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Curie Symmetry Principle: Does It Constrain the Analysis of Structural Geology?

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Abstract. Curie symmetry principle, the causality relation between the symmetry of the causes and the resultant effect, has often been invoked to infer the composite deformation history of geological bodies in the Earth's crust. This principle provides a powerful constraint for predicting an unavailable past physical condition; the effects at the macroscopic level must be the same or higher symmetry than the intersection of the causes. However, in nonlinear phenomena, it is shown that the resultant effect selects a lower symmetry than the intersection of causes, which logically opposes the principle. Here, we introduce a symmetry breaking principle where the symmetry group of the effect is included in the intersection of the causes, and derive a new nonlinear phenomenological equation to formulate the symmetry breaking phenomena. The new principle suggests that the anisotropic pattern may not be necessary to consider the anisotropic causes or the multiple deformation history under nonlinear phenomena.

1. Introduction

Through geological time, the Earth's crust has suffered a sequence of intermittent deformations and its history is recorded in deformed rocks as superimposed permanent strain. Structural geology deals with these deformations of crustal rocks and one of its aims is to infer from observations of the geometrical rock texture, the strain or stress history of deformation (e.g., HATCHER, 1995). Thus, the causality relation gives a powerful constraint in structural geology where past physical information is not available. Symmetry aspect of this causality relation is the Curie principle of symmetry (CURIE, 1894; JAEGER, 1920;

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PRIGOGINE, 1947; SHUBNIKOV, 1956; NYE, 1969; CHALMERS, 1970; SHUBNIKOV and KOPTSIK, 1974; KOPTSIK, 1983). This principle lays down the necessary relation between the symmetry of the cause and the resultant effect in a physical process, thus, allows us to predict possible and forbid impossible properties of the geological system (PATERSON and WEISS, 1961; TURNER and WEISS, 1963; HROUDA, 1973; LISTER and WILLIAMS, 1979; WEISS and WENK, 1985; TWISS and UNRUH, 1998).

Pierre Curie was the first to formulate the symmetry principle which is composed of three parts (CURIE, 1894):

1. *If certain causes yield the known effects, the symmetry elements of the causes should be contained in the generated effects.*

2. *If the known effects manifest certain dissymmetry* (absence of symmetry elements), *this latter should be contained in the causes which have generated those effects.*

3. *The converse to these two previous propositions is not true, at least in practical; i.e., the effects may have higher symmetry than the causes which generate these effects.*

JAEGER (1920) has restated these statements as follow: *the effects may occasionally have the same or a higher symmetry than the causes, but the last cannot have a higher symmetry than the effects produced*. In structural geology, the Curie principle has been firstly introduced by SANDER (1930) and has been invoked as a basic rule that "*Whatever the nature of the contributing factors, the symmetry that is common to them cannot be higher than the symmetry of the observed geologic texture, and symmetry elements absent in the texture must be absent in at least one of the contributing factors* (PATERSON and WEISS, 1961)". Consequently, the Curie symmetry principle does not allow us to infer the symmetric cause for interpreting the lower symmetric (anisotropic) pattern in the resultant effect in a physical and geological process.

In irreversible thermodynamics, the causality relation parallels the phenomenological equation between the tensorial flux (F_i) and force (X^j):

$$F_i = L_{ij}X^j, \tag{1}$$

where L_{ij} is the phenomenological coefficient (ONSAGER, 1931a, b). In an isotropic system, using the Curie principle, PRIGOGINE (1947) has pointed out that the tensorial flux (effect) can not 'interact' and 'couple' with the tensorial force (cause) whose rank differs by an odd number. This statement is the phenomenological aspect of the Curie principle. In an anisotropic system, KATCHALSKY and CURRAN (1965) have suggested that the principle breaks and then the symmetry of the causes can produce the lower symmetry in the effect. But, we can represent that the Curie principle does not break by taking the anisotropy of the system ($L_{ij} \neq L_{ji}$) into account. However, STEWART and GOLUBITSKY (1992) and STEWART (1995) have suggested that even in an isotropic system, the principle can be broken by microscopic asymmetric disturbances, which can trigger an instability of the fully symmetric state. This shows the collapse of the Curie principle of symmetry. Thus, if the principle breaks under the onset of an instability, previous geological study based on the Curie principle must be reconsidered in terms of the deformation history of geologic bodies.

In this paper, we redraw the Curie principle of symmetry by group theory and present an application of the redrawn principle to the field of structural geology. Furthermore, examples of the breaking of the Curie principle will be presented. Finally, we propose a symmetry breaking principle as a new constraint for analysis in structural geology and discuss application limits of the Curie principle.

