

Ordering Disorder after K. L. Wolf

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First of all there is physical change... The succession of equilibria is the interplay of the laws of nature, which is really only one law: Order itself. It is never chaotic. It is simply changing. Every change is equally in equilibrium as the one that proceeded it...

Change, nature's concern, is nonconscious.

Change in man is conscious. It has to do with rule and not law.

Rule is made to be changed; the essence of rule is to find a greater rule which can hold people's sensitivities together... The longer it lasts, the stronger is the rule, but the welcome of the new rule is tremendous (WURMAN, 1986).

This quotation is from a publication of mostly recorded talks, extemporaneously delivered by Louis I. Kahn (1901–1974), architecture's most influential and enduring thinker, not to slight practitioner, of the last half of the 20th century. Kahn's concept of *chaos*, touched upon here, is more fully developed in another quotation, met in a different publication. In the context of ordinary language, I know of no better characterization of chaos than it:

Nature cannot change its laws. If it did, there would be no Order whatsoever. There would be what we think is chaos (LOBELL, 1979).

Kahn's use of the word *Order* (usually capitalized) requires special attention. To him, as made clear in the first quotation, *Order* is equivalent to *natural law*. Not only is the consummately symmetrical crystal an integral part of Order, but the cataclysmically exploding star is no less so. Kahn refers to *Order* as "the possibility to be," vis-à-vis his word *Desire*¹, "the will to be": From the macro- to the microcosmic scale, the dual components of *Order* and *Desire*—when, and only when, concordant—beget *Existence*.

These corollaries may be construed from Kahn's contemplation on men's rules:

There are no theories of art (or architecture), only programs—open programs—that can lead to evolving styles of individual artists (Palladio, Mondrian) or to

styles of long-lasting eras of cultural (Romanesque), as well as of short-lived movements (Art Nouveau).

Theories are built on laws of nature. Programs are built on rules of men².

From a set of mindfully chosen rules, the open program evolves by jettisoning spent rules, rule by rule, and wresting, out of need, rules with new promise, rule by rule.

On the one hand, the contemplation on and articulation of laws of geometry and mathematics and, on the other hand, the search for and application of well formulated rules for designing (derived, as Kahn recognizes, from “nature’s laws”) are what largely engage interdisciplinary groups such as represented by Katachi (The Society for Science on Form) and ISIS-Symmetry (International Society for the Interdisciplinary Study of Symmetry). These shared efforts, however, must not be dissipated on any attempt to reintegrate science and art in the manner that human curiosity and human strivings seemed (from present-day perspective, at least) to have been organized through the ages (up through the Early Renaissance) under one unified entity, *philosophy*; for when science was born with Galileo, science separated irreparably from art. (In other words, don’t be looking for any more Leonardos.) What Katachi U Symmetry, among a host of allied efforts, can facilitate is the ageless transfer between *knowledge* and *practice*, now bifurcated into separate branches of human endeavor that are not equivalents (*dissymmetric* congruents—as two halves of a whole), but reciprocals (*antisymmetric* duals—as the yin and yang.)³

If the *knowledge hunters*—scientists—and the *application exploiters*—artists—are to make strides in an exchange of insights into *Order* (i.e., the gamut of “the possibility to be,” as defined by Kahn), they would do well to strive for common ground; and an important imperative of common ground is common terminology. Today, for instance, when designers describe repetitive patterns, they do best by using the nontechnical language of the crystallographer. Recently, I reviewed the catalog entries of a reputed art historian on the basketry of American Indians. A regular pattern on one basket—a *quincuncial* arrangement (WEYL, 1952), whose international designation is *cm*—was described by the expert as a “scatter pattern”—of all expressions! Obviously he had not acquainted himself with Hermann WEYL (1952) or with WASHBURN and CROWE (1988). WEYL, of course, as SHUBNIKOV and KOPTSIK (1974), has helped immensely in making the subject of symmetry accessible to all in a nontechnical, but specific language. I am certain that almost all who are engaged in Katachi U Symmetry are comfortable with that language and find it to be exceedingly workable.

In many of these recently organized interdisciplinary associations, the memberships have moved from a focus on rigid regularities—passionate preoccupations with crystals and their models, the regular and semiregular polyhedra⁴—to lesser regularities. However, I have noticed that, while many scientist and artist do now speak in shared terms about highly regular symmetries, neither employs a comprehensive, much less unambiguous, terminology about entities that *seem* to be less than symmetrical. We use and hear such words as “disorder,” “broken symmetry,” “irregularity,” “randomness,” and even “chaos.” Many things that are called “broken symmetry” and the like are, in fact, not. Rather they

are often lesser symmetries—that is, *orders* or *degrees* (in other words, different *levels*) of symmetry that are lower than *isometry*. Therefore, I am urging the universal adoption of the classification of the *lesser symmetries* and, in fact, the whole range of *structure*, as formulated by the German chemist K. L. WOLF (1901–1969), an authority on molecular structure.

