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Earthenware Reconstruction Based on the Shape Similarity among Potsherds

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Abstract. This paper proposes an earthenware reconstruction system, which can automatically reconstruct some earthenware from numerous given potsherds in two dimensional grayscale images. The system supposes that given potsherds are thin and moderately flat and small. Some earthenware having three dimensional shape, such as crocks, can be reconstructed by the system within the supposition. The system performs earthenware reconstruction through two phases. At the first phase, potsherds are joining automatically in two dimensions. An efficient joint detection algorithm is proposed at the phase. At the second phase, three dimensional shape is recovered by adequate three dimensional transformation. In this paper, some experimental results of reconstruction from numerous potsherds are also reported.

1. Introduction

In the field of archaeology, the shape of earthenware offers significant knowledge of not only the era when it is used, but also ancestor's life style and cultural interchange. Thus, it is very important to reconstruct earthenware from excavated potsherds. Reconstruction of earthenware, however, is attended with difficulties, due to a numerous excavation of potsherds, weathering and lack of parts. The reconstruction, therefore, imposes a heavy task on archaeologists, and a numerous earthenware has not been reconstructed yet. In order to reduce the task of the reconstruction, we have developed a system, which can automatically reconstruct earthenware from given potsherds by KANOH (2000).

In general, earthenware should be reconstructed in three dimensions, because potsherds have three dimensional shape. We are taking our stand on that some earthenware shaped in three dimensions can be reconstructed from potsherds in two dimensional images, if the earthenware consists of numerous potsherds, and if each of the potsherds is adequately small. In this paper, at the beginning of this research, we suppose that potsherds, whose character in shape are restricted to thin and moderately flat and small (e.g., see Fig. 1), are given, and we consider reconstructing earthenware from the potsherds in two dimensional

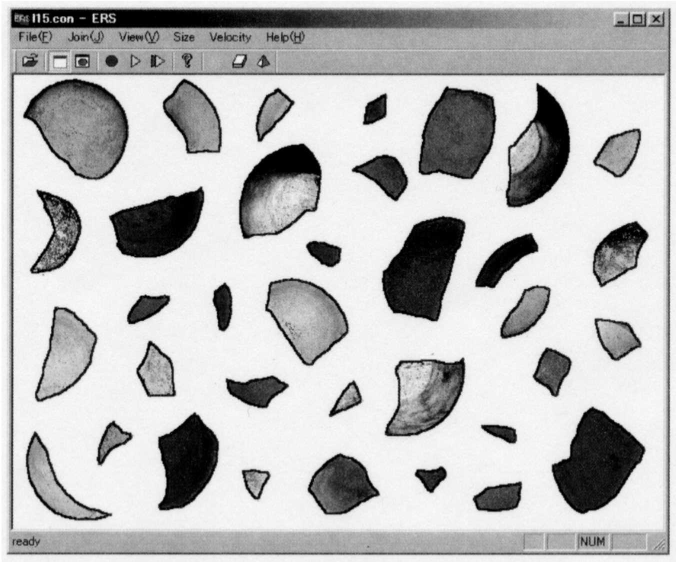

Fig. 1. Images of potsherds.

grayscale. In this paper, firstly as techniques for reconstruction in two dimensions, we propose a method for detection of salient feature and calculation of salient value from contours of potsherds, and then propose an efficient joint detection algorithm considering complexity of contours. We then move on to a method for recovering three dimensional shape. We have implemented an earthenware reconstruction system with the proposed algorithm and method, and shown good performance in reconstruction of a cylindrical clay figurine (called ento-haniwa in Japanese), in addition to flat objects, such as plates, pot-lids, flat parts of earthenware, and so on.

2. System Overview

The section gives a brief description of our earthenware reconstruction system executable on Windows PC. A full detail of the system is given in the later sections. Figure 2 shows an overview of the system. The system has two phases: *pieces-joining phase* and *shape-recovery phase*. In our system, given numerous potsherds are reconstructed into some earthenware through the two phase with the following procedures.

Phase 1: pieces-joining

1) Detection of contours

The system detects contours from input images of potsherds. In the procedure, we adopt the g-snake model proposed by LAI and CHIN (1993, 1995), which can extract a contour of any object from an observational image.