2. Symmetry Principle and Its Application to Structural Geology

Symmetry can be formulated by group theory (WEYL, 1952). According to SHUBNIKOV (1956), Curie's statements can be reduced to a superposition principle by group theory as follows:

$$G_{\text{effect}} \supseteq G_{\text{cause1}} \cap G_{\text{cause2}} \cap \cdots \cap G_{\text{cause}n} = G_{\text{medium}} \cap G_{\text{field}} \tag{2}$$

where G_{effect} is the symmetry group of an effect associated with causes 1, 2, $\cdots, n$, the symbols $\supseteq$ denotes the equal or the inclusion in a group, $\cap$ is the intersection and G_{cause1}, G_{cause2}, $\cdots, G_{\text{cause}n}$ are the symmetry groups of causes 1, 2, $\cdots, n$ (SHUBNIKOV, 1956; SHUBNIKOV and KOPTSIK, 1974; LISTER and WILLIAMS, 1979; KOPTSIK, 1983). Here, the symmetry group of causes can also be divided into the group of the medium factor G_{medium} and the applied field factor G_{field} as a physical sense (SHUBNIKOV and KOPTSIK, 1974). This equation means that the intersection of the symmetry group of each cause must be included in the group of the effect.

In an application example of the Curie principle, the intersection between causes (the symmetries of the medium and the applied field) will show orthorhombic symmetry when the mechanically isotropic medium is subjected to an orthorhombic stress field. Therefore, the symmetry of the resultant effect must be at least of orthorhombic symmetry. In a macroscopically isotropic system, the symmetry of the effect actually shows an orthorhombic or higher axial ellipsoidal shape under the linear constitutive relation such as Hooke's law. This is consistent with the Curie principle.

Moreover, we can explain the method of how to apply a principle to the problem in structural geology. Figure 1 illustrates a recumbent fold that has monoclinic symmetry as an effect. The factors of the causes include both the stratified plane with an infinite fold axis and the unknown past stress field. Thus, in order to infer the past stress field, using the Curie principle, this monoclinic fold pattern must be interpreted as reflecting the monoclinic (shear) stress field of the cause, or the overprinting of a later structures due to changes of the stress field. That is why the Curie principle forbids the effect at the macroscopic level to have a lower symmetry than the intersection of causes. This means that the lower symmetry of the effect requires a lower symmetric cause or a multiple deformation history under a macroscopically isotropic and linear system.

Although we represented the powerful constraint of the Curie principle under an isotropic medium and a linear constitutive field, it is lately reported that some nonlinear phenomena can not be constrained by the Curie principle even in a macroscopically isotropic system. Consequently, we outline in the next section, some examples of the breaking of the Curie principle.

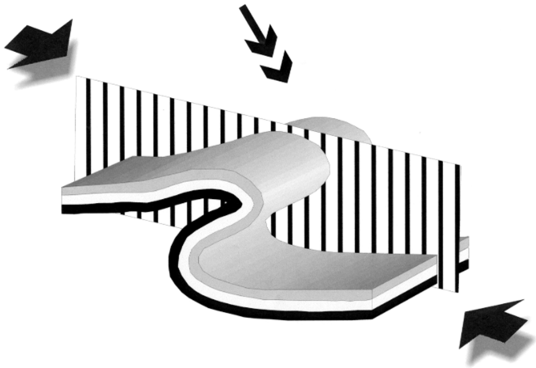

Fig. 1. Schematic diagram showing recumbent folds which are overturned to such an extent that the limbs are essentially horizontal. These monoclinic geological structures are found in the cores of mountain ranges such as the Canadian Rockies, Alps and Himalayas and probably record shortening of the Earth's crust associated with plate convergence. Its formation is considered as reflecting the shear stress field (solid arrows with shadow) or the overprinting of a later structure due to changes of the stress field (double arrow). The vertical plane is a reflection plane in a monoclinic pattern and this fold pattern has a two fold axis along this plane.