The levels of symmetry of Wolf's hierarchic classification (WOLF and WOLFF, 1956)

Table 1. Levels of structure, after Wolf and Wolff, *Symmetrie*.
(I = invariant; V = variant)

	position	size	angle	shape	certainty	rule
autometry	I	I	I	I	I	I
isometry	V	I	I	I	I	I
homoeometry		V	I	I	I	I
syngenometry			V	I	I	I
katametry				V	I	I
hypometry					V	I
heterometry						V
ametry						

Table 2. A typology of mapping, reconstituted from March and Steadman, *The Geometry of Environment*.

mapping	invariant	position	length	angle and ratio	parallelism	cross-ratio	neighborliness
identity	•	•	•	•	•	•	•
isometry		•	•	•	•	•	•
similarity			•	•	•	•	•
affinity				•	•	•	•
perspectivity					•	•	•
topology							•

are these: *isometry*, *homoeometry*, *syngenometry*, *katametry*—four levels of *diminished invariances*. Below these four, Wolf placed *heterometry* and *ametry*—two levels of structure with no pervasive invariances.

Similar to a table (Table 1) that I had earlier construed out of Wolf's text—where he had advanced his classification of structure—a table (Table 2) was published by MARCH and STEADMAN (1974), short of two decades after the publication of Wolf's *Symmetrie*. March and Steadman's hierarchic typology, though quite similar to Wolf's, is intended to serve a somewhat different, yet allied, purpose: classifying the modalities of the two-dimensional representation of two-and three-dimensional figures—termed “mapping” by its authors. Among many striking similarities, the differences between the two constructs suggest that each was independently developed.

My attention was first drawn to a category in the March and Steadman table that seemed to vindicated a point that I had pressed during my first meeting with Wolf in 1965; for when I put together my table, based on Wolf, it struck me that there was a fifth invariance atop his four lessening levels of invariances: the absolute invariance of *identity*. Wolf's first response was that *identity* was merely an *operation* (*eine Deckoperation*)—not a full-blown type of structure. However, in not dismissing my point outright, Wolf said that he would reconsider it and get back to me. He never did get back, and he died four years after this meeting; therefore, the welcomed impression of vindication, albeit tenuous, when I found that March and Steadman had placed *identity* at the top of their table. In deference to Wolf, I have replaced *identity* with the pseudoGreco-term *autometry*⁵.

Though March and Steadman's focus, as noted above, was on a classification of mapping types, while Wolf's focus was on a classification of levels of structure (*Grade der Artverwandtschaft geometrischer Gebilde*), a comparison of the two tables is informative. Wolf's *isometry* corresponds well enough with March and Steadman's *isometry*, keeping in mind the differences between their respective objectives; and Wolf's *homoeometry* corresponds, in a like manner, with March and Steadman's *similarity*. It is at Wolf's third level, *syngenometry*, that the March and Steadman table becomes especially instructive. Actually, Wolf's *syngenometry* corresponds to March and Steadman's *affinity*. But Wolf's definition of *syngenometry* is too narrow in his incorporation of *only* affinity; it excludes types of structures that retain essential properties—ones that should preclude them from being demoted to Wolf's fourth level, *katametry*.

Linked to the historical development of descriptive and projective geometry, *perspectivity* and *topology* are treated by March and Steadman in their table, after *affinity*, as two additionally distinct mapping types. All three, however, can be neatly folded, as a hierarchy of subtypes, into Wolf's *syngenometry*. March and Steadman made the germane point that “an affine projection is a special case of *perspectivity* when the center of the perspective is at infinity” and, further, that projections cast “onto curved or irregular surfaces produce topological transformations.” Clearly, then, March and Steadman's justly differentiated mapping types, *perspectivity* and *topology*, belong, along with *affinity*, in a wider circumscription of Wolf's *syngenometry*.

Affine projections, perspectivity, and topological transformations, however, to be consistent with Wolf's definition of *syngenometry*—as types of symmetry vis à vis types of mapping—must be serial in nature, not just any embodiment at all of the three projective types. An analogous example serves to clarify: If there are a number of congruent figures

in any number of random arrangements, their instances will not constitute symmetric (i.e., isometric) structures. Only when these figures are arranged such that they conform to one or another of the several organs of symmetry (mirror axes, rotational axes, translational vectors, etc.) will they meet the conditions of isometry. In like manner, *continuous deformation*, both spatial and formal, is the common *operative* element of serially organized structures that would validate any one of these three subtypes as a member of Wolf's third level of symmetry, i.e., *syngenometry*.

The convergence of Wolf's classifications with March and Steadman's classifications ceases here: Wolf's next categories, *katametry*, *heterometry*, and *ametry*, have no practical place in March and Steadman's typology of the mapping of figures—where, by definition, general shape is conserved. Imperatively, general shape is *not* conserved in Wolf's last three levels.