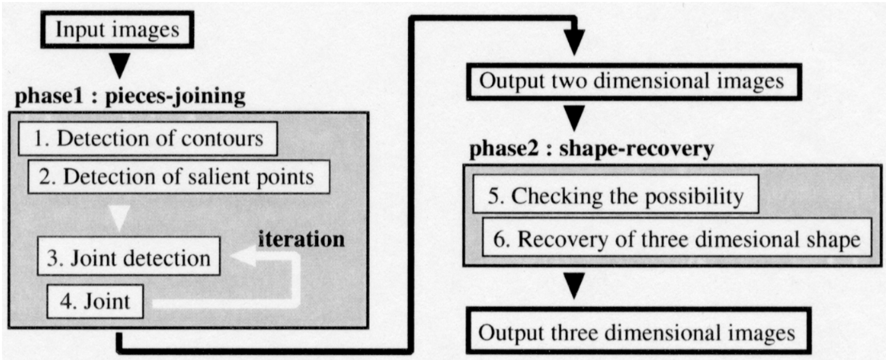

Fig. 2. An overview of the earthenware reconstruction system.

2) Detection of salient points
Each of contours is divided into some expanded curves called *sub-contour* by the salient points in the procedure. These sub-contours are candidates for joints.
3) Joint detection
For each sub-contour, salient value is calculated, and then, for all pairs of sub-contours, similarities are evaluated by the salient values. The most similar pair is selected after that.
4) Joint
If the similarity of selected pair satisfies some condition, two potsherds having these sub-contours are joined.

The system executes the iterations of series of procedures 3) and 4), and finally, outputs some plane images of reconstructed earthenware.

Phase 2: shape-recovery
5) Checking the possibility
In this procedure, the plane images generated at the above phase are checked about the possibility of three dimensional shape recovery. Once some images pass the check, the system proceeds to the next procedure with the images; otherwise the system terminates with the plane images of reconstructed earthenware.
6) Recovery of the three dimensional shape
The system recovers three dimensional shape of the earthenware from the plane images with adequate three dimensional transformation.

After this phase, the system outputs three dimensional images of reconstructed earthenware.

3. Detection of Salient Points

Contours detected from given potsherds should be divided into sub-contours by salient points. In our system, the sub-contours are considered candidates for joint, and the shape of sub-contour is utilized as a criterion for joint detection. Accordingly, dividing contours is important in terms of performance of the system.

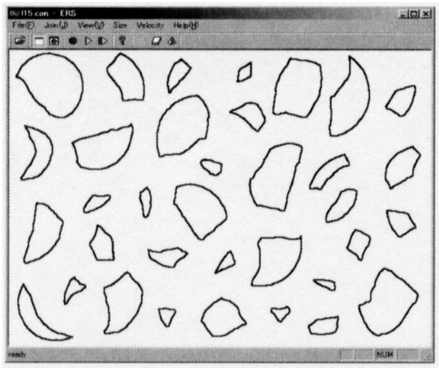

Fig. 3. Contours of potsherds.

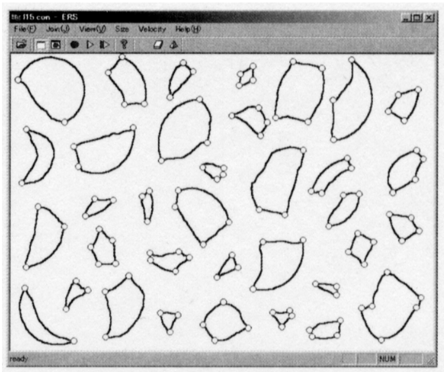

Fig. 4. Divided contours by salient points.

Paying attention to reconstructed earthenware, a contour of a potsherd is almost divided into joints by salient points such that the points become vertexes constructing a polygon which approximates the contour with as few vertexes as possible. We, therefore, adopt ROSENFELD and JOHNSTON (1973)'s method for detecting salient points having above property.

Figure 4 shows the result of detecting salient points from contours shown in Fig. 3.

