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Symmetry versus Katachi in Animal Organisms

Sz. BÉRCZI[1,2] and B. LUKÁCS[1,3]

[1]*Matter Evolution Scientific Subcommittee of the Hungarian Academy of Sciences, Hungary*
[2]*Faculty of Science, Department of G. Physics, Eötvös University,*
H-1117 Budapest, Pázmány Péter s. 1/a, Hungary
[3]*Central Research Institute for Physics, RMKI, H-1525 Budapest-114, P.O. Box 49, Hungary*

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Abstract. We demonstrate that for animal organisms there seems an inverse relationship between symmetry of the form and abilities of the organism. A “very perfect” body could do almost nothing; therefore probably it could not even plan any action either. In the maximally symmetric animal body almost nothing would exist inside to be expressed outside.

1. Introduction

One would call an animal symmetric if its form were symmetric (at least in general sense: not only the visible form, but the general quality of the body too). For example an animal would be spherically symmetric if arbitrary rotation along 3 different axes still brought points to identical points. Then the body would be a perfect sphere, but also internal organisation would happen on spherical shells, the color of the animal might change only between concentric shells etc. (spherical symmetry prohibits extruding limbs and so on). It is easy to see that any more complicated animal could not work if it were so symmetric. The katachi of a spherical animal would be practically nil. If almost nothing is inside then what to be expressed outside?

However, less symmetry and more katachi can be imagined. Still we learn from Evolution that “animals are more potent”, their internal will can be fulfilled more when they are less symmetric. Therefore animal symmetry is somehow contrary to potentialities. To demonstrate this now we perform a bird’s eye overview of terrestrial animal evolution. Beginning with all benefits of a very symmetric animal (it can be easily coded genetically), we expect symmetries decreasing, and katachi (inner topology) increasing, becoming more complicated during evolution.

2. On Killing Symmetries

Obviously on of the most easily tractable aspect of an animal form (in mathematical sense) is continuous symmetry. For this purpose General Relativity has adapted a late 19th century mathematical tool, The Killing symmetry (and Lie-derivative), and we can directly apply it to our target. Note that General relativity, being a theory where coordinates are always labels only, must apply strong tools when trying to orietate itself in generally curved spaces.

Consider a Riemann space of n dimensions. There is a coordinate system $\{x^i\}$ on the manifold. A domain of the space is filled with matter. This matter is characterized by a set of fields $\{\Psi_\alpha(x^i)\}$. This set contains the relevant data of the matter, say Ψ_1 may be the density, Ψ_2 the colour, Ψ_3 classifies the tissue at x^i as liver, muscle, kidney, etc. The domain is generally finite, but we restrict ourselves to the interior and neglect the boundaries. An infinitesimal motion can, e.g., be written as

$$x^{i\prime} = x^i + \varepsilon^I K^i(\boldsymbol{x}) \qquad I = 1, \ldots, N,\ i = 1, \ldots, n$$

where awhere $K^i(\boldsymbol{x})$ stand for N different vector fields, i runs in the coordinate space, I in a parametric one, the ε^I-s are infinitesimally small parameters of the motion, and in General Relativity a summation is automatically meant if an index occurs twice, above and below.

Now take An infinitesimally small triangle and let it move in the above way. In any curved spacetime the triangle will remain geometrically similar (angles invariant) if

$$L_{K^{g}{}_{ik}} = K_{i;k} + K_{k;i} = \Omega_{g_{ik}}$$

(where L_K is the Lie-derivative along K^i, and identical, if $\Omega = 0$, otherwise not. If such a K^i exists, it generates a conformal Killing motion; if $\Omega = 0$, it is simply a killing motion (EISENHARDT, 1950). What was told so far is valid for continuous symmetries; reflections, etc. can not be approached in this way.

There are Riemann spaces without any symmetry at all, so in them no object can show any continuous symmetry. On the other hand, the maximum number of independent Killing fields is $n(n+1)/2$, for conformal Killing fields $(n+1)(n+2)/2$. This maximal number appears in spaces of constant curvature.

3. Possible Continuous Animal Symmetries

By the Killing vector technique it is easy to list all possible continuous animal symmmetries, because between E(3) (homogeneity + symmetry) and complete lack of symmetry there are finite possible steps. The problem is treated in LUKÁCS (1993a). But we shall see that for our present purposes it is enough to start from a stage where the animal body is axially symmetric. This is: rotational symmetry around one axis.

