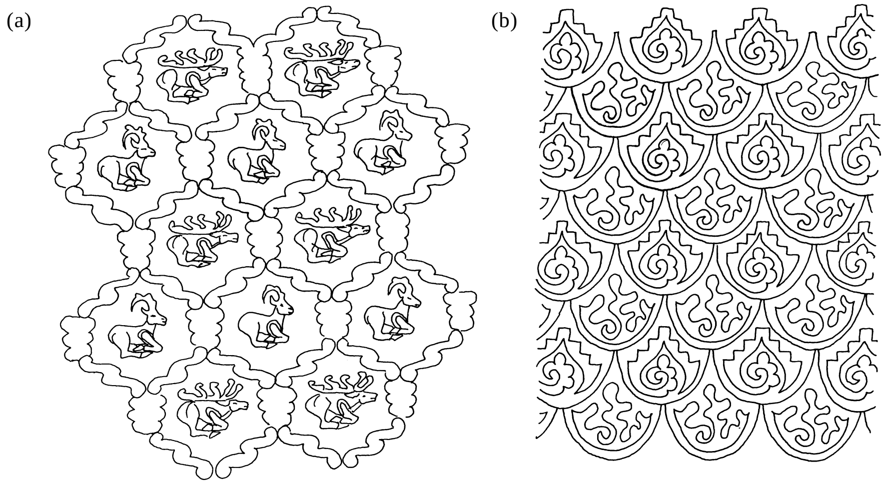


Fig. 6. Scythian wallpaper patterns from kurgan excavations: a. Kelermes, b. Pazyrik.

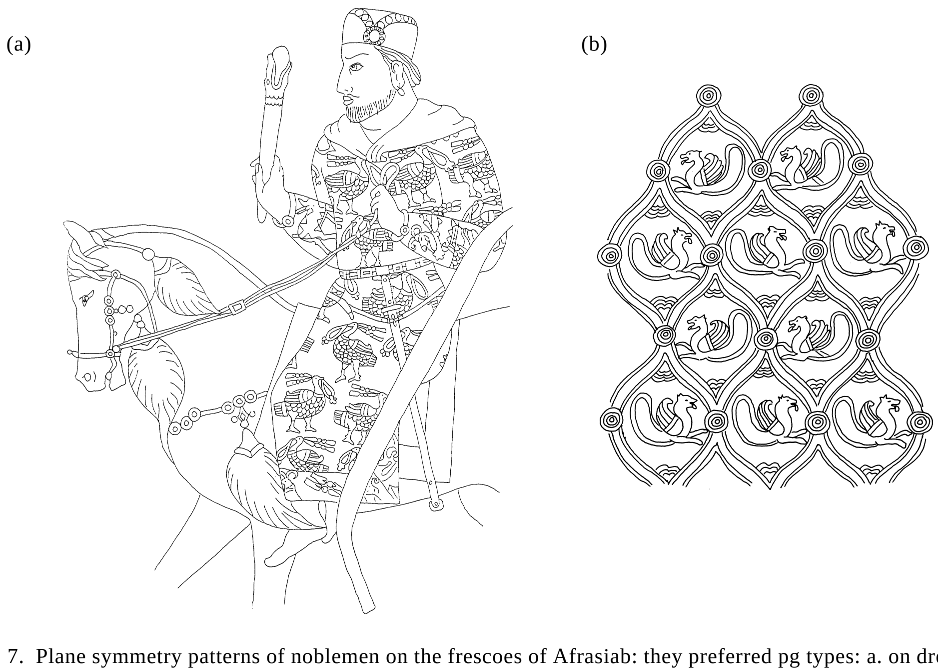


Fig. 7. Plane symmetry patterns of noblemen on the frescoes of Afrasiab: they preferred pg types: a. on dress of a rider, b. details of another fresco dress design (Sogdian art from Central Asia).