4. Joint Detection

4.1. Salient value

We give the P-type Fourier descriptors proposed by UESAKA (1985, 1986) to salient values for shape of sub-contours. Using the descriptors, we evaluate similarity between two sub-contours. The descriptors are represented by the Fourier transform of curves and characterized as follows:

- It is applicable to expanded curves.
- Two endpoints of a reproduced curve always correspond to two endpoints of the original one, respectively.
- Information about a certain shape of an original curve is concentrated in low frequency range.
- A Curve is represented by only the angular function.

In earthenware reconstruction, the shape of a sub-contour is not completely equal to the shape of its joining pair because of weathering and so on. The P-type Fourier descriptors are, therefore, applicable to joint detection. A salient value $c_N(k)$ of a sub-contour c is calculated by low-pass spectrum with cut frequency N of Fourier transformation as follows:

$$c_N(k) = \frac{1}{n}\sum_{i=0}^{n-1}\omega(i)\exp(-j2\pi ik/n) \quad (k = -N, \ \ldots, \ N), \tag{1}$$

where $\omega(i)$ is the P-type expression of c and n is an enough large natural number. For more precise, please refer to UESAKA (1985, 1986).

Using the salient value, similarity $\varepsilon_{a,b}(\phi)$ between sub-contour a and b can be evaluated by the following equation:

$$\varepsilon_{a,b}(\phi) = \sum_{k=-N}^{N}\left|a_N(k) - \exp(j\phi)b_N(k)\right|^2 \quad (\phi = 0, \ \ldots, \ 2\pi), \tag{2}$$

where $a_N(k)$ and $b_N(k)$ denote the low-pass spectrums of P-type Fourier descriptors of a and b, respectively.

4.2. Shape similarity

The P-type Fourier descriptors are invariant under translations, and changes of the scale. Rotations of an original curve are represented by the descriptors multiplied by $\exp(j\phi)$. It follows from Eq. (2) that b becomes more similar to a when $\varepsilon_{a,b}(\phi)$ gets nearer to zero. It also follows that $\varepsilon_{a,b}(\phi) = 0$ when the shape of b is completely equivalent to the shape of a.

In our system, we, therefore, define the shape similarity between a and b by the following equation:

$$ss(a,b) = \frac{1}{\min_\phi \varepsilon_{a,b}(\phi)}. \tag{3}$$

The above salient value and similarity are also adopted by the existent work in HORI (1999).

4.3. JDID

In our system, an efficient joint detection algorithm considering complexity of contours, JDID proposed by KANOH (2000), is implemented. The algorithm reduces computational cost of joint detection drastically, therefore numerous potsherds can be reconstructed at once. For a contour, the algorithm considers some contours approximating it by different precision, *shape-precision*, and suitably selects one approximation for calculating shape similarity by complexity of the contour.

For each sub-contour c, the algorithm considers sequential lines $s(c)$s with case that shape-precision $sp_S = 0, ..., \overline{sp_S}$ (shown in Fig. 5), where $\overline{sp_S}$ is upper bound of shape-precision. It should be noticed that for all c, $s(c)$ can approximate c more accurately with case that sp_S is higher. Here, $s_i(c)$ denotes the sequential line of sub-contour c with its shape-precision $sp_S = i$.

Figure 6 shows an outline of search trees constructed by JDID. The search begins at the lowest shape-precision. In the figure, the leftmost tree, for example, shows joint detection from two potsherds A and B. The leftmost branch in the tree, for example, shows

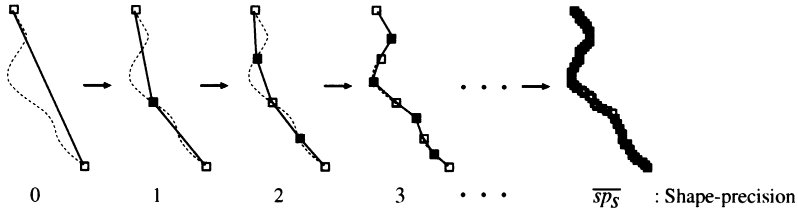

Fig. 5. Sequential lines with different shape-precisions.