3. Nonlinear Phenomena and Symmetry Breaking

Figure 2 illustrates the buckling phenomena of a mechanically isotropic spherical shell (YAMAGUCHI and FUJII, 1977; FUJII, 1980; FUJII and YAMAGUCHI, 1980). The factors of cause are the spherical shell with an infinite fold axis and an axial pressure loaded on the shell top. When the shell deforms nonlinearly beyond an elastic regime, the buckling phenomenon occurs as a result of non-elastic deformation. The resultant symmetry of the spherical shell selects a symmetry group with a finite fold axis, depending on the structural parameter of the shell (the ratio of the radius and the height of the shell). In this case, three fold axis is possible symmetry group, which is lower symmetry group than that of causes. This actually shows that the symmetry element of the effect is lower than that of the intersection of causes. This logically opposes the Curie principle. Buckling phenomenon of a plastic sheet (Fig. 3) provides a similar example of symmetry breaking phenomena (STEWART and GOLUBITSKY, 1992). The factors of cause include both the plastic sheet with an infinite fold axis and an orthorhombic stress field. Thus, if we apply the Curie principle to this phenomenon, it will predict an orthorhombic symmetry to the intersection of causes. But, one reflection plane is absent in the effect, even though it exists in the intersection of causes. So, we can suggest that the Curie principle dose not work in nonlinear phenomena as well as in the previous example.

Laboratory data is presented below as another example of the symmetry breaking phenomenon. BORRADAILE and ALFORD (1987) conducted a triaxial compression test to irreversible piezo-magnetization under an overall coaxial (orthorhombic), homogeneous and macroscopically ductile (non-elastic) conditions. Therefore, the permanent strain (ε_{ij}) as an effect relates linearly with coaxial stress as a cause in the viewpoint of total deformation theory (HILL, 1950). In this experiment, the starting materials are magnetically

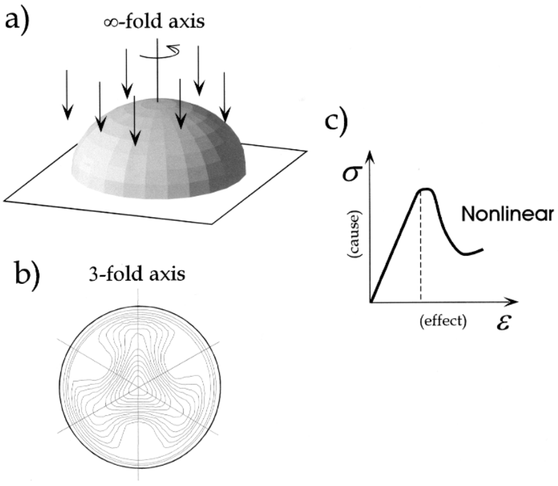

Fig. 2. Hemispherical shell buckling. (a) The axial pressure, such as a snow, loads on the top of the isotropic hemispherical shell which has a infinite fold axis. (b) View from the top of the resultant pattern of the buckling shell showing the 3 fold axis. (c) Stress-strain curve for deformation of the spherical shell loaded under a constant axial pressure. Elastic behavior occurs along the straight line portions of the curve. At stresses greater than the elastic limit, the shell will buckle as a result of non-elastic deformation and the stress-strain relationship becomes nonlinear. Each coordinate can regard the vertical axis as a cause and the horizontal axis as an effect.

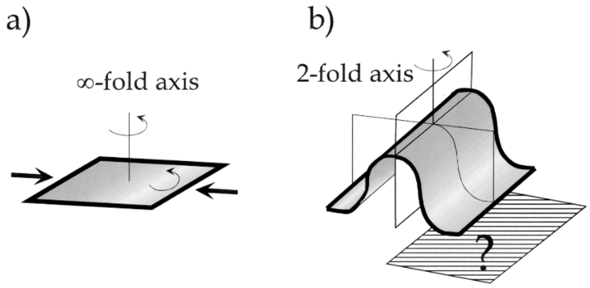

Fig. 3. Buckling of a plastic sheet. (a) Plastic sheet which has a infinite fold axis is subjected to an orthorhombic stress field under an elastic regime. (b) Buckling occurs as a result of non-elastic deformation when the plastic sheet deforms beyond an elastic regime.

and mechanically homogeneous-isotropic sandstones at a macroscopic level. Under the described conditions, BORRADAILE and ALFORD (1987) summarized the quantitative relationship between the ellipsoid of the initial magnetic susceptibility $\breve{\chi}_{mn}$, the resultant one χ_{mn} and the permanent strain as follow:

$$\chi_{mn} / \breve{\chi}_{mn} = M^{ij} \varepsilon_{ij} \tag{3}$$

where M_{ij} is Borradaile-Alford's empirical matrix (BORRADAILE and ALFORD, 1987; NAKAMURA and NAGAHAMA, 1997a, b). Equation (3) can be recognized as a phenomenological equation, if we regard a change of susceptibility as a flux, the stress-induced permanent strain as a force and Borradaile-Alford's matrix as a phenomenological coefficient. The factors of cause are the homogeneous-isotropic sandstone and the induced permanent strain under an orthorhombic stress field. By applying the Curie principle, the resultant symmetry must be orthorhombic. However, the Borradaile-Alford's matrix obviously is an asymmetric matrix:

$$M^{ij} = \begin{pmatrix} 0.066 & -0.002 & -0.014 \\ 0.001 & 0.065 & -0.025 \\ -0.004 & -0.010 & -0.175 \end{pmatrix}.$$

