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Information Complexity of Laminar Chaotic Mixing Field Produced by Two Parallel, Rotating Cylinders

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Abstract. The purpose of the present work is to numerically analyze information complexity of the laminar chaotic mixing field produced by two parallel, rotating cylinders from a viewpoint of optimization of mixing field. The new method for estimating complexity presented is based on the information theory and the nonlinear theory. A very simple flow system consisting of two parallel rotating cylinders was investigated. A Poincaré section was constructed by tracing the trajectories of tracer particles in order to reveal the final mixing pattern. As a measure of spatial homogeneity of particle dispersion, information entropy was calculated. It has been found that the information entropy can suitably evaluate mixedness of laminar mixing fields.

1. Introduction

Mixing is one of the most fundamental and important operations in industrial chemical processes and a wide variety of mixing equipment has been commercially available in a wide variety of impeller configurations, size and operation. The purpose of optimization of mixing equipment and operation is to minimize the input energy and time to reach a desirable mixing state.

There are two phases of approaches for investigation of mixing in chemical engineering, i.e. analytical phase and synthesis one (YOSHIKAWA, 1979).

In the analytical phase, for understanding mixing phenomena, an extensive interest has been taken in nonlinear dynamic theory to fundamentally study the mechanism of mixing motion in simplified two-dimensional creeping flows from a theoretical point of view since the pioneering work of AREF (1984), OTTINO *et al*. (1988) and OTTINO (1989). INOUE and HIRATA (2000) pointed out that an entanglement motion between stable-manifolds and unstable-manifolds near hyperbolic fixed points and turnstile lobes plays an important role for mixing. They also suggested that Poincaré mapping is very useful and effective for the study of fluid mixing and is similar in dispersion pattern to an ultimate mixed state because this mapping visualizes invariant manifolds and KAM curves on a Poincaré section. The chaotic mixing theory has given significant advances in the understanding of fundamental mixing-effective motions.

Regarding the synthesis phase, a rather wide gap between basic theoretical development and industrial practice still remains for designing and/or optimizing practical mixing devices. Some difficulties arise in the process of designing these mixing devices:

(1) three-dimensional mixing field,
(2) complexity of the flow field produced by a complex form of practical agitators,
(3) how to estimate of mixing state.

Regarding the spatial difficulty, it is necessary to establish an appropriate method for estimating mixedness depending on the relevant parameters.

The purpose of the present work is to obtain an estimation function of mixedness in laminar basic flow in order to construct a new design concept for mixing device from the viewpoints of the chaotic mixing theory. The new estimation method presented here utilizes Poincaré mapping based on the above-mentioned chaotic mixing theory and on the information theory.

2. Flow Model

A simple flow system consisting of two parallel rotating cylinders was investigated as an idealization of a stirred vessel, shown in Fig. 1. This flow system, firstly introduced by AREF (1984), produces two co-rotating point vortices. The two co-rotating cylinders with an outer diameter, d = 15 mm, were used instead of point vortices in order to compare with the experimental results. These two cylinders were separated by a fixed distance $2a$. They were revolved on and off periodically and alternately with a constant time period T. This imaginary flow model is based on the assumption that while one of the cylinders is on, the other does not exist in the flow field. The rotation of both cylinders was clockwise. The rotational displacement, P, was given in the round number of rotation per one cycle as a driving protocol. The velocity profile around a rotating cylinder was obtained by analytically solving the Navier-Stokes equations under the conditions of zero radial velocity and of infinite outer boundary. In order to obtain the final mixing patterns, a Poincaré section was constructed by tracing the trajectories of tracer particles after each cycle of the driving protocol.

3. Experimental Details

In order to confirm the validation of the above imaginary model, the time-dependent streaklines and/or Poincaré sections were observed by using a flow-visualization technique. The cylinders were 15 mm in outer diameter. The cylindrical vessel was 377 mm in inner diameter. The origin (0, 0) of the coordinate system corresponds to the center of the cylindrical vessel. The two cylinders were located at (x, y) = (–33 mm, 0 mm) and (33 mm, 0 mm), respectively. Silicone oil (viscosity μ =10,000 cP, Newtonian fluid) was used as working fluid. These two cylinders were separated by $2a$ = 66 mm as shown in Fig. 1. In order to eliminate the effect of the bottom of the cylindrical vessel on flow field, water

