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A Many-Body Disk Model of Slip Phenomena and Disk Contact Networks—A Spin System with Frustration on Irregular Lattices

Takayuki HIRATA[1], Hideki GOTO, Tatsuaki YOSHIMURA, Atsushi OGAWA and Yoshifumi HARADA

Department of Human and Artificial Intelligent Systems, Faculty of Engineering, Fukui University, 3-9-1 Bunkyo, Fukui 910-8507, Japan
[1]*E-mail: d970062@icpc00.icpc.fukui-u.ac.jp*

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Abstract. A many-body disk model of slip phenomena that was one of the system having frustrated states was investigated from the viewpoint of disk contact networks. Experiments were carried out by using acrylic resin disks packed into an annular cell that consisted of an outer rotating cylinder and an inner fixed cylinder. The time series of torque at the inner cylinder was measured as a macroscopic behavior of the system, and the motion of individual disks was monitored by a CCD video camera recorder as a microscopic elementary process. Distributions of *n*-path polygon loops in the disk contact networks and the coordination number of the disks were obtained.

1. Introduction

Sliding friction has been attracting a great deal of attention from scientists (PERSSON, 1998). A large earthquake is an example of frictional sliding. In earthquake faults, there is a fault gouge between fault surfaces, which plays an important role in fault sliding. In general, some fragments generated by wearing lie between slip surfaces. Experiments with granular materials showed that stick-slip occurs (NASUNO *et al.*, 1997; CLAUDIN and BOUCHAUD, 1998). Stick-slip phenomena also emerge in the many-body disk model.

A many-body disk model of slip phenomena has $1/f$ fluctuation that is one of the most exciting topics in nonlinear physics (CLAEYS and SIMOEN, 1997). $1/f$ fluctuation is not yet well understood, and there is no general model to explain its occurrence. Self-organized criticality (SOC) where the power law size distribution appears is also an exciting topics in nonlinear physics (BAK *et al.*, 1987). The size distribution of stick-slip events of the many-body disk model also obeys a power law indicating a SOC phenomenon (HIRATA, 2000).

In this study, we constructed the disk contact networks by analyzing the video image data. Our aim is to understand macroscopic behaviors based on a microscopic elementary process.

2. A Many-Body Disk Model and Experiments

The annular cell that consists of the outer rotating cylinder and the inner fixed cylinder is shown in Fig. 1. One hundred and forty-eight or one hundred and fifty-two disks were packed between two co-axial cylinders with 70 mm and 140 mm in diameter at the packing fractions 0.74 or 0.76, respectively. Each disk consisted of an acrylic resin cylinder with a diameter of 8 mm and a height of 10 mm. A rubber belt with a thickness of 6.0 mm was mounted on the inner cylinder in order to make contact with each disk.

The outer cylinder was rotated by a motor that was controlled in units of 1.0×10^{-3} degrees. The experiments were carried out at a constant rotation rate of 1/80 Hz (angular frequency/ 2π) for a period of 5000 s. The torque at the inner fixed cylinder was measured as a time-series. The maximum frequency response of the torque sensor and its amplifier was 3 kHz. The linearity of the torque measurement was ±1% The output of the torque measurement system was fed into a digital voltmeter connected to a PC via a GP-IB. The sampling frequency of the digital voltmeter was 10 Hz. The time series of torque observed at the experiment is shown in Fig. 2, in which $1/f$ fluctuation is observed (HIRATA, 1999).

3. Disk Contact Networks

A frustrated state is one characteristic of the many-body disk model of slip phenomena; a disk whose direction of rotation cannot be determined only by examining the directions of rotation of neighboring disks under shear stress is known as a frustrated disk. The frustrated state of the disks depends on the disk contact network which is defined as a

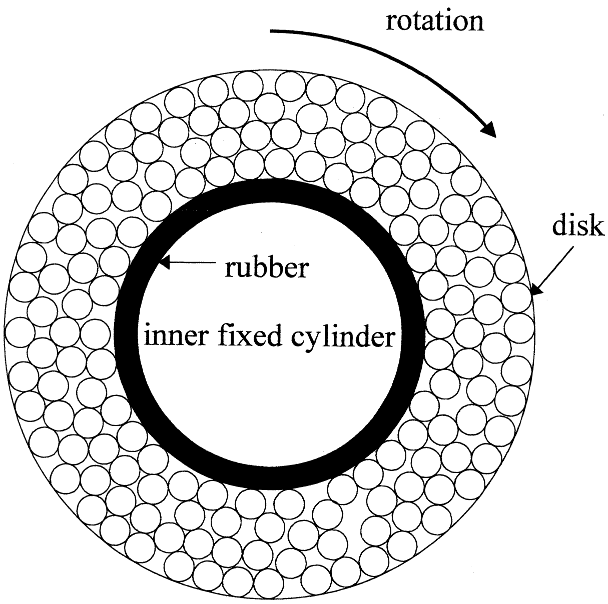

Fig. 1. Experimental setup. An annular cell consists of the outer rotating cylinder and inner fixed cylinder.