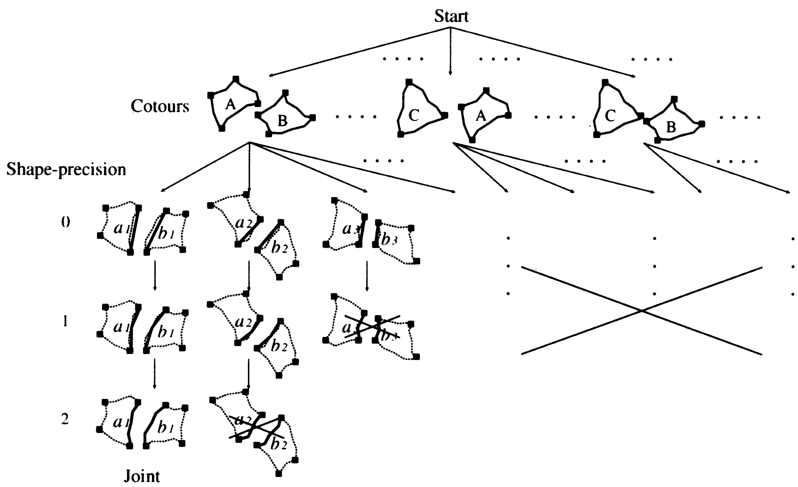

Fig. 6. An outline of search trees constructed by JDID.

a comparison of shape between two sub-contours a_1 and b_1 in A and B respectively. In the branch, a node at the depth of i ($i > 0$) has two sequential lines $s_{i-1}(a_1)$ and $s_{i-1}(b_2)$. A node at the deeper depth has sequential lines with their higher shape-precision. The search proceeds in the breadth-first manner with iterative deepening (KORF, 1985a, b). Node expansion at the depth of i behaves intuitively as follows:

i) calculate shape similarity $ss(s_{i-1}(a), s_{i-1}(b))$, and then judge whether the pair is a joint or not,

ii) from the above judgment, delete nodes containing inconsistent pairs (see nodes marked with $\times$ in Fig. 6),

iii) go on in either of following three ways according to the number of remaining nodes at the same depth:

zero: terminate the search in failure,

one: terminate the search in success, and detect the pair of sub-contours in the node as joint,

more than one: generate a descendant node, which contains pair ($s_i(a)$, $s_i(b)$) of sub-contours, with their shape-precision $sp_S = i$.

The search iterates the above node expansion in breadth-first manner with successively heightening shape-precision.

In the last of this section, let us discuss the of JDID for our earthenware reconstruction system. Although there generally exists more than one joint pair in given potsherds, JDID detects only one joint pair at once. In other words, our system performs reconstruction of earthenware by executing JDID successively for each step of joining. At one step, JDID dose not consider global evaluation (i.e., considering not all combination of sub-contours for all steps of joining) for selection of the most similar pair, but considers local evaluation (i.e., considering all combination of sub-contours at the one step). Earthenware reconstructed by our system, thus, cannot always have an optimal combination of joint pairs, while the most similar pair selected by JDID is the best in each step, that is, our system finds quasi-optimal solution. Considering global evaluation for selection causes that earthenware reconstruction becomes a NP-hard problem, and it is almost impossible to propose an optimal algorithm which can solve the problem in adequate execution time. JDID, therefore, aims to detect a pair efficiently, which can be adequate joint pair for earthenware reconstruction. For more precise about JDID algorithm, please refer to KANOH (2000).

5. Three Dimensional Shape Recovery

In this section, we describe a technique for shape-recovery phase in our system: recovery of three dimensional shape from plane images of earthenware reconstructed at pieces-joining phase (see Sec. 2). In general, earthenware having solid shape should be reconstructed from potsherds represented in three dimensions. The technique is an attempt for our system to reconstruct earthenware, having solid shape, from two dimensional images of its potsherds. In our system, it should be noticed that all input potsherds have character restricted in shape: thin and moderately flat and small.

In shape-recovery phase, by two procedures, a plane image of reconstructed earthenware is transformed into a three dimensional image representing solid shape of the earthenware. The procedures are described briefly in the following sections.

