Analytical Approach for Oscillation Properties of Soft Materials

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(Received April 19, 2000; Accepted May 19, 2000)

Keywords: Modified Brusselator Model, The Equation of Motion, Soft Materials, Fabric Pendulum, Free Damped Oscillations

Abstract. It is well known that the Brusselator model reveals simple oscillations and damped oscillations by the suitable parameters. However there is no chemical reactor system that corresponds to the Brusselator model. We have a good real physical system that corresponds to the Brusselator model under some conditions. The system is a free damped oscillation of soft material and described by an equation of motion. In this paper, we show the relation between the Brusselator model and the physical system. We also estimate the experimental data of the physical system.

1. Introduction

We consider a modified Brusselator model in an open system.

$$\begin{aligned}&A \to X,\\&B + X \to Y + D,\\&2X + Y \to (2 + \varepsilon)X,\\&X \to E.\end{aligned}\tag{1}$$

The fluxes A and B are assumed to be constant. X and Y are the intermediates, and X changes Y into X by the autocatalytic reaction. Free parameter ε reveals to a "strength " of the autocatalytic process and this modified Brusselator model is reduced to the original Brusselator model (PRIGOGINE and LEFEVER, 1968; PRIGOGINE *et al.*, 1969) when $\varepsilon = 1$. We show briefly that this modified Brusselator model is related to a physical phenomenon described by the equation of motion under some special conditions.

The rate equations concerning X and Y are

$$\frac{dX}{dt} = A - (B + 1)X + \varepsilon X^2 Y,$$

$$\frac{dY}{dt} = BX - X^2Y. \tag{2}$$

Here, we take rate constants are equal to 1 for simplicity.

The stationary solution is given by the controllable positive parameters A and B, and free parameter ε.

$$(X_0, Y_0) = \left(\frac{A}{B(1-\varepsilon)+1},\ B\left(\frac{B(1-\varepsilon)+1}{A}\right)\right). \tag{3}$$

Time developments of the fluctuations (x, y) around this stationary solution are approximated to the following linearized equations

$$\frac{dx}{dt} = L_{11}x + L_{12}y,$$

$$\frac{dy}{dt} = L_{21}x + L_{22}y, \tag{4}$$

where Jacobian's elements are

$$L_{11} = (2\varepsilon - 1)B - 1, \qquad L_{12} = \varepsilon X_0^{\,2},$$

$$L_{21} = -B, \qquad L_{22} = -X_0^{\,2}. \tag{5}$$

We can choose $L_{11} = 0$ under the condition $B = 1/(2\varepsilon - 1)$. We also get another condition $\varepsilon > 1/2$ because B is positive. In this choice, $\mathrm{tr}J = -X_0^{\,2} < 0$ and $\det J = \varepsilon BX_0^{\,2} > 0$. Therefore, if the condition $(-\mathrm{tr}J)^2 - 4\det J < 0$ (or $A < 2[\varepsilon/(2\varepsilon - 1)]^{3/2}$) is satisfied, then a damped oscillation occurs.

After the normalization $\varepsilon X_0^{\,2}y \equiv y'$,we can scale $L_{12} = 1$. Finally, by the replacements

$$\varepsilon BX_0^{\,2} \equiv \frac{k}{m},\ X_0^{\,2} \equiv 2\gamma, \tag{6}$$

we get the equation of motion

$$\frac{dx}{dt} = y',$$

$$\frac{dy'}{dt} = -\frac{k}{m}x - 2\gamma y' \tag{7}$$

or

$$m\frac{d^2x}{dt^2} + 2m\gamma\frac{dx}{dt} + kx = 0. \tag{8}$$

This is just the equation of motion at each position for the free damped oscillation of soft material as shown in Sec. 2. The modified Brusselator model around the stationary solution was related to the physical phenomenon described by the equation of motion under some special conditions. The previous controllable parameters A and B correspond to the damping coefficient γ and the elastic constant k. Here, m is the mass of soft material. Therefore, we can also estimate the controllable parameters in the modified Brusselator model by means of analyzing the free damped oscillation of soft material.

2. The Equation of Motion for Soft Materials

Soft materials can be changed in the form by the external forces contrary to rigid bodies and we have extended the equation of motion for soft material phenomenologically from that for rigid body by considering this changeability in the form of soft material (MORIKAWA, 1995).