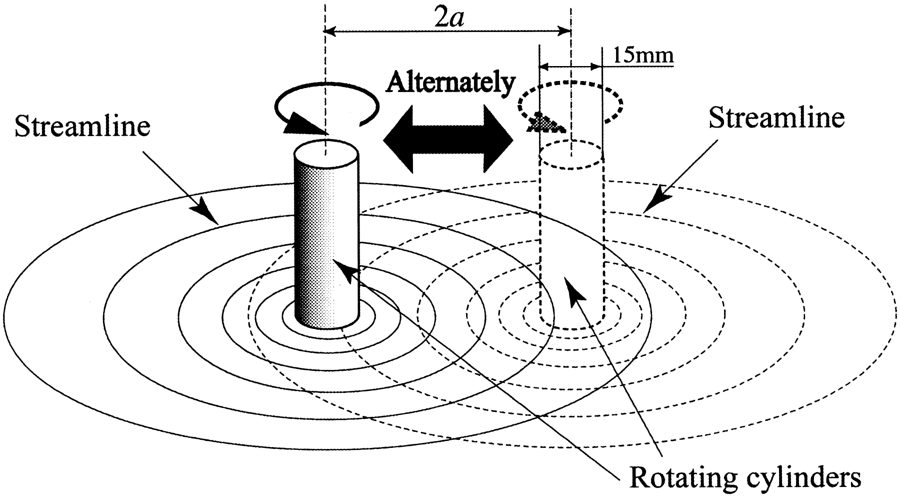

Fig. 1. Schematic illustration of two-dimensional flow model.

regarded as sufficiently low viscosity fluid was laid below the silicone oil. For the sake of investigation of streaklines, small droplets of silicone oil colored by dyestuff were aligned on the y-axis. The visual data of streakline patterns were taken by a digital camera. In order to obtain a Poincaré section, the trajectories of several tracer solid particles (Amberlite, 1 mm dia.) were observed by the same digital camera.

4. Estimation Method

Entropy can be a measure characterizing complexity of the pattern and/or the symbol sequence (KANEKO, 1989). OGAWA and ITO (1975) introduced the entropy in the information theory to qualify mixedness in a mixing device. In general, the information entropy indicates the degree of homogeneity of probability distribution. In the present work, the information entropy was introduced to estimate the degree of homogeneity of tracer-mark distribution as a Poincaré section based on the chaotic mixing theory, which corresponds to the mixing state.

After getting a sufficient number of marks of tracer particles as a Poincaré section, the information entropy was calculated as a measure of spatial homogeneity of particle dispersion by the following procedure. The flow field was partitioned into $m \times n$ small square boxes having an equal area. The probability of marks of tracer particles in the (i, j) box, $P_{i,j}$, was calculated. The information entropy, S, was defined as follows:

$$S = -\sum_{j=1}^{n} \sum_{i=1}^{m} P_{i,j} \log P_{i,j}. \tag{1}$$

When tracer marks are concentrated into a small number of boxes, the information entropy gives a small value. On the contrary, the entropy becomes large when tracer marks are dispersed evenly over the whole region. This suggests that the information entropy can be a measure in the homogeneity of mixing fields.

5. Results and Discussion

5.1. Mixing patterns and Poincaré section

Figure 2 shows an experimental result of streakline patterns (a), (b) and Poincaré sections (c), (d). In this series of experiment of streakline patterns, the total round number of each cylinder rotation is fixed at 32, which means the same input power for each run. Just after one cycle with revolution number P = 1 of each cylinder, an orderly structure dominates in the flow field. This result suggests insufficient mixing at this case. It can be seen from the streakline pattern that 'stretching' and 'folding' motion is efficiently iterated at P = 4. Consequently, a disorderly structure, called chaotic sea, grows in the Poincaré section. It has been found that the mixing in chaotic sea is enhanced.