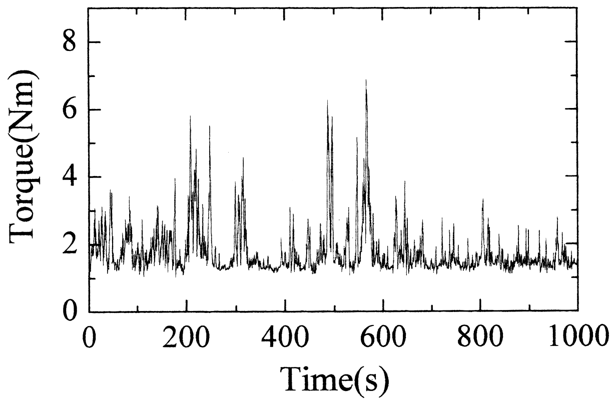

Fig. 2. Time series of the torque measured at the inner fixed cylinder.

network generated by connecting the centers of the disks in contact. Since the disk configuration changed during experiments, the disk contact networks varied dynamically.

Let us consider a relationship between the behavior of individual disks and the disk contact network. For simplicity, individual disks are assumed to have two states: clockwise and anticlockwise rotations. A frustration occurs in an odd path loop in the disk contact network. For example, in a triangular network, all disks are in frustrated state. This is equivalent to a system of Ising spins with anti-ferromagnetic interaction on a triangular lattice. The frustrated disks, whose number depends on the topology of the disk contact network, is considered to govern the resistibility to shear stress. Therefore, the amplitude of fluctuation of stick-slip is related to the resistibility of the many-body disk system to shear stress.

In anti-ferromagnetic spin systems on irregular lattices, the Hamiltonian (total energy) of the system is defined by

$$H = -\sum_{n.n} J_{ij} S_i S_j \tag{1}$$

where J_{ij} is –1 as the interaction is anti-ferromagnetic type. To obtain the disk contact network is the first step to investigate the spin system on irregular lattices. HIRATA and YOSHIMURA (I999) pointed out that the minimum value of total energy strongly depended on networks, and a migration of disk caused a large change of networks. Evolution of disk contact networks, which is related to the time series of torque, along with the viewpoint of the Ising spin system on irregular lattices, will give us a clue for understanding the many-body disk model.

The disk contact networks are shown in Fig. 3. As the disks were packed into the annular cell randomly, the disk contact networks included various n-path polygon loops. Five hundred and one disk contact networks were obtained by analyzing the five thousand second video image data at every 10 second. The number of n-path polygon loops included in the networks was counted. Distribution of n-path polygon loops averaged for 501 disk contact networks is plotted against n in Fig. 4. The sum of 3, 4, and 5-path polygon loops reaches more than 95%. The averages of n for the packing fractions 0.74 and 0.76 were 3.83 and 3.70, respectively. This result is plausible, since the average path of polygon loops approaches 3 (the closest disk packing) with increasing packing fraction.

The percentages of n-path polygon loops changed during the experiments. The frequencies of n-path polygon loops are plotted against time in Fig. 5. The averages of total loops for the packing fractions 0.74 and 0.76 were 91.21 and 104.72 with the variance of 45.93 and 30.71, respectively. The fluctuation of the data for the packing fraction 0.74 was larger than one of 0.76, which may be attributed to larger disk migration (HIRATA and YOSHIMURA, 1999).

The distribution of coordination number is shown in Fig. 6. In the case of counting the coordination number of the disks, the contacts between the disks and the inner and outer cylinders are taken into account. The number of disks with coordination number less than 3 were negligible, which suggests that each disks are supported each other and there is no

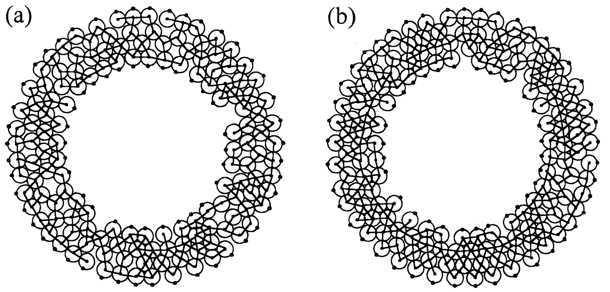

Fig. 3. Disk contact networks: a) the disk contact network obtained at the experiment of packing fraction 0.74, b) one of 0.76.