Therefore, the effect (left hand side of Eq. 3) shows a lower symmetry than orthorhombic. This result also suggests that the causality relation between causes and effects is coupled by the operation of the asymmetric matrix. NAKAMURA and NAGAHAMA (1997a, b) have revealed theoretically that this operation matrix is asymmetric as a function of both the secant modulus for a nonlinear strain history and an irreversible magnetostriction tensor, and thus, is dependent on strain: $M^{ij}(\varepsilon)$. Therefore, this experimental example also shows the breaking of the Curie principle; the symmetry of the effect is of a lower symmetry than the intersection of causes. Consequently, these examples described above suggest that a lower symmetric pattern may not be necessary to consider lower symmetric causes or multiple deformation history.

4. Discussions and Conclusions

When a mechanical system deforms nonlinearly beyond an elastic regime (such as a dynamic instability), the symmetry of the observed effect selects a lower symmetry than that of the intersection of causes. Therefore, we can propose a symmetry breaking principle by group theory:

$$G_{\text{medium}} \cap G_{\text{field}} \supset G_{\text{effect}} . \tag{4}$$

Equation (4) indicates that the symmetry group of the effect is a subgroup of the intersection of causes. A similar formulation has also been proposed in physics by KOPTSIK (1983). This group theoretical formulation is also applicable to symmetry prediction of pattern in nature, such as meandering rivers and sand dunes. This is obviously contradictory to the Curie principle, which states that the effect must be of the same or higher symmetry than the intersection of causes. However, it does not fail the powerful constraint for symmetry argument. These results are an indication of the limits of application of the Curie principle. We can apply the Curie principle to linear problem under the linear constitutive equation. However, in nonlinear cases where a dynamic instability is triggered, the Curie principle breaks and the application of the symmetry breaking principle is necessary, providing a new powerful constraint for the analysis of nonlinear problem. This symmetry breaking principle has already been discussed in terms of the group representation theory, bifurcation theory and pattern formation (SATTINGER, 1977, 1978, 1979; YAMAGUCHI and FUJII, 1977; FUJII, 1980). Therefore, we can conclude that the symmetric cause may generate a lower symmetric pattern and it may not be necessary to consider the multiple deformation history when interpreting an anisotropic geologic texture. This principle will open a new window for structural analysis of geological bodies that have suffered nonlinear phenomena.

The phenomenological equation is represented by a homogeneous linear relation (1). ONSAGER (1931a, b) have shown that the phenomenological coefficient is symmetric ($L_{ij} = L_{ji}$), in terms of the principle of microscopic reversibility at equilibrium, which expect the past and future behavior of the process to be same. However, the experimental magnetic result indicates that Borradaile-Alford's matrix as a phenomenological coefficient is asymmetric (i.e., $M^{ij} \neq M^{ji}$). Because the matrix is a strain dependent, Eq. (3) suggests that a change of magnetic susceptibility increases nonlinearly with strain during non-elastic deformation. Therefore, to interpret symmetry breaking phenomena phenomenologically, it is necessary to extend the linear relation (1) to a nonlinear phenomenological equation at the onset of instability: $F_i = \left(L_{ij} + \tilde{L}_{ij}\right)X^j$ where $\tilde{L}_{ij}$ is an anti-symmetric tensor. This idea is tentative and gives a new hypothesis which is not consistent with the principle of microscopic reversibility at equilibrium. Sattinger (1977) has shown that a nonlinear differential equation at equilibrium possesses bifurcation solutions after the onset of instability and selects a lower symmetry group (e.g., hexagonal cellular motion in Benard convection, the buckling of sphere). Therefore, the nonlinear extension of Eq. (1) with an anti-symmetric tensor satisfies a bifurcation problem and can be regarded as a new phenomenological equation to be able to apply to symmetry breaking phenomena.

By modifying the previous statement (PATERSON and WEISS, 1963) using the symmetry breaking principle, we can propose a new constraint directly applicable to the interpretation of tectonically deformed geological bodies: "if the nature of the factors contributing to a deformation triggers the dynamic instability beyond the elastic deformation, the symmetry that is common to them can be higher than the symmetry of the observed geologic texture, and symmetry elements present in the contributing factors may disappear in the observed texture."

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