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# Pend-Art: A New Branch of Computer Art

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**Abstract.** Superposition of more than one single harmonic oscillation in both $x$- and $y$-direction of the plane yields an infinite family of curves. This is a kind of generalization of the classical *Lissajous* curves. The variety of such curves may serve not only displaying abstract forms but conceptual contents as well. This can be the source of a new branch of computer art.

Let us take a double pendulum, i.e. a construction such that a second pendulum is linked to the end of a pendulum. The linkage is constructed so that the planes of swing of the pendulums are perpendicular to each other. Projecting the motion of the end of the second pendulum to a horizontal plane, one obtains a kind of curves named after the last century French physicist *J. A. Lissajous*. In fact, such curves are well known to physicists and engineers, and prior to the general use of computers, they could be extensively studied using oscilloscopes.

Now using more than one pendulum in both planes of swing, one obtains curves that can be regarded as a kind of generalization of the classical *Lissajous* curves. Moreover, our demonstrative physical model can be replaced by a mathematical model, which in turn makes possible the use of computer for displaying these curves.

This is the technical background for starting a new branch of computer art, to which I coined the name "PEND-ART". Some of the first pieces are shown here. They have been made using *Maple V* program. The mathematical conditions to generate these drawings are as follows:

(a) both the $x$- and $y$-coordinate is a function of the same (say, time) parameter, so that every pattern consists of just a single curve;

(b) both of these functions are a finite linear combination of sine and/or cosine functions (one such summand in these linear combinations describes the motion of a single pendulum, so that the coefficient in the summand corresponds to the amplitude of swing);

(c) the ratio of the periods of any two sine or cosine functions is a rational number, (this means that the frequency of each pendulum is an integer multiple of a base frequency), so that the curve is always closed (i.e. each pendulum returns to its starting point).

To take an example, the curve of the "Eye" (see Fig. 1a) can be given as

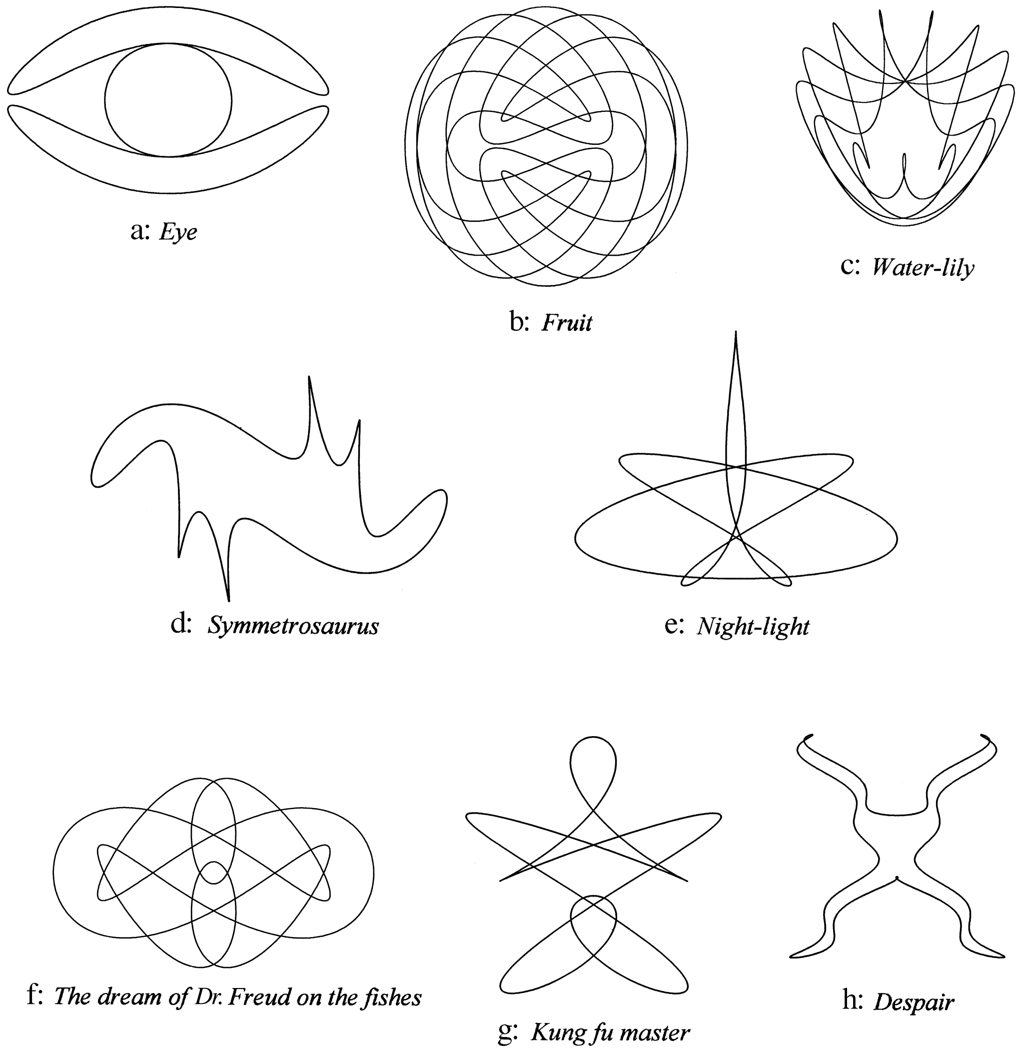


Fig. 1. Examples of PEND-ART. a: “Eye”, b: “Fruit”, c: “Water-lily”, d: “Symmetrosaurus”, e: “Night-light”, f: “The dream of Dr. Freud on the fishes”, g: “Kung fu master”, h: “Despair”

$$x = \cos(t) - 2\cos(3t),$$
$$y = 0.875\sin(t) + 0.6125\sin(5t).$$

The aim of the author is to search for the possibilities of displaying either abstract forms or conceptual contents. Although the mathematical tools have been intentionally restricted to a degree as is given above, the wealth of forms that opens to us is astonishing.

In this realm of “visual mathematics”, of course, there is some danger of banalities.

Aside from the classical *Lissajous* curves (which themselves occur in an infinite variety and in fact are beautiful patterns as well), our curves as defined above are closely related to another family of well-known curves, namely, to *epicycloids*, *hypocycloids*, *epitrochoids* and *hypotrochoids*. Such curves are produced by a point fixed in a definite manner to a circle rolling along the exterior or round the inside of another circle. Some of them were known even in the antiquity. In fact, *Ptolemy*, the great astronomer applied these to describe the apparent motion of celestial bodies. On the other hand, nowadays they can also be obtained by drawing toys made for children. These are undoubtedly beautiful (and highly symmetrical) flower-like patterns, but we use them at the very most in modified form.

Thus, the border of the world of all these classical curves is definitely transgressed and "PEND-ART" is defined to discover newer and newer forms. At the same time, symmetry is an intrinsic feature of these drawings implied by our method. Hence, we feel that they together provide a beautiful representation of both *Katachi* and *Symmetry*. Accordingly, we hope that the computer, frequently regarded or even accused as an inanimate instrument, (in fact, in itself it is!) helps in this way, too, to build further the bridge between the "Two Cultures".