The equation of motion for the free damped oscillation of rigid body is written by the moment of inertia I under the assumption that the resistance is proportional to its velocity

$$I\frac{d^2\theta}{dt^2} + c_I h^2\frac{d\theta}{dt} + mgh\theta = 0, \tag{9}$$

where $h = l/2$ and l is the length of rigid body. If rigid body has uniform mass distribution, the moment of inertia is proportional to its mass $I = m\,(l^2/12 + h^2)$. The displacement at each position u on rigid body is approximated $Xu \fallingdotseq u\theta$ $(0 \le u \le l)$ within the small amplitude oscillation. Then the equation of motion at each position u is derived

$$m\frac{d^2Xu}{dt^2} + 2m\gamma\frac{dX_u}{dt} + kX_u = 0, \tag{10}$$

$$k = m\omega_0^2,\ \omega_0 = 2\pi\nu_0.$$

We have extended this equation of motion to that of soft material phenomenologically considering its changeability in the form

$$m\frac{d^2X_u}{dt^2} + 2m\gamma_u\frac{dX_u}{dt} + k_uX_u = 0. \tag{11}$$

The elastic coefficient $k_u = m\omega_{0u}^2$ $(\omega_{0u} = 2\pi\nu_{0\mathrm{u}})$ and the damping coefficient γ_u have the suffix u which reflect the changeability in the form of soft material. The proper frequency

ν_{0u} has the relation with the oscillation period T_u and the damping coefficient γ_u

$$\nu_{0u} = \left[\left(\frac{1}{T_u}\right)^2 + \left(\frac{\gamma_u}{2\pi}\right)^2\right]^{1/2}. \tag{12}$$

Thus, we can analyze the oscillation characteristics by estimating the oscillation curves.

In the ideal case without damping, the proper frequency is given by

$$\nu_0 = \frac{1}{2\pi}\left[\frac{g}{l}\right]^{1/2} \quad \text{(simple pendulum)}, \tag{13a}$$

$$\nu_0 = \frac{1}{2\pi}\left[\frac{g}{L}\right]^{1/2}, \quad L = \frac{\left[\frac{l^2}{12} + h^2\right]}{h} \quad \text{(physical pendulum)}. \tag{13b}$$

We can compare the experimental values with these theoretical ones.

3. Experiments

3.1. Oscillation materials

We picked up several cotton fabric elements as soft materials and an acrylic plate as rigid body. All fabric elements were cut out to be rectangular with the width 5 cm from the cloth materials.

We cut out 4 samples (sample A–D) with the same length 20 cm from the same cloth material 1. These samples were expected to have the same characteristics. We cut out the fabric strips with the 5 types of length (5, 10, 20, 40, 60 cm) from the cloth material 1. We also prepared the fabric strips with the same length 20 cm from 4 kinds of cloth materials (1–4). Each thickness is 5.65, 3.20, 3.33, 6.56×10^{-2} cm and each weight is 1.50, 0.81, 0.65, 2.27 g respectively. The acrylic plate with the length 20 cm and the width 5 cm was prepared to compare with the fabric strips.

3.2. Oscillation experiments

The upper end of each fabric strip was fixed to a holding rod in a horizontal direction that could rotate freely on its axis. Each fabric strip was hung down and oscillated freely in the air centering round the rod just like the pendulum. Therefore, we call this as “fabric pendulum”. In some cases, we put on several weights at the lower free end of the fabric strips.

The acrylic plate was also fixed to the holding rod. We hung down and oscillated the acrylic plate freely in the air as the fabric strips.

The oscillation amplitude was restricted within the narrow range in order to realize the small angle approximation.

The control of the temperature and the humidity is crucial because cloth materials contain the air inside and also water except for special cases. The temperature 21 ± 2°C and the humidity 65 ± 5% were kept during the whole experiments.

4. Results and Discussions

The oscillation characteristics depend on the position of soft material. In this paper, we focus on the oscillation properties at the lower free end of soft material.

The discrepancy of the oscillation characteristics between 4 samples from the same cloth material 1 became smaller as mass ratio between the weight and the fabric strip increased. We pick up the sample D and exhibit the damping coefficient and the proper frequency against mass ratio (Fig.1). Fabric pendulum was sensitively affected by the weights. The oscillation characteristics became rapidly smaller and closer to those of simple pendulum as mass ratio increased.

Figure 2 reveals the damping coefficient against the length of the fabric strip without weights from the cloth material 1. Typical statistical error is shown for the material 1 with the length 20 cm. The damping coefficient showed a trend to be smaller and closer to the theoretical value of simple pendulum beyond the statistical errors as the length of the fabric strip increased.

It was found that the changeability in the form and the size effect of soft material on the oscillation characteristics decreased as the length of the fabric strip increased. This is the similar effect to the mass ratio’s increase as shown above.