7. Reduction of Plane Patterns: Cutting out Double-Threads (Old Hungarian Art)

Comparison of Table 2 with Fig. 1 shows the redundancy of the plane symmetry groups with respect to the given classification of double frieze types. The redundancy is used in Fig. 8, where a family of patterns are arranged in a generating relations table. In the center is the popular double frieze of the Old-Hungarian art. It was both considered as bands cut out from a plane symmetry pattern, and as a double frieze pattern constructed from a

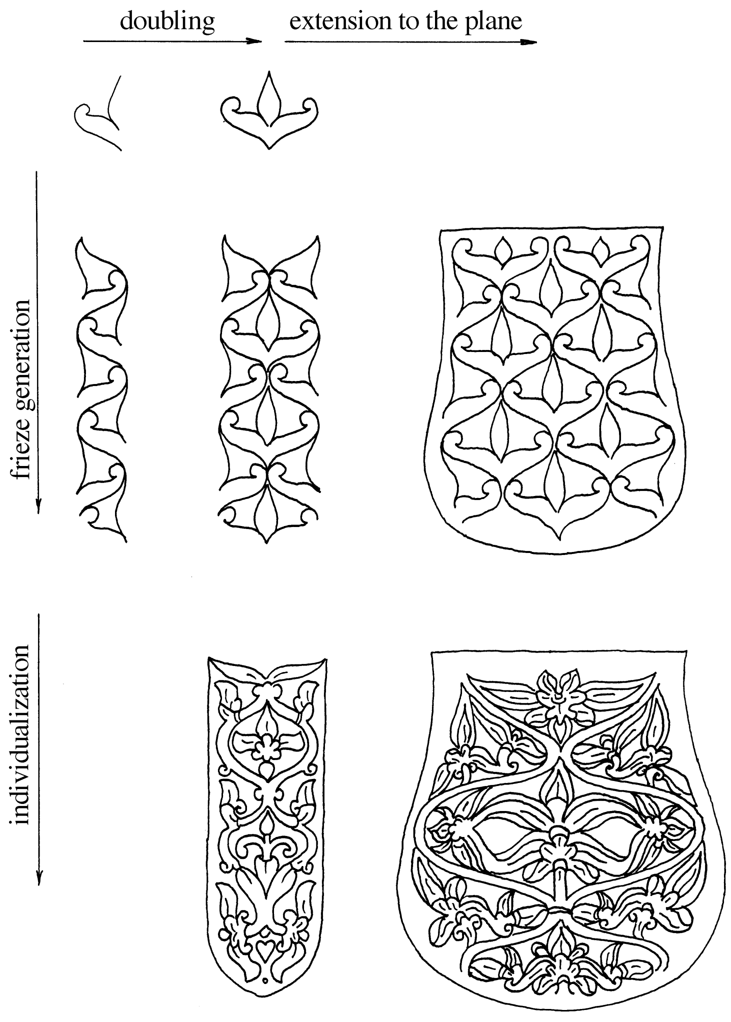


Fig. 8. Relation between constructional ways from simple to complex and back. m-g type double frieze can be cut out from cm type wallpaper pattern and can be built up from mirror symmetric palmettes or from the glide reflection generated frieze pattern by mirror reflection (Old-Hungarian art from the Volga-Kama region, Kaukazus and Carpathian Basin).

glide reflection generated basic frieze. This means that one kind of pattern discovered from technology could serve as the sample pattern for different constructional directions. This fact emphasizes the role of the classification of double friezes given earlier in the paper because it is more exact in distinguishing some ornamental frieze patterns than their plane symmetry patterns (they may be cut out from more then one of them).

A relation table arrangement of different ornamental patterns from the Old-Hungarian art shows that more then one way of constructional freedom exist from repeating units up to the plane patterns. (The mirror symmetric doubling of the g type frieze and its extension to cm type wallpaper patern was early recognized by LÁSZLÓ, 1943b.)

