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Symmetry Properties of Spatial Distribution of Microfracturing in Rock

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Abstract. We measure quantitatively symmetry and entropy regarding symmetry of spatial distributions of acoustic emissions generated by microfracturings during creep before ultimate whole fracture of a rock specimen. In the measurement, symmetry properties of the two-dimensional discrete Walsh functions are utilized, and the two-dimensional discrete Walsh transform is applied to the spatial distributions. We find that the spatial distributions of acoustic emissions are rich in double symmetry. Moreover, it is found that the richness in double symmetry increases and the entropy decreases with the evolution of microfracturing process. This decrease of the entropy comes from the increase of the richness in double symmetry. Thus we conclude that the microfracturing process evolves under constraint of the increase of the richness in double symmetry.

1. Introduction

Spatial distributions of earthquakes or acoustic emissions (AEs) seem to be complex or disorder. From the viewpoint of the concept of fractal geometry (MANDELBROT, 1982), we can characterize such spatial distributions (e.g., KAGAN and KNOPOFF, 1978; HIRATA *et al.*, 1987; NANJO and NAGAHAMA, 2000). The fractal dimension is a tool to understand the difference of such spatial distributions. For example, HIRATA *et al.* (1987) found that the spatial distributions of AE, which occurred during creep before ultimate whole fracture, show the fractal structure, and the fractal dimensions decrease with the evolution of the microfracturing process. It was pointed out that extrapolation from laboratory measurement provides the prediction of occurrence of large earthquakes by the decrease of the fractal dimensions. However, to our knowledge, no paper has reported the success of the prediction of large earthquakes by monitoring the decrease of the fractal dimensions.

It is natural to analyze symmetry properties in terms of a continuous scale rather than in terms of 'yes or no' (e.g., VAN VALEN, 1962; YODOGAWA, 1982; PALMER, 1986; PALMER and STROBECK, 1986; NAGY, 1990; ZABRODSKY *et al.*, 1992). YODOGAWA (1982) introduced a method to measure quantitatively symmetry of a pattern consisting of dots, and an entropy regarding symmetry, which is called symmetropy. However, no one has applied it to a pattern in nature. This method may be useful to monitor the spatial distributions of earthquakes or AE events. If the spatial distributions of earthquakes or AE events show symmetry, then symmetry properties put such the spatial distributions under constraint.

WHYTE (1949, 1974) discussed 'unitary principle' which states that nature is a field in which tendencies to order and to disorder contend each other, and that the former includes the later. Moreover, it was pointed out that the order and the disorder are characterized by symmetry and by entropy, respectively. That is, the unitary principle is manifested by tendency to symmetry despite the increase or the decrease of entropy in nature. However, as pointed out by DINGLE (1949), the tendency to symmetry was never checked by experimental studies.

In self-organized criticality (SOC) model of earthquakes, ITO and MATSUZAKI (1990) took ENYA (1901)'s idea. ENYA (1901)'s idea is that main shock disturbs strain distribution, and aftershocks occur to decrease the heterogeneity of the strain distribution in the earth's crust. This process is called entropy relaxation, in contrast, the process taken by the SOC model of BAK and TANG (1989) is called energy relaxation. ITO and MATSUZAKI (1990) derived that the fractal dimensions of the spatial distribution of earthquakes decrease toward critical steady state. This result agrees quantitatively with that found in rock fracture experiment (HIRATA *et al.*, 1987). However, the relation between entropy and the evolution of fracturing process to the critical steady state has never been examined.

Here, we examine whether spatial distributions of AE published in HIRATA *et al.* (1987) show symmetry, using the method introduced by YODOGAWA (1982). Then, we discuss the evolution of microfracturing process.

2. Two-Dimensional Discrete Walsh Transform

In this paper, according to YODOGAWA (1982), we quantify symmetry of a pattern, using the two-dimensional discrete Walsh transform. To do it, we utilize that the two-dimensional discrete Walsh functions are divided into the four types of symmetry: vertical symmetry, horizontal symmetry, centrosymmetry, and double symmetry (Fig. 1). We quantify these four types in a pattern. Moreover, we measure entropy regarding symmetry. This entropy is called symmetropy.