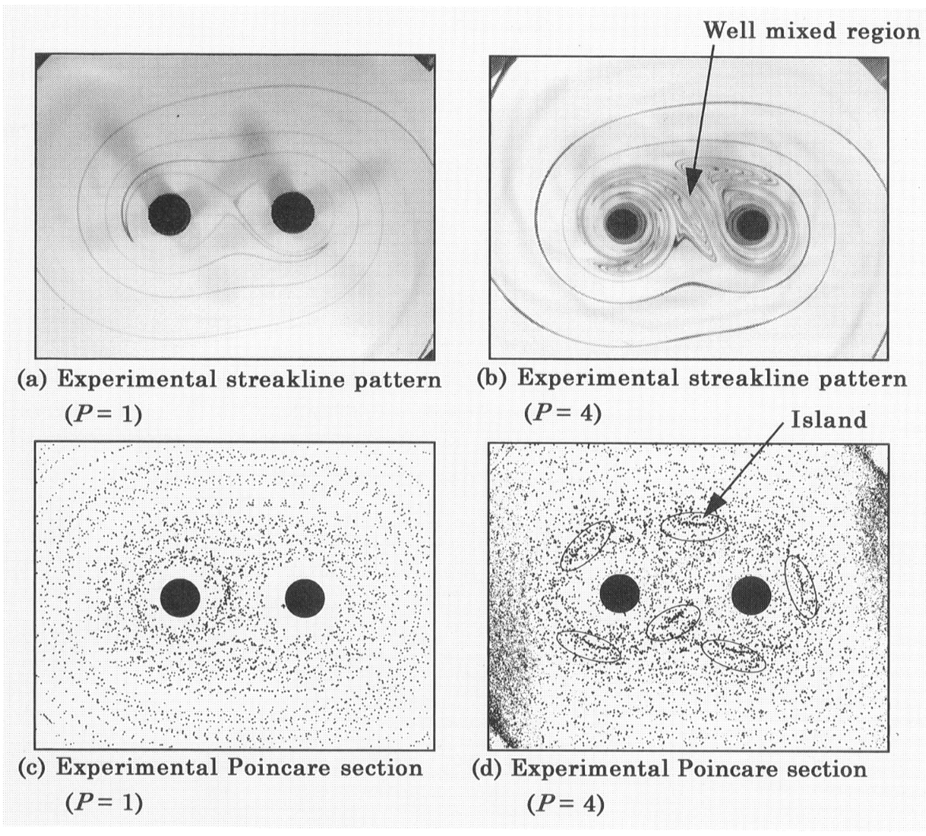

Fig. 2. Experimental streakline patterns and Poincaré sections.

Figures 3 and 4 show numerical Poincaré sections after 10,000 cycles obtained by varying revolution number *P*. In Figs. 3 and 4, the 16 or 32 particles were evenly aligned on the *x*-axis ($-0.8 \le x \le 0.8$) as the initial position, respectively. Owing to the assumption that while one of the cylinders is on, the other one does not exist in the flow field, the disorderly structure can be seen even within the space of the two cylinders. Nevertheless, the numerical results give a good agreement with the experimental ones. Comparing with Figs. 3 and 4, it has been found that the area of chaotic region seems to be independent of initial number particles if the Poincaré section has a sufficient number of trajectory marks of tracer particles. The small orderly structure around the chaotic mixing region, i.e. island structure, can be easily detected when tracer particles are densely distributed in both the experimental and numerical Poincaré sections at $P = 4$ (see Figs. 2(d) and 4(c)). The chaotic (disorderly) region which implies a well mixing region grows with increasing the revolution number *P*. It can be considered that this model is valid and useful for discussion on two-