8. Unusual Constructions: Extension of the Set by Coloring (Hanti-Manysi Art)

In human dressing the coat is frequently adorned by ornaments. Running along the edges the frieze pattern is dublicated at the front edge of the coat (Fig. 5). This coat edge pairing and doubling resulted in mathematical invention in another communal art. This is the Hanti-Manysi Art in Western Siberia. Their color frieze mosaic structures have specific conditions, which we formulate shortly. 1. If no color is considered the Hanti-Manysi frieze pattern is formed by one single continuous line. 2. This line contains repeated congruent sections along the line. 3. If the half of the mosaic is regularly colored: the single line separates the pattern to two half-planes. On side of the half-plane is colored, other is white. The repeating congruent section became 2 times enlarged. The three basic cases shown on the figure exhibit the esentially inventious character (very similar to the Escherian mosaics) of the Hanti-Manysi art. The figure also points the fractal like development: a possibility involved in Hanty-Manysi art (BÉRCZI, 1985) (Fig. 9).

This extraordinary structure is alone in the steppe art, but double colored friezes occurred in some communal arts. The Greek, Scythian, Chinese and Hanti-Manysi arts were collected in Fig. 10. We can see that the rich set (COXETER, 1985a, 1985b) was only partly inhabited by ornamental double colored frieze constructions. The more complex set of patterns is constructed, the more easy the distinction is to find intuitive mathematical discoveries in different communal arts.

9. Two Intuitive Summaries in Archaeology and Architecture of Hungary

The occurrence of two different double frieze patterns on the Szeged-Kundomb belt mount makes this object interesting to analyse. The 10 cm long object contains 12 repeated (flower like) units. From intuitive mathematical point of view it is a masterpiece because the goldsmith could prove us that he knows well the four basic congruencies in the plane. Moreover, he used them both in the construction of the frieze threads (g, 2) and doubling (m, 2). There are other relations between the double frieze patterns, and the elements on the object which are left for pleasure of reader to discover (Fig. 11a).

I have found only one similarly intuitive summary of pattern knowledge in architecture of Romanesque art of Hungary. This is a church window tympanon in Halmágy, Alsó-Fehér County, Transylvania, from the XIIIth C.A.D. On it there are four archs of friezes, all different. From the outer one they are g, mg, t, and m types, and because mg contains half turn this object also exhibits the knowledge of all basic frieze types on one place (Fig. 11b).

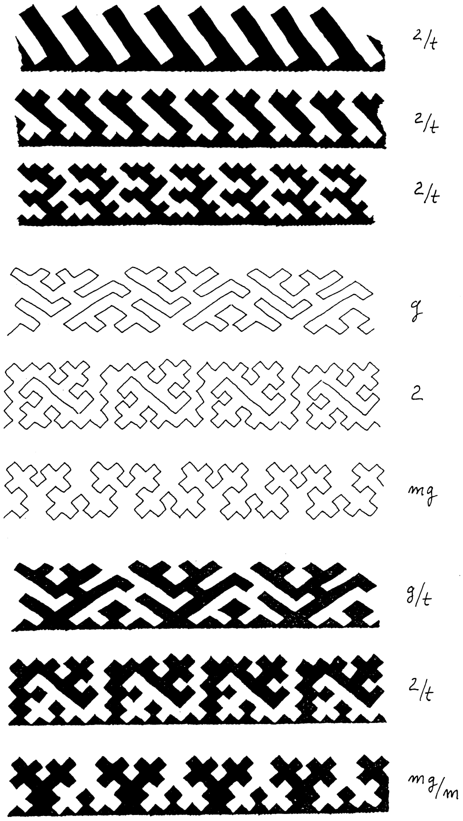


Fig. 9. Hanti-Manysi color frieze mosaics: a. their fractal developments, b. without coloring a single line forms the frieze pattern with repeating sections, after coloring this line separates the colored and uncolored half planes (Western Siberian art).