Figure 3 presents the damping coefficient against mass ratio for the fabric strips with the length 20 cm from 4 kinds of cloth materials. The damping coefficient became smaller and closer to the theoretical value of simple pendulum as mass ratio increased as shown in Fig.1. The oscillation properties were slightly different between the heavier and thicker fabric strips from the material 1 and 4 and the lighter and thinner strips from the material 2 and 3.

These are interpreted from the transition of the changeability in the form of the fabric strip. The fabric strip has suffered some resistances due to the air existence and the inner frictions, and the oscillation has been damped. Within the small mass ratio region, the inner frictions dominate its damping and the oscillations of the thinner fabric strips were damped faster than the thicker ones. The damping coefficients of the fabric strips were larger than that of the acrylic plate. As mass ratio increases, the changeability in the form of the fabric strip decreases and the air resistance dominates its damping. Therefore, the oscillations of the acrylic plate were strongly damped than those of the fabric strips.

We can estimate the controllable parameters in the modified Brusselator model corresponding to the free damped oscillation of soft material by Eq. (6) from the experimental data. In the fabric strip with the length 20 cm without weights, $\varepsilon \fallingdotseq 0.50$, $A \fallingdotseq 78$, and $B \fallingdotseq 220$. The fluxes are larger than those of the typical chemical reactor system by the order of 2. This is a reflection of the fact that $k/m >> \gamma$ in fabric pendulum.

5. Conclusions

It is well known that there is no chemical reactor system that corresponds to the Brusselator model. The modified Brusselator model linearized by fluctuations corresponds to the equation of motion for the free damped oscillation of soft material under the conditions $B = 1/(2\varepsilon - 1)$, $A < 2\,[\varepsilon/(2\varepsilon - 1)]^{3/2}$, $\varepsilon BX_0^{\,2} = k/m$, and $X_0^{\,2} = 2\gamma$.

We also estimated the oscillation properties of soft materials. Fabric pendulum was

