# Dynamic Polystring Transformahedra Modeling

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**Abstract.** Dynamic Polystring Transformahedra Modeling (DPTM), U.S. Patent 5,316,483 is a new modeling concept in the exploration of 3-dimensional space in general and polyhedra specifically. Polyhedra are built with tubes passing through a polyhedral core and colored loops of string through those tubes. The color loops of string define the edges of a resulting polyhedron. The color patterns mirrotate at opposite ends of the tubes as well as opposing faces of the polyhedra due to intrinsic structural requirements of the modeling process. The polyhedra deform and/or interpenetrate as a result of sliding the tubes inline (inline translation). This paper will look at a few important embodiments of DPTM and is meant to introduce the concept, but is not comprehensive.

## 1. Introduction

Transformahedra broadly are polyhedra models which change shape. Polystring refers specifically to the following models, in which string loops are used to define paths and edges of these transforming polyhedra.

Dynamic models of polyhedra are built with tubes passing through a central polyhedron core (Fig. 1). Through these tubes, strings can be run in varying looped patterns which follow the edges of another polyhedron. Furthermore, the tubes (constant length axes) can slide in line through the cores, varying the face shape, as well as volume of the resulting constructed polyhedron. The polyhedra deform and/or interpenetrate as a result of sliding the tubes inline (inline translation). This differs from previous transformational models, in that face sizes and edges transform while axes remain constant.

The resulting color patterns of the loops mirrotate at opposite ends of the tubes as well as opposing faces of the resulting polyhedron due to intrinsic structural requirements of the modeling process. The looped patterns may relate to knot theory as multiple patterns rotated in space. As tube number increases, string looping and color patterning becomes more varied and complex. Various symmetry patterns result.

## 2. History of Development

Dynamic Polystring Transformahedra Modeling (DPTM) is a new concept in the exploration of 3-dimensional space in general and polyhedra specifically. Traditionally, polyhedra have been seen as solid, static objects. Over the centuries, with new developments in technologies for modeling, the ways in which we look at these shapes changes. Early examples in stone and metals exist. Later conceptions include the Platonic solids in early Greek thought from about 400 BC. Plato saw the 5 regular solids as “ideals” and related them to elemental constituents of the world, hence the name Platonic solids. The Renaissance artists, most notably, Leonardo da Vinci, produced wooden edge based models as well as the drawings to produce them. Kepler used them to order the planetary movements, while Euler determined intrinsic relationships of faces, edges and vertices. In the 20th century, with the development of space frames, geodesic and “tensegrity” structures, notably by Kenneth Snelson and R. B. Fuller, the emphasis moves from the “hard” fixed edges of polyhedra to interior “hard” compression struts with exterior “soft” flexible cables in tension following the edges of polyhedra in their static form.

Dynamic Polystring Transformahedra Modeling extends this concept by turning the compression struts into tubes through which the cables may pass. The cables then become continuous loops. By fixing the angular displacement of the tubes (i.e. perpendicularly through the faces of various polyhedron cores) and passing the loops through the tubes, the loops follow the edges of various polyhedra; in general, the dual of the core polyhedron. Sliding the tubes inline (translate) through the core, the loop edges will vary in size. Furthermore, by coloring the loops, various symmetrical arrangements become evident in the structuring of the polyhedra.

## 3. Explorations of DPTM-Rules of the Game

*Definitions:*

A tube is considered straight, non flexible, has length with only two openings at opposite ends through which the loops may pass.

An edge is the position traversed by the loop, outside of the tubes between two tubes’ ends, and corresponds to the edges of polyhedra.

A loop is a flexible length which is closed and unknotted, as in a loops of string in the models.

*Rules:*

1. One loop may pass through any given tube only once, but is not required to pass through all tubes.
2. Any loop which passes along the same edge as another loop (i.e. between the same two openings of two tubes) is considered redundant and is eliminated.
3. Tubes are, in general, the same length unless noted, as in multiple core models.
4. Tubes, in general, pass through the same central core, except in multiple core models and combinations.

## 4. DPTM Models

### *4.1. Interpenetrating tetrahedrons*

Octahedral core with 4 equal length tubes, 3 colored loops passing through each tube once (Fig. 1).

Attributes: The 4 tubes follow the diagonals of a cube. When the tubes are centered at the core, the loops follow the edges of two interpenetrating tetrahedra, similar to the stella octangula. When the tubes are displaced through the core, one tetrahedron will expand as the other contracts (Fig. 2).