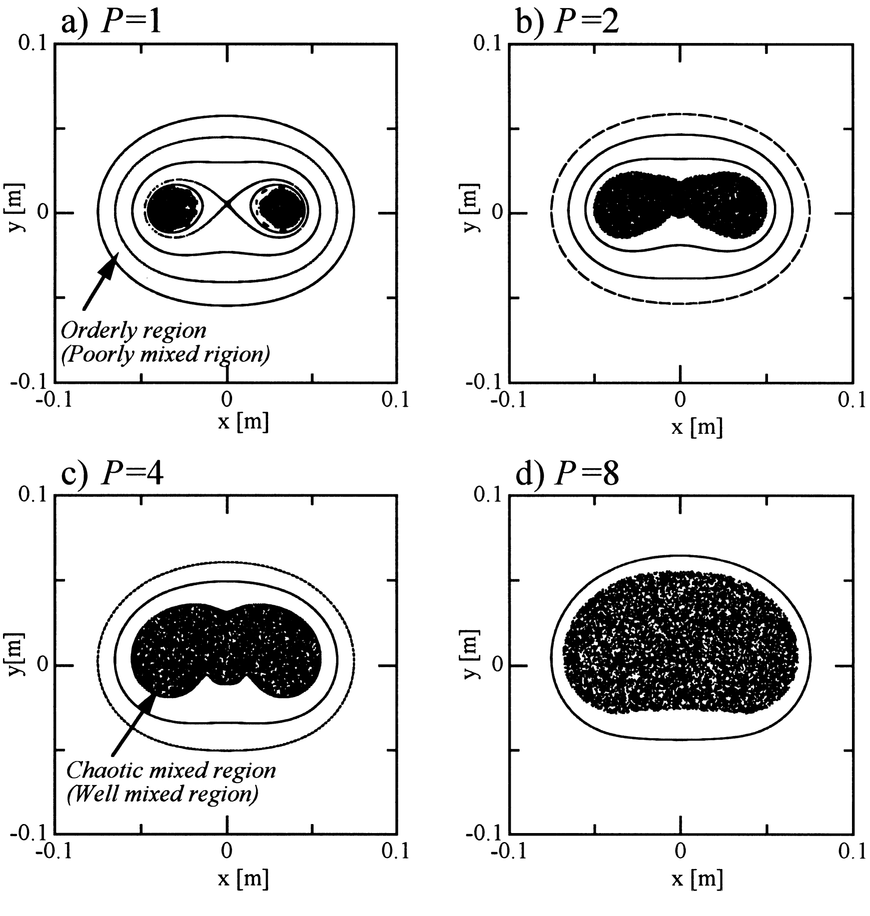

Fig. 3. Poincaré sections after 10,000 cycles (16 particles).

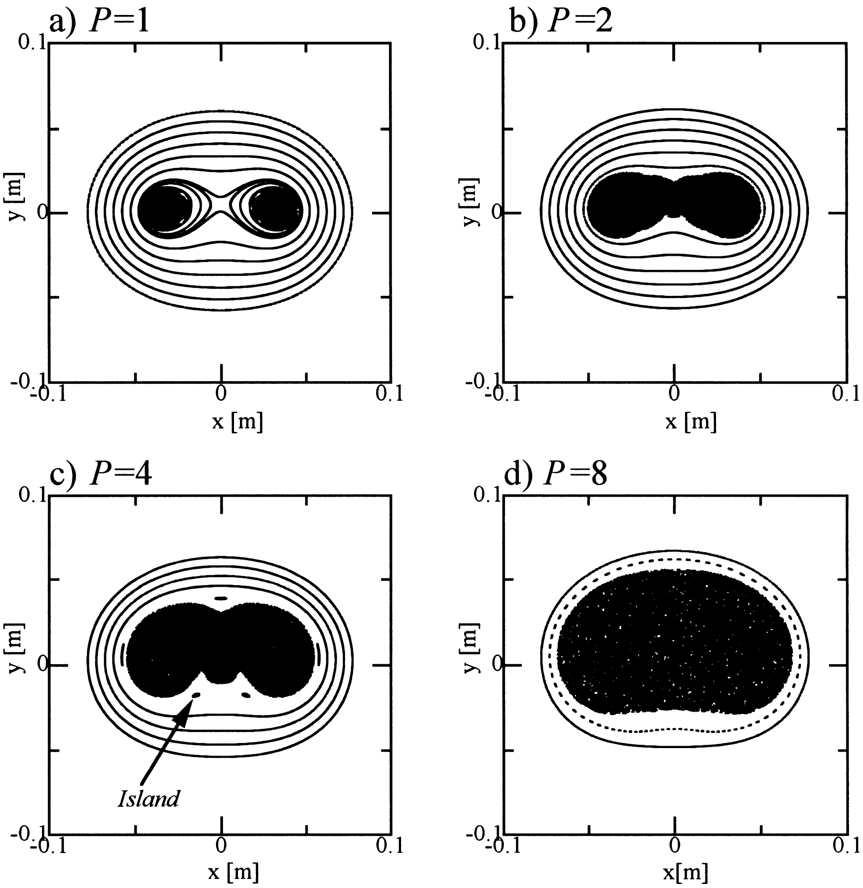

Fig. 4. Poincaré sections after 10,000 cycles (32 particles).

dimensional laminar mixing. The numerical results obtained by the model will be discussed in the following sections.