Patterns used in this paper are restricted to rectangular matrix, each consisting of $M \times N$ rectangular cells, where $M = 2^q$ and $N = 2^r$ (q and r are positive integers). The two-dimensional discrete Walsh transform of a pattern is given by

$$a_{m,n} = \frac{1}{MN} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} x_{i,j} W_{m,n}(i, j), \tag{1}$$

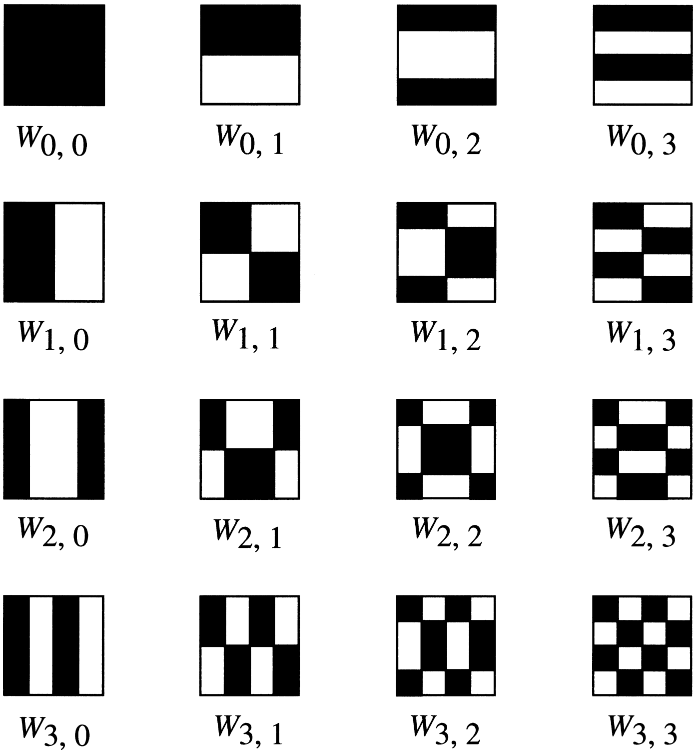

Fig. 1. The first 16 of the two-dimensional discrete Walsh functions $W_{m,n}$ ($M = N = 4$). Black and white represent 1 and −1, respectively. $W_{m,n}$ is vertically symmetric when m = even and n = odd, is horizontally symmetric when m = odd and n = even, is centrosymmetric when m = odd and n = odd, and is doubly symmetric when m = even and n = even, where $W_{0,0}$ is excepted.

$$m = 0,\ 1,\ 2,\ \dots,\ M-1 \text{ and } n = 0,\ 1,\ 2,\ \dots,\ N-1,$$

where $x_{i,j}$ is the value of gray level of a pattern in the j-th row cell in the i-th column, $W_{m,n}(i, j)$ is the value (1 or −1) of the (m, n)-th order of the two-dimensional discrete Walsh function in the j-th row cell in the i-th column (Fig. 1), and $a_{m,n}$ is the two-dimensional Walsh spectrum. If there are just two gray levels: 'black' and 'white', we usually represent $x_{i,j}$ by 1 and 0.

Symmetric component P_k (k = 1, 2, 3, 4) quantifying the four types of symmetry is given by

$$P_k = \sum_{m,n} \left(a_{m,n}\right)^2 / K, \tag{2}$$

where $K = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} (a_{m,n})^2 - (a_{0,0})^2$ and $\sum_{k=1}^{4} P_k = 1$. Vertically symmetric component P_1 is given when m = even and n = odd. Horizontally symmetric one P_2 is given when m = odd and n = even. Centrosymmetric one P_3 is given when m = odd and n = odd. Doubly symmetric one P_4 is given when m = even and n = even. The sum is taken over all ordered pairs (m, n) for $0 \le m \le M - 1$ and $0 \le n \le N - 1$ where $a_{0,0}$ is excepted.

If the value of a certain component is larger than the values of the other components, the original pattern is rich in symmetry. If the values of the four components are almost equal each other, the original pattern is poor in symmetry.