We do not deal here with conformal symmetry. An example was shown in LUKÁCS (1993a). We only note that such a possibility is that the structure is spiralic, with proportionally growing elements. Conformal motions bring in scale factors etc., for such similarities in biology see e.g. D'ARCY-THOMPSON (1917).

4. Discrete Symmetries

We told earlier that discrete symmetries need other approach. Now we may distinguish two types of discrete symmetries: reflections (which, of course, must be the symmetries of space too) and repetitive motions, when a continuous motion is not a symmetry of the organism, but with a finite step it is.

The first case is simple. The eclidean space has 3 independent mirror symmetries, so an organism can have 3, 2, 1 mirror symmetry planes, or non at all.

The second case results in subcases, because the continuous symmetry might be either a translation, or a rotation, or their helical combination. Translation symmetry in discrete steps means that the body is composed of identical cells of length D. Discrete rotational symmetry is possible only in $360°/n$ steps. Discrete helical symmetry, in contrast, is possible in any step.

5. The Evolutionary Tree of Animal Symmetries

In a classical work SIR THOMAS BROWNE (1646) denied the existence of a two-headed snake of the ancient Romans called Amphisbaena, telling that there is no species without bottom, top, front, back, left and right (BORGES, 1967). Then the two-headed Amphisbaena cannot exist since "for the senses being placed at both extreams, doth make both ends anterior, which is impossible". Archaeology and even study of recent acoelomates disprove this statement, and now one can guess that the degree of animal symmetries is decreasing in evolution, due to increasing specialisation and complexity. (Katachi versus symmetry?) To give only a partial reasoning, remember that all Metazoa start from a single cell, so its all cells should be equivalent, therefore the body fully symmetric. If not, some mechanism breaking this symmetry is needed.

The starting point will be full E(3) symmetry, and on Fig. 1. time goes by from above to down. After some early steps discussed in) HAECKEL (1878) and BÉRCZI *et al.* (1993) we arrive at the Gastrula stage. The animal is axially symmetric around the only opening, the protosoma. This stage is seen in embryology.

However the last continuous symmetry must cease because an axial symmetric body cannot well orientate itself. E reasonable compromise is a body woth a cross section of regular polygon. Cnidarians' and ctenophorans' symmetry has frozen here (leftmost). However strong swimmers' body must have been elongated ("streamlined form") and at the end a bilaterally symmetric worm has been formed (middle), with front (mouth, eye etc.) versus back (anus, maybe tail etc.), ventral (toward soil, legs if exist etc.) versus dorsal (no legs etc.) sides. The only reflection symmetry which does not necessarily vanish is left/ right symmetry. (As we saw, SIRTHOMAS BROWNE did not believe even in this symmetry.)

It is better to increase the body to be bigger for not being eaten easily and more complicated for versatility. The central lineage is the naïve straightforward way followed by Mollusca but it is dispreferred by information theory. It is better to multiply in identical