5.1. Checking the possibility

This procedure checks the plane image about the possibility of three dimensional shape recovery. The check is whether a contour of the plane image contains any joint pair of sub-contours and the pair is configurationally consistent, or not. The procedure can be done efficiently by utilizing our proposed JDID algorithm. If some joint pair is found, the plane image is cognizable as a figure to which a solid object is developed.

5.2. Recovery of the three dimensional shape

In this procedure, the system recovers three dimensional shape of the earthenware from the development figure with adequate three dimensional transformation. By the transformation, a development figure behaves intuitively as follows. Firstly, the pairs in the figure detected by the above procedure are joined, and the figure has one closed-space constructed in three dimensions. And then, the figure is gradually modified so that the volume of its closed-space becomes the largest.

In case that the figure is to grow into cylinder shape or cone shape (such like a paper cup), the transformation can achieve drastic efficiency by means of some geometrical characteristics. The specialized transformation is shown in Appendix.

6. Experimental Results

We have implemented earthenware reconstruction system in C++ on a PentiumIII 500 MHz running Windows PC. We have also made some experiments of earthenware reconstruction from excavated potsherds. This section shows two representative experiments verifying effectiveness of our system; one is for efficiency in pieces-joining phase, and another is for performance of shape-recovery phase. In the experiments, parameters for our system, $N = 5$ and $n = 64 \times 2^i$, where i is a value of shape-precision.

6.1. Efficiency in pieces joining

In order to verify efficiency of pieces-joining phase, we have given 35 potsherds shown in Fig. 1 to our system. Figure 7 shows earthenware reconstructed by our system. In this particular example, the result indicates that our system can correctly and completely reconstruct 8 pieces of earthenware from given potsherds.

We have, moreover for effectiveness of JDID, compared execution time of our system with a naive system, by changing the number of potsherds. In the naive system, joint detection is executed with shape-precision fixed on the upper bound. Figure 8 shows the experimental results. The solid line in the figure shows the runtime by our system, while the broken line is those by the naive system. The results indicates that our system embedded with JDID becomes faster than the naive system as the number of potsherds increases. Our system, for example of 35 potsherds shown in Fig. 1, takes 104.9 (sec.) to perform all reconstruction, and the execution time is about 7.6 times faster than naive system.

6.2. Performance of three dimensional shape recovery

We have also made an experiment for recovery of three dimensional shape of cylindrical clay figurines. Our aim of this experiment is to ascertain that our system can correctly recover the three dimensional shape from given potsherds. This experiment,

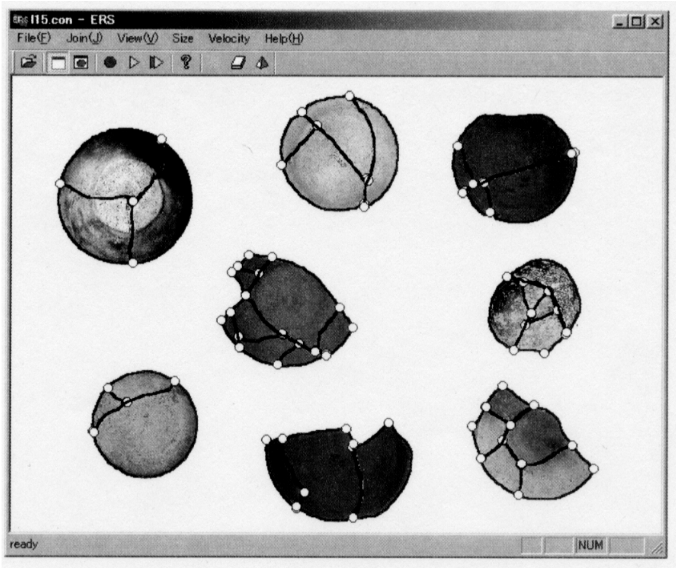

Fig. 7. Output images.

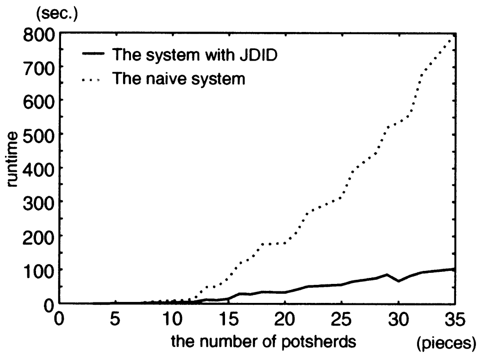

Fig. 8. Experimental results.

therefore, requires a complete set of potsherds, which can show the perfect shape of reconstructed earthenware in advance. We, thus, took photographs* of a figurine reconstructed by archaeologists, and made input images of potsherds by cropping the photographs along their contours. After the above manner, 39 potsherds shown in Fig. 9 are inputted to our system.