10. Whirls, Weaves, Fractals: Celtic, Viking and Japanese Art

Whirls in streams, animals in sinusoid motion (lizards, serpents) and rolling fighting animals were favourite dynamic phenomena of the arts all over Eurasia. In the Western parts many of these scenarios (especially those with reptiles) metaphorically represented the world beneath us: darkness, the waters and death. Especially great number of serpents and dragons appear in Viking and Celtic art. In the Eastern part of Eurasia the dragons are friendly spirits. Sometimes they arose on the crests of weaves in Japan (BÉRCZI *et al.*, 1999). Seaside communities liked these whirls and weaves and formed remarkable patterns from them. As a closing example we show two examples from the fractal-like developments. We have seen the connections here of the Hanti-Manysi art (Fig. 9). The Celtic mirror from Desborough, Great Britain, and Hokusai's weaves from Japan both shows the fractal character of the sea, also proving the role of natural forms in directing intuition of artistic design all over Eurasia (Fig. 12).

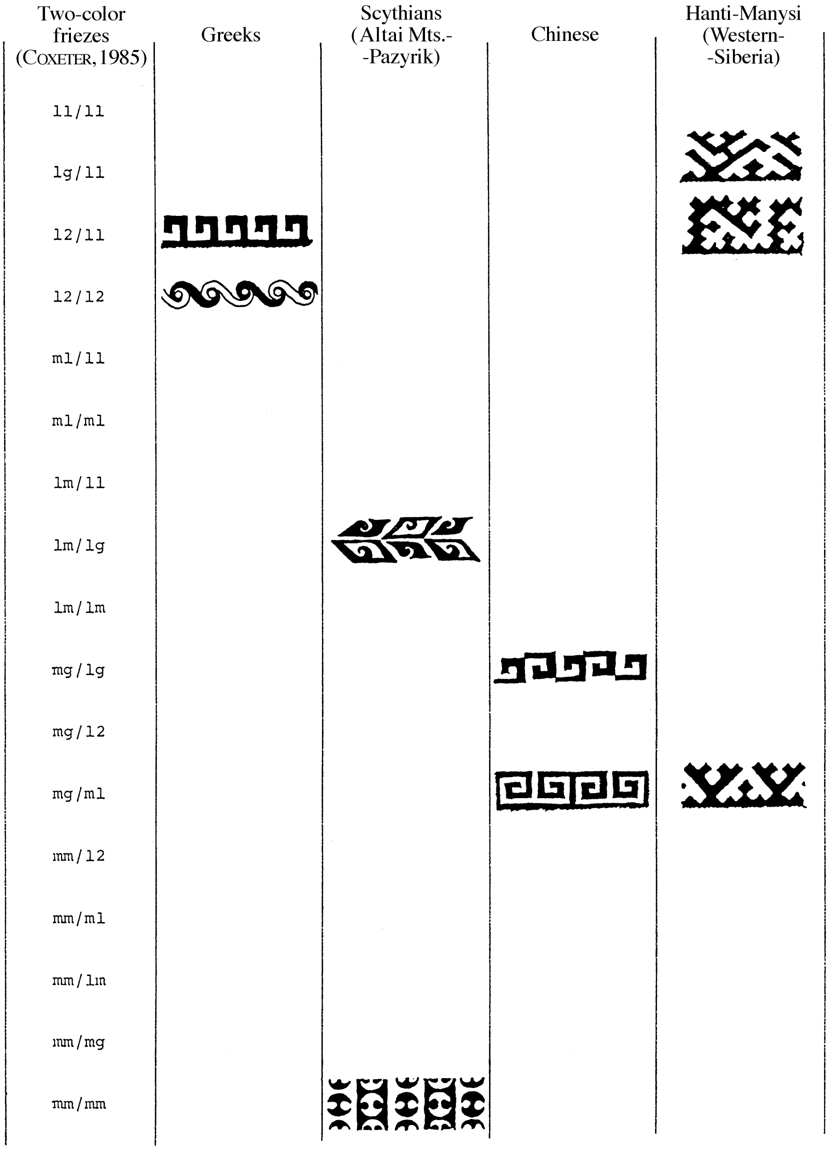
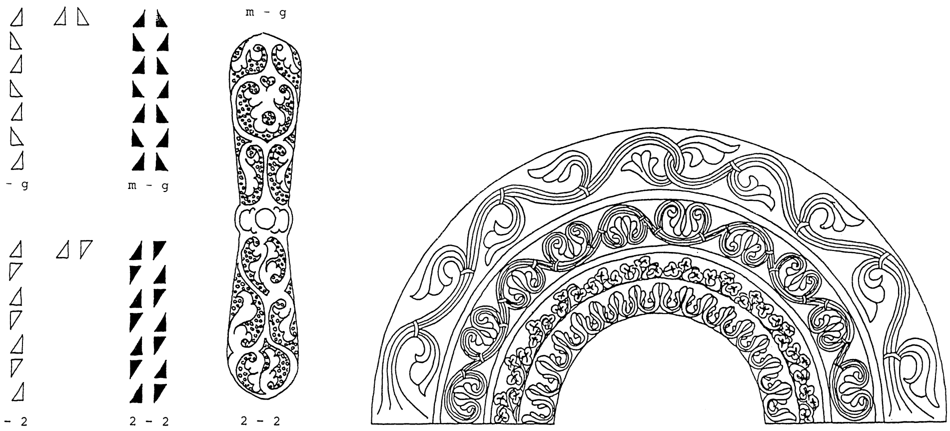
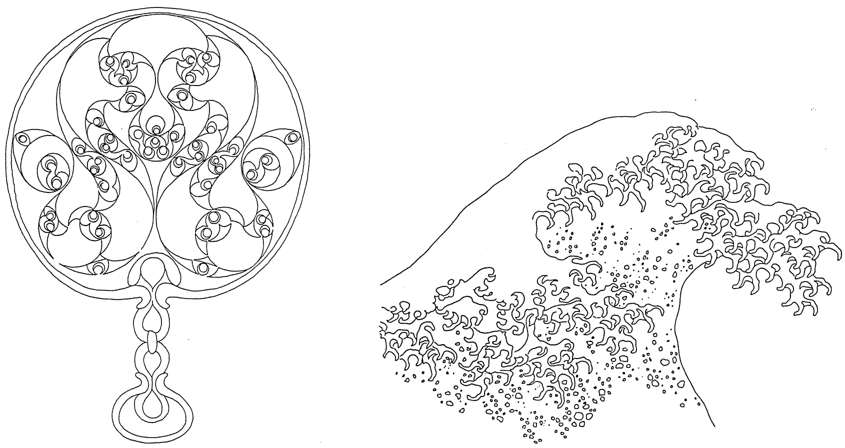