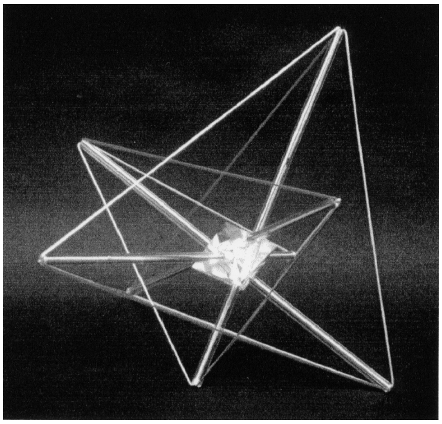

Fig. 1. DPTM with Octahedron core, 4 tubes, 3 loops. Two interpenetrating Tetrahedrons.

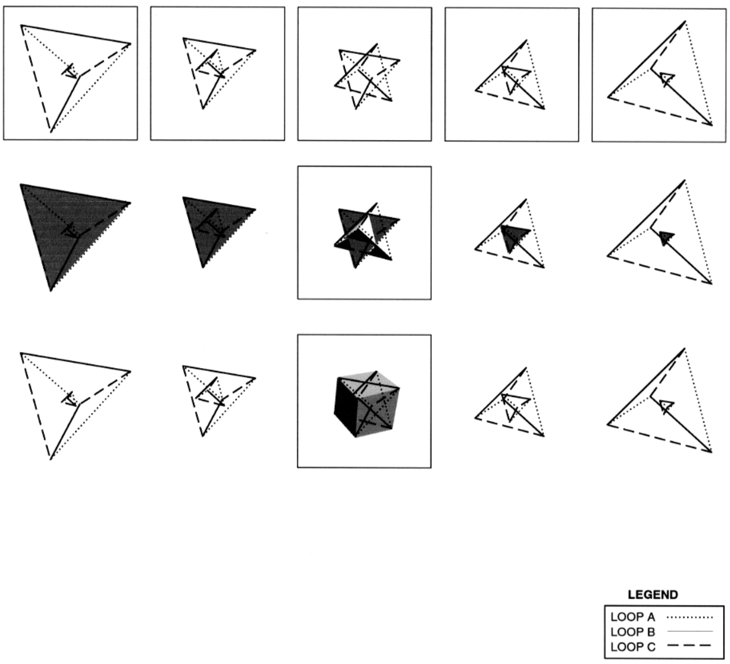


Fig. 2. DPTM Intersecting Tetrahedra Ideal Sequence (4 tubes (not shown)-3 loops (3 per tube)).
Note: Tubes not shown, loop passes to diagonal corner.

In the expanded configuration of the model (the tubes moved tetrahedrally out from the core center), one will see the opposing edges of the tetrahedron being the same of the three colors, so that each of the 6 edges of the tetrahedron are paired in the 3 colors and rotated. In the smaller tetrahedron this is mirrotated, i.e. if in the larger tetrahedron loops A, B, and C respectively are in a clockwise relation, then in the smaller tetrahedron loops A, B, and C are in a counter-clockwise relation. This is due to the tubes moving diagonally through the cube to the opposing face.

In the centered configuration of the model (each tube's midpoint at the core center), the loops follow the edges of two equal tetrahedrons. Looking at the patterning of the loops, one notices that each loop traverses diagonally 4 of the 6 faces of a cube (Fig. 3). The 4 diagonals of one loop arrange themselves orbitally around the cube; each arrangement following the *X*, *Y*, or *Z* axis respectively. Furthermore, in order for the model to expand as described above, these loops must form a Borromean link configuration (i.e. loop A is exterior to loop B while interior to loop C and so on)

*4.1.1. Special case of interpenetrating tetrahedrons*

In an idealized form (Fig. 2), where the tubes have no diameter and pass exactly through the center of the cube and where the loops are able to pass through each other, there is one condition in which loop length for all three loops remain constant as the tubes are translated. This condition occurs when all tubes are moving at a constant rate with respect to the center. In other cases, i.e. when only one or two tubes move as the others are stationary, the loops stretch. This was pointed out to the author by John Conway of Princeton University.