*At the shooting, the figurine is settled on a turntable aligned plumb in the face of a camera, and photographs are taken from the whole direction.

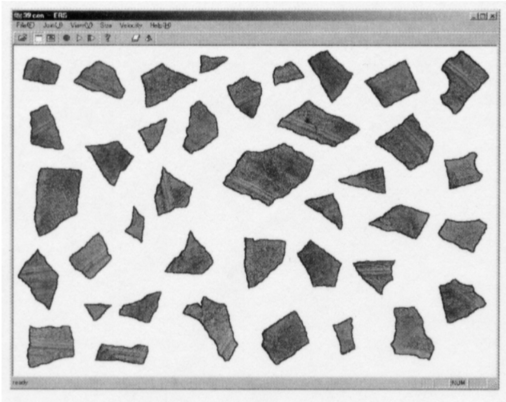

Fig. 9. Input images.

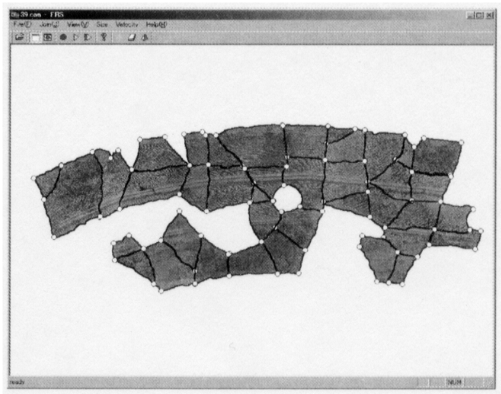

Fig. 10. An output image.

Figure 10 shows the result of pieces-joining phase for the potsherds. Figures 11 and 12 show the result of three dimensional shape recovery from the plane image in Fig. 10. In this particular example, the result shows satisfactory performance as three dimensional shape recovery.

7. Conclusion

In this paper, we proposed an earthenware reconstruction system, which can automatically reconstruct earthenware from given potsherds. As methods for improving the system performance, we proposed an efficient joint detection algorithm (JDID) considering complexity of contours, and implemented a method of recovering three

Fig. 11. The three dimensional image of a reconstructed figurine.

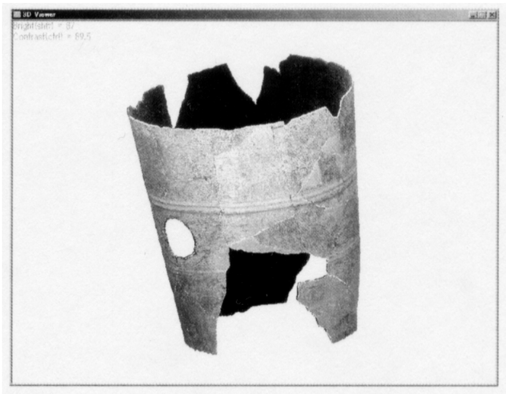

Fig. 12. The three dimensional image of a reconstructed figurine (another angle).

dimensional shape of reconstructed earthenware. We also made some experiments on reconstructing many excavated potsherds, and reported two representative results. One result shows high performance of correct and complete pieces joining with JDID. Another indicates satisfactory performance as three dimensional shape recovery of a cylindrical clay figurine.

In general, it is difficult to implement an efficient, accurate and robust three dimensional transformation which can recover the solid shape from an arbitrary development figure. We have, presently, considered an energy minimization method as a key strategy to solve the above problem, and we will introduce the method into our earthenware reconstruction system in forthcoming papers.

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Appendix: Three Dimensional Shape Recovery Using Geometrical Characteristic

A development figure can be transformed into cylinder shape or circular cone shape (such like a paper cup) by the following three dimensional coordinate transformation.