When we apply the entropy function in information theory to P_k (k = 1, 2, 3, 4), symmetropy S is given by

$$S = -\sum_{k=1}^{4} P_k \log_2 P_k. \tag{3}$$

S is explained as follows:

(1) A pattern can be considered as a zero-memory information consisting of the four types of symmetry, each occurring with a probability whose value equals the corresponding P_k (k = 1, 2, 3, 4). Here, an information source is called a zero-memory source if successive symbols emitted from the source are statistically independent (ABRAMSON, 1963).

(2) S means the entropy of such an information source, and can be considered as a quantitative and objective measure of symmetry.

3. Fracture Experiment

In this paper, we analyze the 9 spatial distributions of AE events generated by microfracturings within rock, published in HIRATA *et al*. (1987). Here, we briefly review the rock fracture experiment and the results of HIRATA *et al*. (1987).

A constant stress fracture experiment of Oshima granite was carried out at the confining pressure of 40 MPa. The rock specimen is a cylinder 50 mm in diameter and 100 mm long (Fig. 2a). The creep is divided into three stages: primary creep, secondary creep, and tertiary creep. 353, 273, and 1438 events of AE detected during the primary, secondary, and tertiary creep, respectively, showed fractal properties. The fractal dimensions decreased from 2.75 via 2.66 to 2.25 as creep progresses. HIRATA *et al*. (1987) showed 3 orthographic projections at each of the three stages (Fig. 2a). AE events distributing within the three-dimensional rock are projected on 3 planes: one plane has a normal vector which is perpendicular to circular (top) side of the cylinder; and each of the two others has a normal vector which is perpendicular to straight parallel sides of the cylinder, and the planes are situated on the left of and on the right of the cylinder.

4. Procedure and Results

We regard the AE events as circles with finite diameter. The circles are equal to circles to draw AE events in HIRATA *et al*. (1987). We analyze the spatial distributions of AE plotted within rock. $(M, N) = (64, 32)$ for the spatial distributions projected on the planes on the left of and on the right of the specimen (Fig. 2b), and $(M, N) = (32, 32)$ for the spatial distributions projected on the plane on the top of the specimen. If we find a part of or whole of one or more of circles in a cell of (i, j), then $x_{i,j} = 1$, otherwise $x_{i,j} = 0$ (Fig. 2b). According to YODOGAWA (1982), we estimate P_k ($k = 1, 2, 3, 4$) and S for the 9 spatial distributions of AE.

Table 1 shows the estimated values of P_k ($k = 1, 2, 3, 4$) and S. $0.09 \le P_1 \le 0.20$, $0.09 \le P_2 \le 0.21$, $0.08 \le P_3 \le 0.22$, $0.40 \le P_4 \le 0.70$, and $1.35 \le S \le 1.92$. For each of the 9 projected spatial distributions of AE, P_4 is larger than P_k ($k = 1, 2, 3$). Moreover, the ranges of P_k ($k = 1, 2, 3$) almost equal each other and the upper limit of P_k ($k = 1, 2, 3$) is 0.22, while

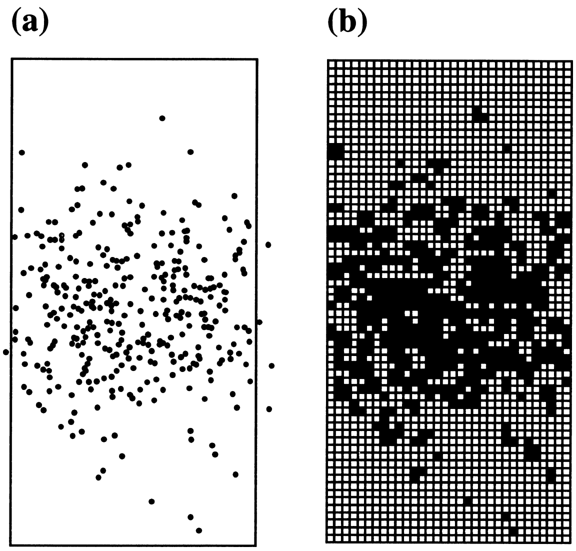

Fig. 2. An example of the spatial distribution of AE events occurring in a rock specimen. (a) 273 events occurring at primary creep are projected orthographically on the plane of a normal vector, which is perpendicular to straight parallel sides of the cylinder rock specimen (modified from HIRATA *et al.*, 1987). The plane is situated on the left of the rock specimen. (b) 64×32 cells cover a rock specimen. Black cells contain a part of or whole of one or more of AE events. 1 and 0 are given to black and white cells, respectively.