Fig. 10. Double-color friezes in some Eurasian arts: Greek, Scythian, Chinese, Hanti-Manysi.



(a) (b)

Fig. 11. Two interesting finds about the concentrated representation of friezes: the patterm knowledge was exhibited: a. the biscuit formed belt mount of Szeged-Kundomb, Csongrád C. from the 7–8. C.A.D., b. the window tympanon from Halmágy, Alsó-Fehér C., Transylvania, 12–13. C.A.D. (Carpathian Basin art).



(a) (b)

Fig. 12. Weaving and whirling of the sea was the common natural origin of fractal arts we can recognize in the Desborough mirror design in the Celtic past of Great Britain, Western Eurasia, and Hokusai's beautiful fractal waeves in Japan, Eastern Eurasia.

11. Summary

We have seen in artworks that in many communities that operations with technological roots became triggers of intuitive design. By them patterns were generated from pre-existing ones reduction, extension and coloring. Recognizing the complementary nature of katachi and symmetry in respect of time and space we constructed a new pattern building and extending algorithm. First basic friezes were developed by local operations, then their doubling (also by local operations) extended them. We formed the basic set of these doubled and extended patterns and projected the rich heritage of steppe art onto this mathematical background. In Celtic, Viking, Scythian, Avar-Onogurian, Old-Hungarian arts, Japan, China, Parthia, Sogdiana, and the Western Siberian Hanti-Manysi arts and that of classical Greeks we found rich sources of intuitive pattern discoveries which need further years of studies with introducing new and new mathematical pattern construction recognitions (JABLAN, 1995, 1999).

REFERENCES

PRINCE MIKASA TAKAHITO (1996) Development and modification of the "Holy Symmetrical Design" West and East of the "Silk Road", in *Katachi U Symmetry* (eds. T. Ogawa, K. Miura, T. Masunari and D. Nagy), Springer, Tokyo, pp. 81–94.

BAKAY, K. (1997, 1998) *Östörténetünk Régészeti Forrásai I, II* (*The Sources of the Hungarian History*), Miskolci Bölcsész Egyesület, Miskolc.

BÉRCZI, Sz. (1985) Escherian and Non-Escherian Developments of New Frieze Types in Hanti and Old Hungarian Communal Art, Lect on *Escher Congress*, Rome, 1985 (Publ. in *M. C. Escher*: *Art and Science* (szerk. H. S. M. COXETER *et al*.) pp. 349–358, old North-Holland, Amsterdam, 1986).

BÉRCZI, Sz. (1989) Symmetry and technology in ornamental art of old Hungarians and Avar-Onogurians from the archaeological finds of the Carpathian basin, seventh to tenth century A. D., in *Symmetry 2* (ed. I. Hargittai,. *Computers Math. Applic. CAMWA*, **17**, No. 4–6, 715–730, Pergamon Press, Oxford.

BÉRCZI, Sz. (1990) *Symmetry and Structure Building*, Tankönyvkiadó, Budapest (in Hungarian).

BÉRCZI, Sz., KABAI, S. and PATAKI, T. (1999) Role of Katachi in Development of Abstract Design, Exhib. and Lect. on *KUS2 Symposium*, Tsukuba, 1999 Nov. 17.

COXETER, H. S. M. (1985a) A Simple introduction to colored symmetry, *Internat. Journal of Quantum Chemistry*, **31**, 455–461.

COXETER, H. S. M. (1985b) The seventeen black and white frieze types, *C. R. Math. Acad. Sci. Canada*, **7.5**, 327–332.

CROWE, D. (1975) The geometry of African art II, *Historia Mathematica*, **2**, 253–271

JABLAN, S. (1995) *Theory of Symmetry and Ornament*, Mathematical Institute, Belgrade.

JABLAN, S. (1999) Are Borromean Links So Rare?, Lect. on *KUS2 Symposium*, Tsukuba, 1999 Nov. 15.

LÁSZLÓ GY. (1943a) *The Horse Mount of the Statue of St. George Made by the Kolozsvári Brothers*, Egyetemi Nyomda, Kolozsvár (in Hungarian).

LÁSZLÓ GY. (1943b) Der grabfund von Koroncó und der altungarische Sattel, *Archaeologia Hungarica*, **XXVII**, Budapest.

NAGY, D. (1979) Symmetry and order aspects in art, *System Theory Conf*., NJSZT, 92–118, Sopron.

SENECHAL, M. (1989) Symmetry revisited, in *Symmetry 2* (ed. I. Hargittai), *Computers Math. Applic. CAMWA*, **17**, No. 1–3, Pergamon Press, Oxford, p. 12.

URABE, T. (1999) Seventeen kinds of wallpaper patterns and traditional Japanese patterns, Lect. on *KUS2 Symposium*, Tsukuba, 1999 Nov. 15.

WASHBURN, D. K. and CROWE, K. W. (1988) *Symmetry of Cultures*, University of Washington Press.

YANABU, A. (1999) Katachi versus Symmetry, Lect. on *KUS2 Symposium*, Tsukuba, 1999 Nov. 15.