Let c and d be sub-contours included in the development figure, and let (c, d) be a joint pair of sub-contours detected from the development figure. Further let (v_{c1}, v_{d1}) and (v_{c2}, v_{d2}) be pairs of corresponding endpoints of c and d when c and d are joined, respectively (see Fig. A1).

Firstly, the development figure is rotated on an x-y orthogonal coordinate system, such that a straight line which connects v_{c1} and v_{d1} becomes parallel to x-axis. And then, transformed shape from the figure are prefigured as either a cylinder or a circular cone by the following condition.

$$\begin{cases} \text{a cylinder:} & \left| \dfrac{\left(\left| \overrightarrow{v_{c1} v_{d1}} \right| - \left| \overrightarrow{v_{c2} v_{d2}} \right| \right)}{\left| \overrightarrow{v_{c1} v_{d1}} \right|} \right| < \alpha \\ \text{a circular cone:} & \dfrac{\left(\left| \overrightarrow{v_{c1} v_{d1}} \right| - \left| \overrightarrow{v_{c2} v_{d2}} \right| \right)}{\left| \overrightarrow{v_{c1} v_{d1}} \right|} \geq \alpha, \end{cases} \tag{A1}$$

where α is a parameter.

Secondly, any points in the development figure are mapped onto the three dimensional coordinate system. Let p be an arbitrary point in the development figure laid out above, and

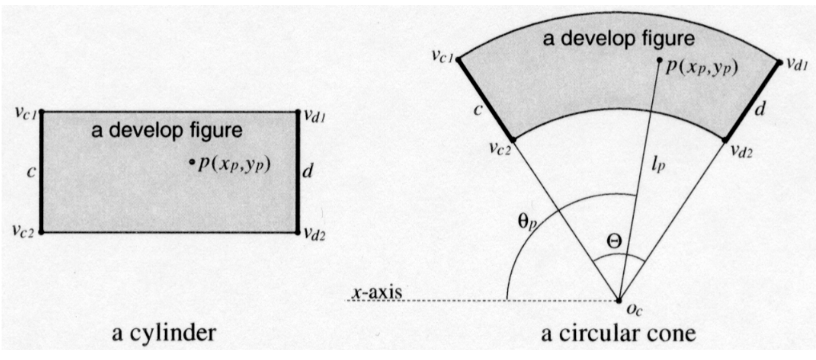

Fig. A1. Two types of cevelopment figure which is applicable to the efficient transformation method.

let the coordinates of p be (x_p, y_p). Further let p' be an arbitrary point in an orthogonal three dimensional coordinate system, and let the coordinates of p' be $(x_{p'}, y_{p'}, z_{p'})$. Then, p is mapped onto p' by either of the following two ways.

The case of a cylinder:

$$\begin{cases} x_{p'} = \dfrac{\left|\overrightarrow{v_{c1}v_{d1}}\right|}{2\pi}\cos(\phi_p) \\ y_{p'} = y_p \\ z_{p'} = \dfrac{\left|\overrightarrow{v_{c1}v_{d1}}\right|}{2\pi}\sin(\phi_p), \end{cases} \tag{A2}$$

where ϕ_p is determined by the following formula:

$$\phi_p = \frac{2\pi x_p}{\left|\overrightarrow{v_{c1}v_{d1}}\right|}. \tag{A3}$$

The case of a circular cone:

$$\begin{cases} x_{p'} = r_p \cos(\phi_p) \\ y_{p'} = l_p \\ z_{p'} = r_p \sin(\phi_p), \end{cases} \tag{A4}$$

where l_p, r_p and ϕ_p are determined as follows:

$$l_p = \left|\overrightarrow{o_c p}\right|, \tag{A5}$$

$$r_p = \frac{l_p \Theta}{2\pi}, \tag{A6}$$

$$\phi_p = \frac{2\pi\theta_p}{\Theta}, \tag{A7}$$

where o_c is the center of a circle including arc $\overset{\frown}{v_{c1}v_{d1}}$, and Θ be an center angle of $\overset{\frown}{v_{c1}v_{d1}}$, and θ_p denotes the angle between x-axis and $\overrightarrow{o_c p}$.

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