Table 1. Estimated values of P_k ($k = 1, 2, 3, 4$) and S.

	Left			Right			Top		
	Primary	Secondary	Tertiary	Primary	Secondary	Tertiary	Primary	Secondary	Tertiary
P_1	0.17	0.17	0.11	0.18	0.14	0.12	0.19	0.20	0.09
P_2	0.16	0.16	0.09	0.15	0.14	0.09	0.19	0.21	0.21
P_3	0.14	0.14	0.11	0.15	0.15	0.09	0.22	0.18	0.08
P_4	0.53	0.53	0.69	0.52	0.57	0.70	0.40	0.41	0.62
S	1.74	1.74	1.38	1.76	1.67	1.35	1.92	1.91	1.50

the lower limit of P_4 is 0.40. That is, P_4 is clearly larger than P_k (k = 1, 2, 3). Therefore, the spatial distributions of AE are clearly rich in double symmetry.

For each of the projections, P_k (k = 1, 2, 3) at primary creep is equal to or larger than that at secondary creep which is equal to or larger than that at tertiary creep, respectively, with the two exceptions. But P_4 at primary creep is smaller than that at secondary creep which is smaller than that at tertiary creep. The exceptions are that P_1 and P_2 at primary creep are smaller than those at secondary creep, respectively, for the projection on the top plane. However, the difference between P_1 at primary creep and P_1 at secondary creep and the difference between P_2 at primary creep and P_2 at secondary creep are small. P_1 and P_2 at primary creep are regarded to equal those at secondary creep, respectively. Therefore, richness in double symmetry increases as creep progresses.

For each of the projections, S decreases as creep progresses. This decrease of S comes from the increase of the richness in double symmetry.

5. Discussion and Conclusions

ITO and MATSUZAKI (1990) submitted the SOC model of earthquakes, taking into consideration the entropy relaxation hypothesized by ENYA (1901), which is different from the energy relaxation hypothesized by BAK and TANG (1989). Successfully, the decrease of the fractal dimension of the spatial distribution of earthquakes toward critical steady state was derived. This decrease of the fractal dimension in the SOC model (ITO and MATSUZAKI, 1990) agrees quantitatively with that found in the rock fracture experiment (HIRATA *et al.*, 1987). The tertiary creep stage in the experiment of HIRATA *et al.* (1987) is near the critical point just before the ultimate fracture. Because of these, we can regard that HIRATA *et al.* (1987) observed the evolution of the microfracturing process approaching to the critical point. However, ITO and MATSUZAKI (1990) did not clear the relation between entropy and the evolution of fracturing process toward the critical steady state.

In this paper, we measured the components of the four types of symmetry (vertical symmetry, horizontal symmetry, centrosymmetry, and double symmetry) and symmetropy for the 9 spatial distributions of AE. To measure them, the two-dimensional discrete Walsh transform was used. We found that doubly symmetric components are larger than other symmetric components. This represents that the spatial distributions of AE are rich in double symmetry. Moreover, we found that the richness in double symmetry increases and symmetropy decreases with the evolution of the microfracturing process. This decrease of the symmetropy is due to the increase of the richness in double symmetry. Therefore, it is concluded that the microfracturing process evolves under the constraint of the increase of the richness in double symmetry.

If the scale-invariant nature of fracturing holds from the microscopic level of microfracturing in rocks to the macroscopic level of earthquakes, the results of the rock fracture experiment can be extrapolated to an explanation of the natural earthquake (HIRATA *et al.*, 1987).

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