Wolf's *katametry* (literally, low measure), the lowest level of symmetric structure, prompts diverse comments. For one, the remaining invariant at this level, after the distinguishing invariances of position, size, angle, and shape (topological) have been stripped away, is *rule of relationship*—that is, a rigorous program that fully determines a structure. There are simple patterns that are immediately detectable to the eye: Wolf's clear-cut series of polygons, beginning with an equilateral triangle and followed by a square, a regular pentagon, a regular hexagon, etc. That *et cetera* is obvious; one easily knows what is next to come by the application of the unmistakable rules (add one new same-sized side to the last generated figure; have all sides circumscribe a circle): a regular heptagon, then a regular octagon, etc., etc. But there are structures whose steadfast rules are not perceptibly discernible. One of the oldest and grandest games of man was to determine the rules of the seven magical moving bodies of the heavens (so important, that our week of the seven deities—the days—was born of it). The ultimate determination of the rules of those heavenly bodies, so confounding over the ages, was the foundation of science. Despite the neat isometric fit of such a science as crystallography, the epitome of the highest level of orderliness, *katametry*, the lowest level of invariance, is where much of science abides.

Hybrid symmetries take their proper place in Wolf's katametric category. A spiral of Archimedes fits this characterization: A structure that rotates and *dilatates* outwardly from a center at an *arithmetic* rate of growth is homoeometric in its unending expansion and isometric in its constant expansion of 1: the structure as a whole is, in the end, neither an *isometry* nor an *homoeometry*, but an hybrid, which can only be admitted into Wolf's classification at the level of *katametry*.

An analogy is offered as an argument for the classification of such hybrids—that is, structures that conflict, with respect to symmetry, in their being invested with characteristics of two or more different levels of symmetry: Consider a group of six pointed stars aligned on a square grid. The resulting periodic pattern type is neither *p6mm* (improper 6-fold), signaled, but not ratified, by the potential of the regular hexagram, nor *p4mm* (improper 4-fold), signaled, but not ratified, by the potential of the square grid—both pattern types accommodating a large count of symmetric properties—but merely *p2mm* (improper 2-fold)—a pattern type with a relatively small count of symmetries.

In the same vein, as above, any structure with instances of modules of symmetry, even predominating instances, that are in no way *regularly* organized (i.e., by rule), must be

classified as *heterometry*. Further commentary on *heterometry* (otherwise, asymmetry) seems to be scarcely necessary in this discourse.

The *programmed design* is a staple assignment of my formative design studio in architectural studies. While the succinct brief for the assignment permits anything from *isometry* on down Wolf's scale of symmetric structures to *katametry*, the exploration of the lowest level has led to some of the more striking results: designs that, determined by indiscernible, though not overly complex rules—indeed, the leaner the rules, the more fulfilling the provocative outcome—appear to be casual or capricious. Programmed randomness? A contradiction, indeed! That is to say, what is often perceived as random may not be random at all, but wholly programmed (e.g., the timing of eclipses).

A few years ago while working with the programmed design assignment in studio, one student's solution brought about the designation of a new category that begged insertion into Wolf's construct: *hypometry*—to be located beneath *katametry*, at which rule is fully determinate, and above *heterometry*, at which there is no integral rule. The student initiated, as required, a program that consisted of a small set of planar pieces and a strict set of rules. This did not lead, however, to only one inevitable arrangement, but to different arrangements—probably an infinite number of them. Such seems to be the case of quasicrystals and Penrose tiling—not regular, yet not haphazard. That is, not just anything goes. Like Penrose tiling, there were stringent limitations in the generation of this student's design, but *some* choice (or chance) was still operative in regard to which would be the next piece to be fitted into the spreading pattern. At the hypometric level, rules still organize a general determination of the structure; but the ultimate outcome is one of *uncertainty*; and there are, therefore, alternative outcomes that are multitudinous in number.

It is clear that Wolf's lowest level of structure, *ametry*, is not a characterization of chaos. Wolf's concept of *ametry* does not have the sweep of Kahn's chaos; Wolf's *ametry* is something else. Though Wolf considered *ametry* to be hypothetical and not achievable, it seems, from his own abstruse description (1956),⁶ that he has portrayed *a field*, an endless entity without figure: a blue sky without a cloud; a sea of grass without an intrusive weed.

The universal adoption of Wolf's classification of structure is urged in order to address the phenomena of lesser forms of orderliness in a more savvy way than resorting to a babel of imprecise synonyms for them. It is to be recognized that beyond absolute regularity there are nuances of decreasing regularities, of reductive invariances: that there is an order to so-called *disorder*.

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Notes

1. Kahn uses interchangeably the word *Psyche*; *Desire*, especially for the purpose here, is more explicit.
2. I have been perplexed at the readiness of colleagues in design to speak about “design theory.” If there truly is a theory of design (or of art), then, those who advance Creationism as an alternative to the Theory of Evolution cannot be denied—they would have a point.
3. This paper from a designer, for example, does not meet scientific standards, but is offered in the hope that observations, cultivated in design, can contribute to knowledge. So too with Architect Kahn’s insights.
4. D’Arcy Thompson (1942) called an undue fixation with these things “inexcusable Pythagoreanism.”
5. *Autometry* and *hypometry* were invented for me by mathematician George Balgoglou, SUNY Oswego.
6. “If—as in the case of *identical sameness*, which in reality is executable only as an idea—there would be a condition where no recognizable Gestalt (or figure) is left in the rudimentary pattern, then, we would have amorphous structure, which is lacking inter-figural and intra-figural shape relationships. This would be—only in concept, as it were—the case of genuine *ametry*.”