*4.1.2. Octahedron core as anti-prism and extensions to other self-intersecting models*

The self intersection of the two tetrahedrons occurs specifically because the number of vertices and faces of tetrahedron is the same. Since the octahedron core can be thought of as a triangular antiprism, allowing 3 tubes to rotate about a central axial tube; the implication is that, theoretically, an infinite number of self intersecting models can be

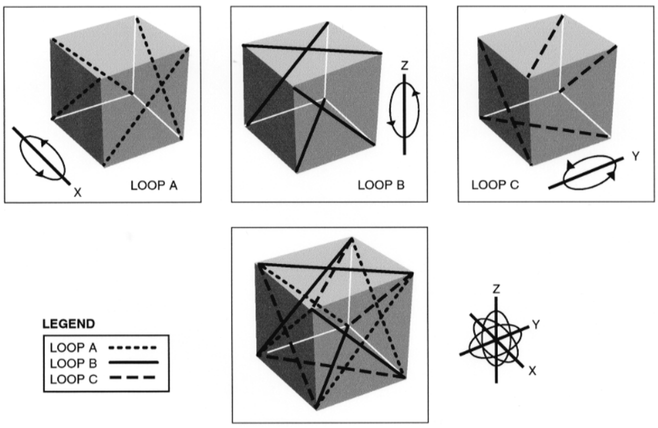


Fig. 3. DPTM Loop Pattern @ Intersecting Tetrahedra (4 tubes (not shown)-3 loops (3 per tube)). Note: Tubes not shown, loop passes to diagonal corner.

made, as the number of tubes around a central axial tube increases. The author has not constructed any models of this type, but the stellated dodecahedron core with 6 tubes (5 around 1), suggest this possibility.

*4.2. The cube*

Octahedral core with 4 equal length tubes, 4 loops (Fig. 4).

Another possible loop arrangement involves 4 loops with 4 tubes, defining the edges of a cube. Here, only 3 loops of the possible 4 pass through a tube at any given time. As the tubes are translated, the cube deforms (Fig. 5).

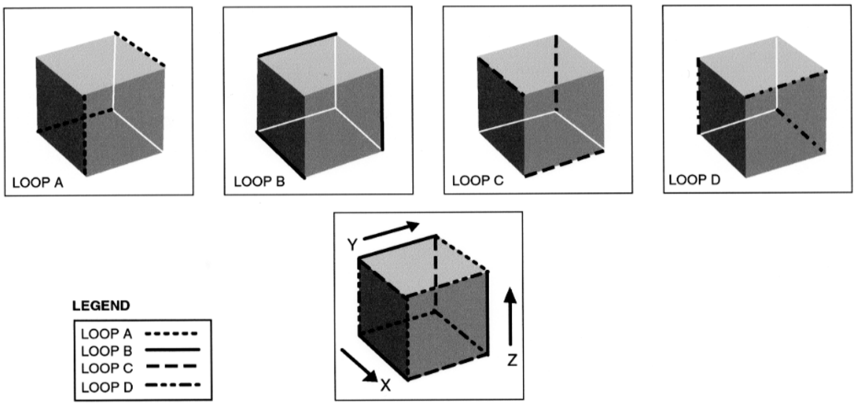


Fig. 4. DPTM Loop Pattern @ Cube Edges (4 tubes (not shown)-4 loops (3 per tube)). Note: Tubes not shown, loop passes to diagonal corner.

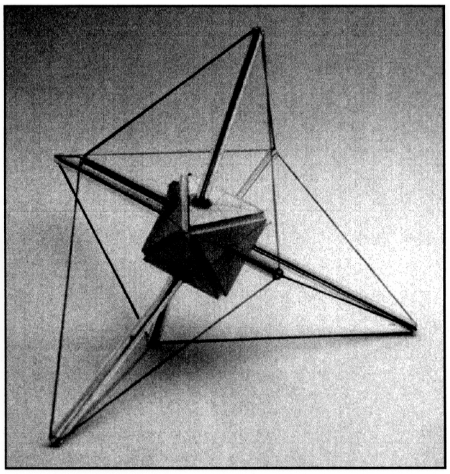

Fig. 5. DPTM with Octahedron core, 4 tubes, 4 loops—Deforming Cube.

### 4.3. *The octahedron*

Cubic core with three equal length tubes, 4 colored loops passing through each tube once (Fig. 6).

This generates the edges of the octahedron. The color edge patterning mirrotate from face to opposite face of the octahedron. This is in general true for all polyhedra that essentially deform but do not interpenetrate. In addition, the 3 edges of each loop pass through *X*, *Y*, and *Z* plane respectvely

### 4.4. *The icosahedron and stellated dodecahedron*

A dodecahedron core, 6 equal length tubes (Fig. 7) Pictured here are two configurations resulting from 6 tubes, the icosahedron and stellated dodecahedron. Both have multiple possibilities for loop arrangement and patterning.

