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Coupled Map Lattice Model for Pattern Formation in Diffusion Field

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Abstract. We have newly simulated a coupled map lattice model of melt growth including surface tension. The tip forms parabolic interface and the dependency of tip velocity on supercooling nearly agrees with theoretical and experimental results.

Pattern formation has been researched as non-linear and non-equilibrium problem. Various patterns grow in many diffusion field experiments such as viscous-fingering, crystal growth and electro-chemical deposition. Dendritic pattern has been studied with many theoretical analyses, experiments and numerical simulations. Recently, a coupled map lattice model (CML) has been succeeded in obtaining various growing crystal patterns (SAKAGUCHI, 1998). In this model, crystalizing process is devided into two processes. One is diffusion process and the other is growing process. However, the effect of surface tension is neglected in this model.

In order to introduce such effect (Gibbs-Thomson effect) to the model, we define interfacial curvature, because the interfacial temperature depends on the curvature. Considering the above, two-dimensional CML model for melt growth is represented as following.

The diffusion process is written as

$$u_{n+1}(i, j) = u_n(i, j) + D(u_n(i + 1, j) + u_n(i - 1, j) + u_n(i, j + 1) + u_n(i, j - 1) - 4u_n(i, j)).$$

The growing process is written as

$$\begin{aligned} u_c(i, j) &= 1 - c_1 K(i, j) \\ x_{n+2}(i, j) &= x_n(i, j) + c_2(u_c(i, j) - u_{n+1}(i, j)) \\ u_{n+2}(i, j) &= u_{n+1}(i, j) + c_3(u_c(i, j) - u_{n+1}(i, j)), \end{aligned}$$

where, $u_c(i, j)$ is interfacial temperature, $x_n(i, j)$ is an artificial variable, $u_n(i, j)$ is temperature on (i, j) site, $K(i, j)$ is the curvature of the interfacial site, n is iteration time and

D is diffusion coefficient. The variable $x_n(i, j)$ works as a counter at the interface and if it goes over 1, the (i, j) site is crystalized. Then $x_n(i, j)$ is an order parameter for the crystalization. The parameter c_1 corresponds to capillary length, and c_3/c_2 is proportional to latent heat. The melting temperature u_m is 1. (The temperature is fixed to u_∞ at the boundary of radius $R_\infty = 200$ far from the original seed.) The value of $u_m - u_\infty$ corresponds to the supercooling in this system.

We suppose that $\Delta \equiv u_m - u_\infty$ represents a degree of non-equilibrium in the system. We have performed simulations changing the variable Δ when $c_1 = 0.1$, $c_2 = 0.3$ and $c_3 = 0.42$. We show a typical pattern in Fig. 1. The tip is parabolic interface (Fig. 2). Tip velocity

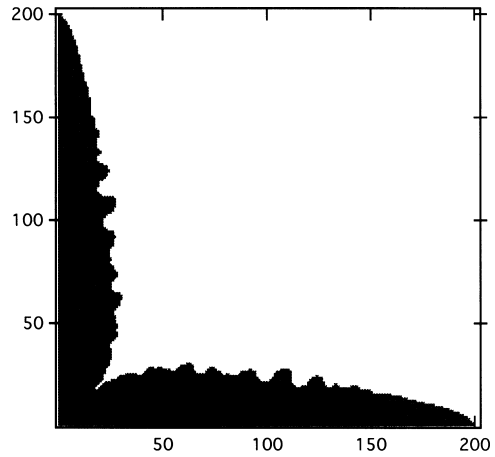

Fig. 1. A typical pattern in $c_1 = 0.1$, $c_2 = 0.3$, $c_3 = 0.42$ and $\Delta = 1$.