5.2. Estimation of mixedness

The information entropy was calculated by varying the area of unit box. The information entropy was also calculated for regularly homogeneous and entirely random states as the two limits of mixing state for comparison with the numerical results. As shown in Fig. 5, at a given area of unit box, the information entropy becomes large with increasing rotation number. In the cases of homogeneous and random mixing states, the tracer marks are dispersed evenly over the whole region. Consequently, it is difficult to discriminate between these two cases. However, the regularly homogeneous state has the largest value and the random state gives the second one in the range of area of unit box less than 10^{-5}. This result suggests that unevenness of distribution of tracer marks in the random state affects on the information entropy in the range of sufficiently small area of unit box. Hence, it is possible to discriminate between these two cases by decreasing area of unit box. These results indicate that this estimation method successfully evaluates mixedness of laminar mixing fields of this kind. However, it should be noticed that the entropy increases with decreasing area of unit box and that the entropy difference decreases with increasing number of tracer particles (compare with Fig. 5 (a) and (b)). This result implies that the information entropy depends on the spatial scale of unit box and the number of tracer particles.

Furthermore, in the range of small scale of unit box, Fig. 5 shows that the information entropy linearly decreased with increasing area of unit box on logarithmic scale in all cases of the two-dimensional laminar chaotic mixing generated from the present flow model. On the other hand, homogeneous and random mixing states do not show such a linear decrease. It has been found that the local scaling behavior of the information entropy for spatial scale has been observed in the chaotic mixing states. This result indicated that mixing states have a fractal structure in the case of chaotic mixing. Therefore, more sophisticated estimation might be possible by obtaining fractal dimension and/or singularity spectrum in this case.

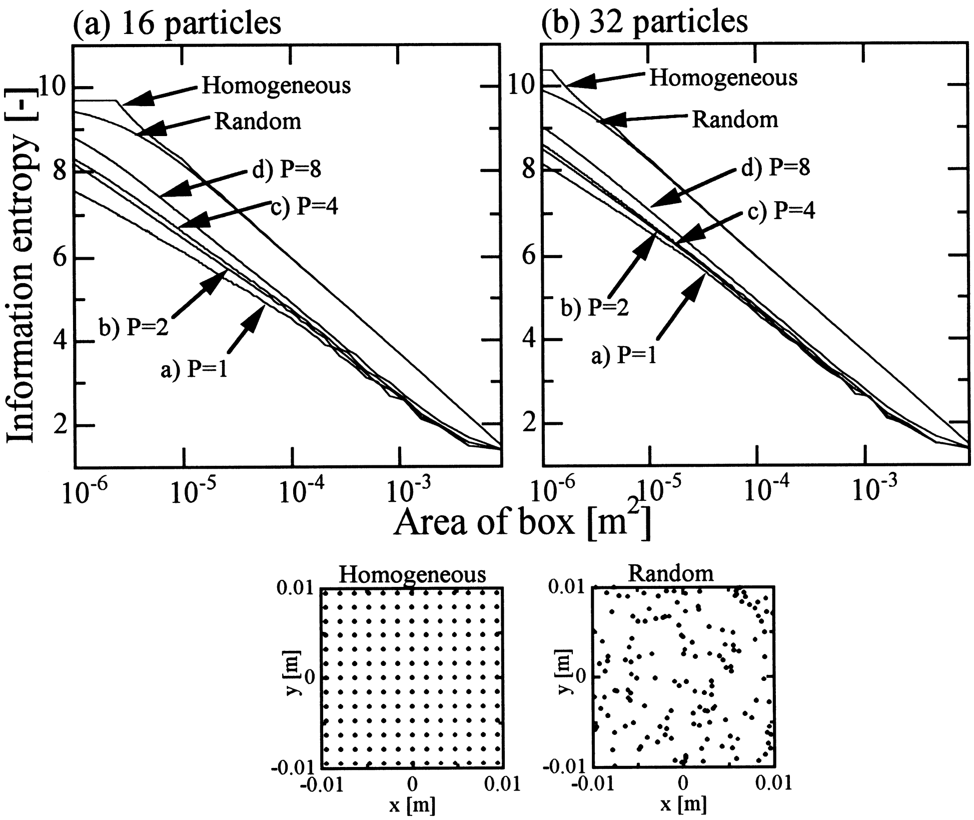

Fig. 5. Information entropy against area of unit box: (a) 16 particles and (b) 32 particles.

6. Conclusion

The following conclusion may be deduced:

The information entropy successfully evaluates mixedness of laminar mixing field and the information entropy can suitably evaluate mixedness of laminar mixing fields.

It is necessary to choose the appropriate values of both the area of unit box and the number of tracer particles in order to obtain the entropy estimation.

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