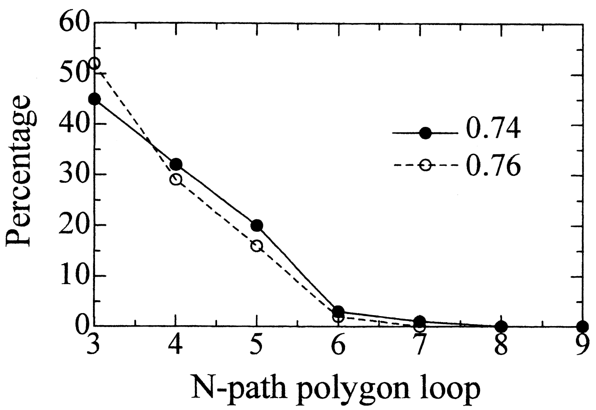

Fig. 4. Distributions of n-path polygon loops. The solid circle and solid line indicates the data of packing fraction 0.74, and the open circle and broken line indicates the data of 0.76.

free space to move. The average coordination number were 3.73 and 3.89 for the experiments of packing fraction of 0.74 and 0.76, respectively. The difference between the distributions for 0.74 and 0.76 is very small. The changes of distributions of coordination number during the experiments are shown in Fig. 7.

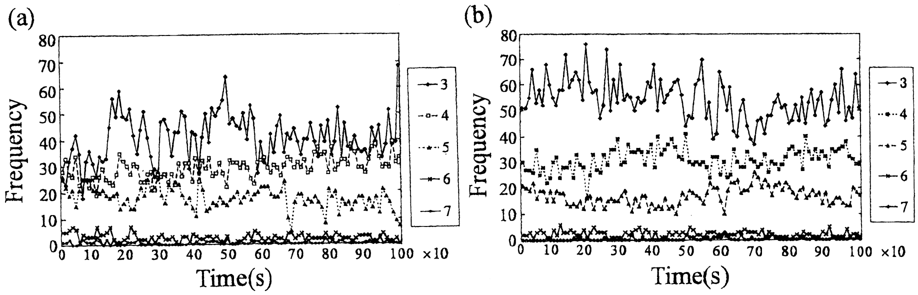

Fig. 5. The frequency distributions of n-path polygon loops were plotted against time. a) the distribution of the experiment of packing fraction 0.74, b) one of 0.76.

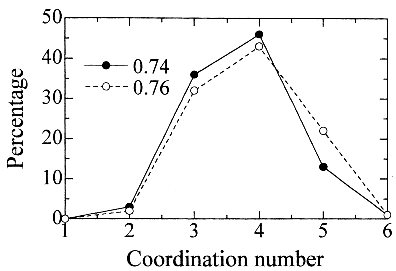

Fig. 6. Distributions of coordination numbers. The solid circle and solid line indicates the data of packing fraction 0.74, and the open circle and broken line indicates the data of 0.76.

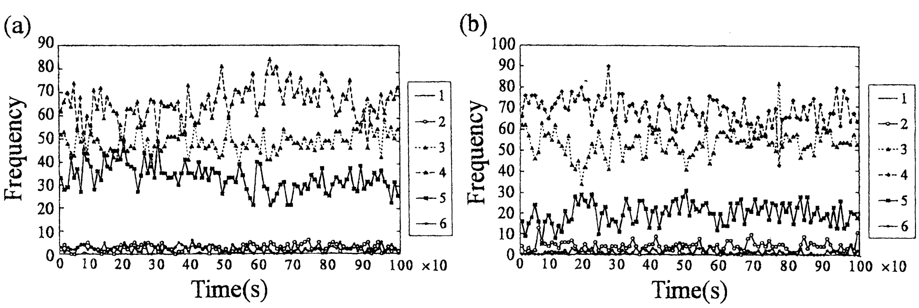

Fig. 7. The frequency distributions of coordination numbers were plotted against time. a) the distribution of the experiment of packing fraction 0.74, b) one of 0.76.

4. Concluding Remarks

We obtained disk contact networks by analyzing video image data and observed dynamical changes of networks. The numbers of *n*-path polygon loops in the networks and the distribution of *n*-path polygon loops were obtained. The fluctuation in the distributions of *n*-path polygon loops become larger for smaller packing fraction. The average path of *n*-path polygon loops decreased with increasing the packing fraction. The average paths of polygon loops were 3.83 and 3.70 for the experiment of packing fractions 0.74 and 0.76, respectively. The distributions of coordination number of disks were also obtained, whose averages were 3.73 and 3.89 for packing fractions of 0.74 and 0.76, respectively. The average of coordination number increased with the packing fraction.

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