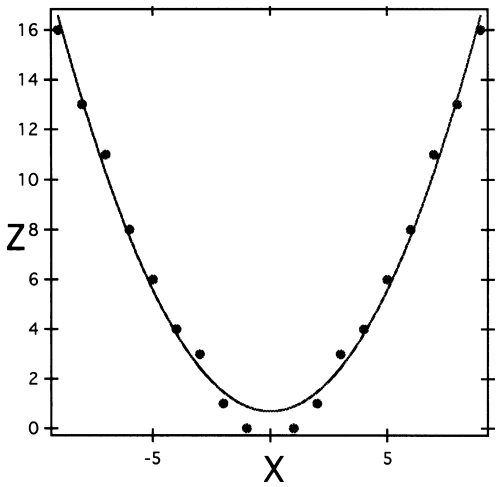

Fig. 2. Fitting curve of the tip and the curvature K is 0.392.

V is proportional to $\Delta^{3.6}$ (Fig. 3). This numerical results is consistent with the theoretical result, $V \sim \Delta^4$ (SAITO, 1996).

For another set of c_1, c_2 and c_3, the tip is parabolic, and the tip velocity V is proportional to $\Delta^{4.2}$ (Fig. 4).

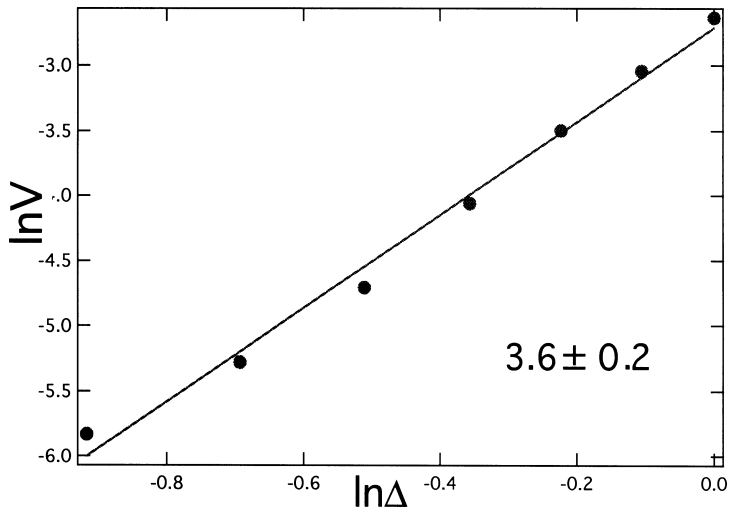

Fig. 3. Δ dependency of V in $c_1 = 0.1$, $c_2 = 0.3$ and $c_3 = 0.42$. This graph shows $V \sim \Delta^{3.6}$.

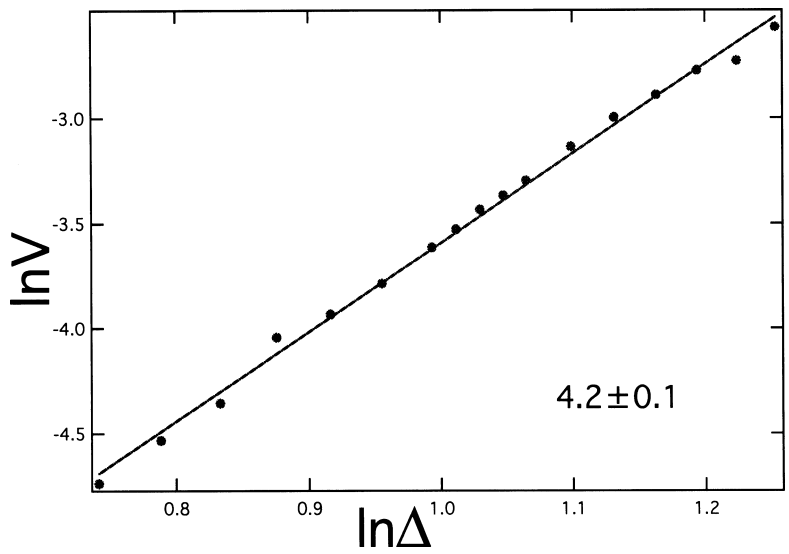

Fig. 4. Δ dependency of V in $c_1 = 0.225$, $c_2 = 0.1$ and $c_3 = 0.5$. This graph shows $V \sim \Delta^{4.2}$.

We consider the model reproduces dendrites qualitatively well. However there are many parameters, in this model such as c_1, c_2, c_3 and Δ. For further study, we want to investigate the global phase diagram changing these parameters.

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