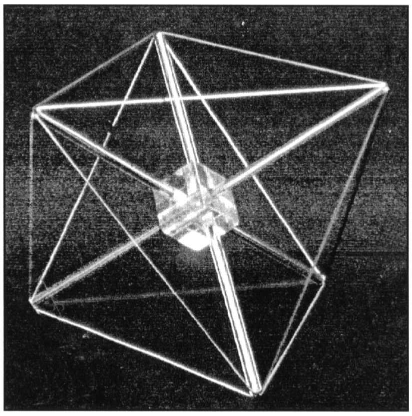

Fig. 6. DPTM with Cube core, 3 tubes, 4 loops—Deforming Octahedron.

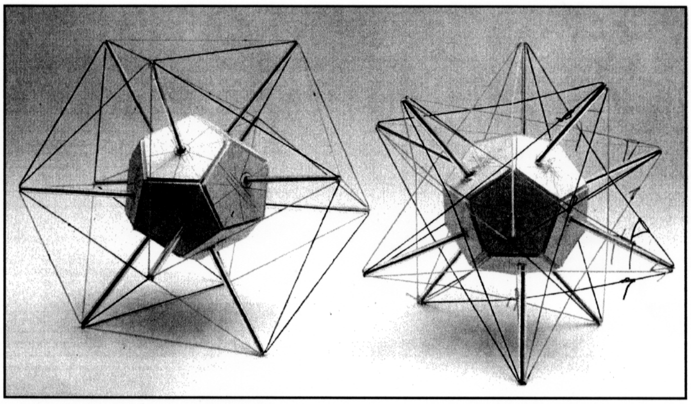

Fig. 7. DPTM with Dodecahedron core, 6 tubes, 2 patterns—Icosahedron and Stellated Dodecahedron.

## 5. Combinations and Multiple Cores

Combinations of the various models may be constructed based on vertex, edge and face bonding of shapes. This effects string patterning. Multiple core models are also possible (Figs. 8 and 9).

## 6. Conclusion

This paper has been an introduction to the possibilities of Dynamic Polystring Transformahedra Modeling (DPTM). As a new concept in the exploration of spatial structures and polyhedra, it offers a rich variety of possibilities for modeling dynamic spatial relationships.

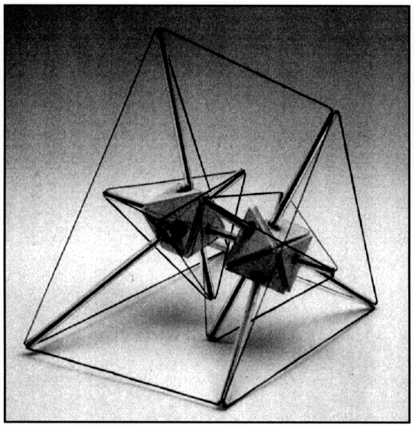

Fig. 8. DPTM with 2 Octahedron cores, 7 tubes, 3 loops—Prism.

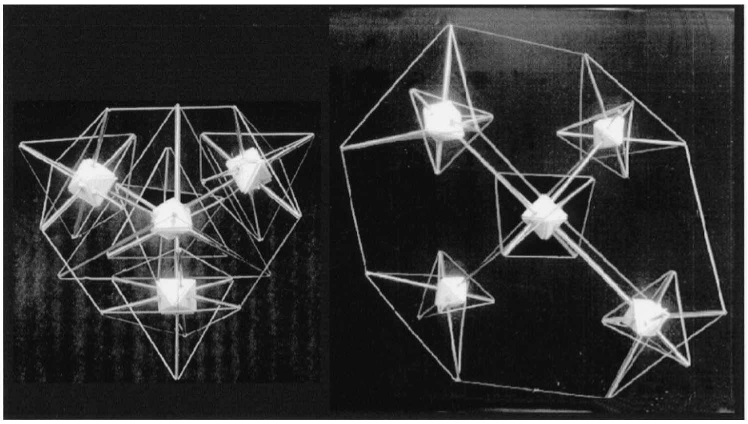

Fig. 9. DPTM with 5 Octahedron cores, 16 tubes, 3 loops—Contracted and Expanded.