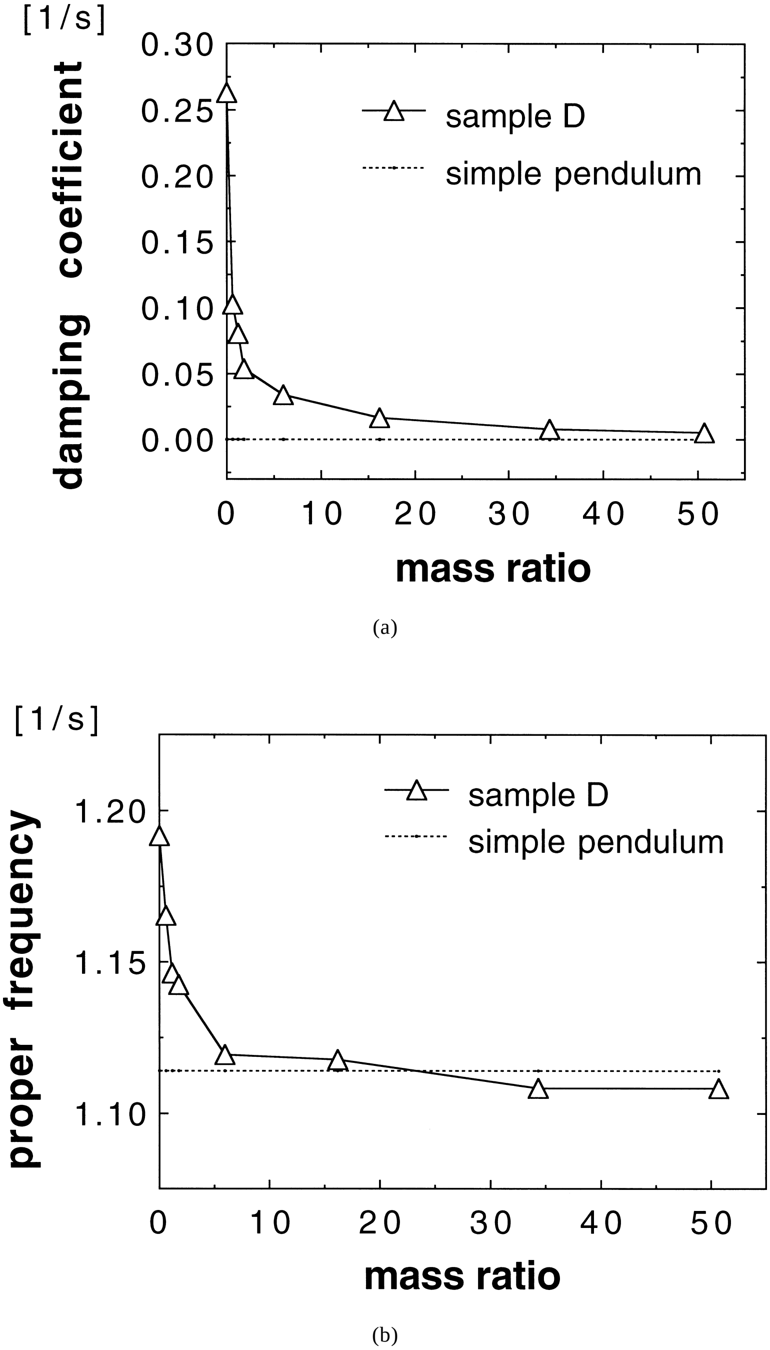

Fig. 1. Damping coefficient (a) and proper frequency (b) against mass ratio for the sample D from the cloth material 1.

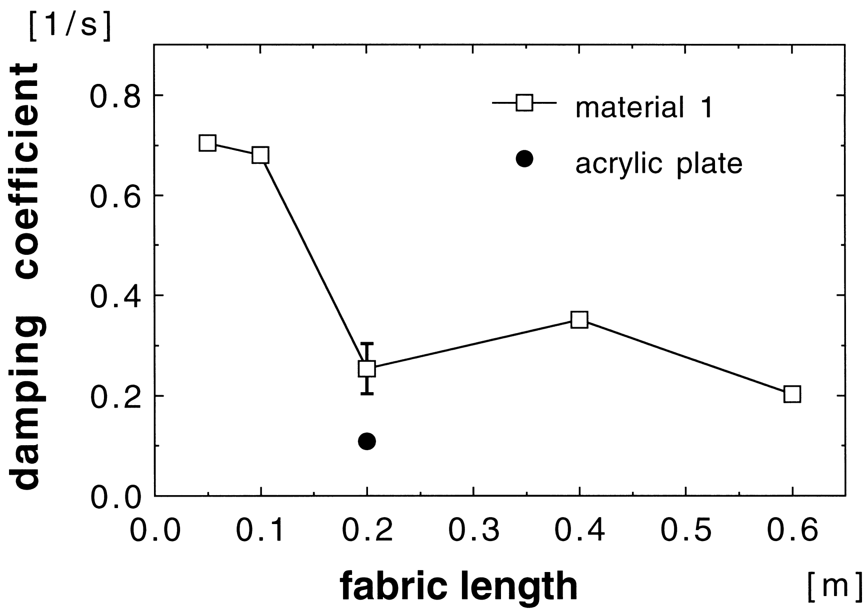

Fig. 2. Damping coefficient against the length for the fabric strips without weights from the cloth material 1.

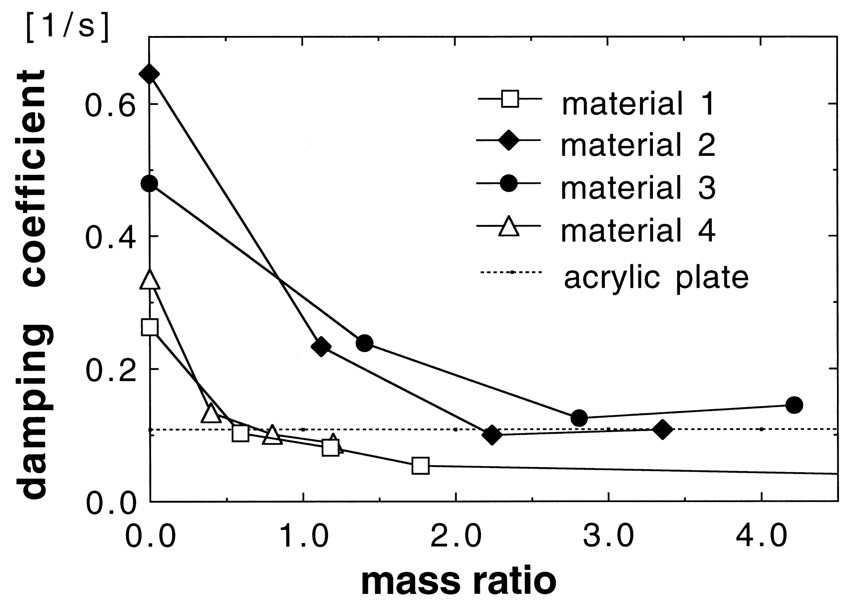

Fig. 3. Damping coefficient against mass ratio for the fabric strips with the length 20 cm from 4 kinds of cloth materials.

found to be closer to simple pendulum when the weight and/or the length increased. The weights improved the oscillation behaviors and the damped oscillation curves became smoothly instead of decreasing the features of soft materials. The extrapolation of mass ratio to 0 may be effective for soft materials accompanied with the large statistical errors experimentally.

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