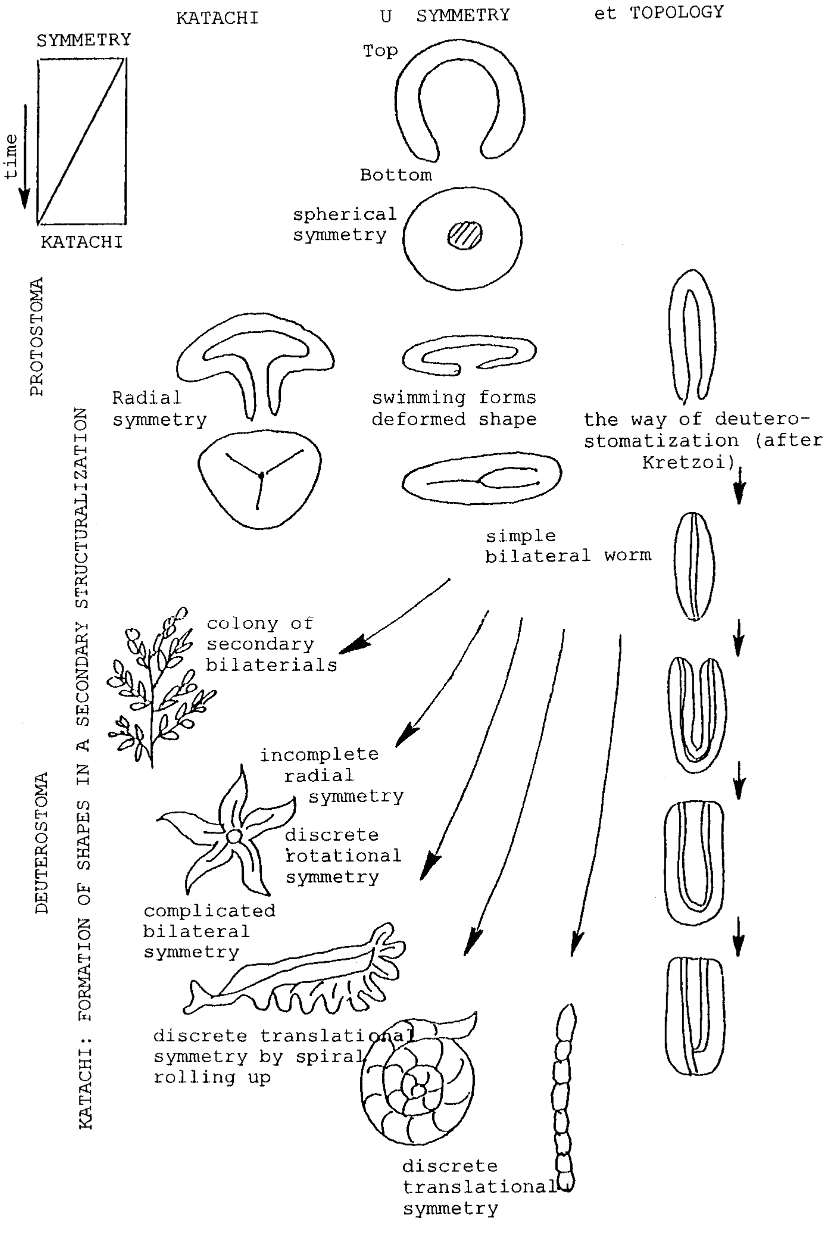

Fig. 1. Katachi, symmetry and topology in the formation of the body shape of animal organisms. After the common line of axial symmetry around the only opening of the animal (protostoma, upper third of the figure) two ways of shape evolution is shown (lower twothirds of the figure). Because axial symmetric body cannot orientate itself well enough, bodies with regular polygonal cross section developed (cnidarians, stenophorans, left side). Bodies of strong swimmer animals evolved toward an elongated, streamlined shapes and finally the worms with bilateral symmetry has been formed (radiation from the central column). The way of deuterostomatisation (right column) is shown in five steps according to the theory of KRETZOI (1964).

modules. The other four ways show such evolutions. The leftmost one is colony forming, seen among Bryozoa, Hemichordata and Tunicata, maybe present in vertebrate evolution. The next is to grow the body in many direction (pseudo-symmetry of discrete rotation), seen among Echinodermata. The two cases on the right are to multiply the body lengthwise. This may lead to homologous metamery (discrete translational symmetry of Annelida) (LAKE, 1990), or the symmetry may be discrete spiral motion (conformal rotation + dilatation), for which the best candidates are Ammonites. Along these lineages symmetry is growing, but not more than the body.

Still there is no chirality. However if the internal gradients are large enough, thermodynamic forces may order the chiral biomolecules and large-scale chirality may emerge. Still, substantial left-right asymmetries seldom appear in Regnum Animalia. A fairly asymmetric animal was *Stylophora*, but it seems to have been a cul-de-sac of evolution. Vertebrate bodies often exhibit *slight* asymmetry (mammal heart, avial oviducts etc.), but generally neither the outward appearance nor the functioning shows too much of this asymmetry. The mammal evolution led to *functional* left-right asymmetry (i.e. the emergence of notions of left and right) only several Ma ago in hominisation.

6. On the Topology of Animals

We did not yet discussed the rightmost column. To do it, we turn to topology.

Topology is a *global* property. In Riemannian geometry the simplest contrast can be formulated for a 2-dimensional surface with Euclidean geometry (Pythagoras' formula is valid). (That is E(2) symmetry.) Still it may be a sheet, a cylinder, the mantle of a cone etc. All different *forms*, and for a working organism connectivity is very important. Let us again follow the evolutionary steps.

Gastrulation makes the topology that of the sphere, anything it was previously. In this topologic stage we can find Cnidaria, Ctenophora, and most Platyhelminthes, except for Nemertoidea.

In Nemertoides appears first the second, anal opening (rightmost, second from upwards). The enteral channel becomes a complete shaft driven through the body, so we have arrived at the topology af sphere minus a simple shaft. (Small openings of the secretion system as proto-nephridia *et al*. are neglected here for simplicity.) This structure remains in the whole Protostomata branch. However *the evolution leading to us was different*.

On the protostomic side the protostoma of the gastruloid stage remains the mouth and the anal opening is new. On the deuterostomic side the mouth is new, and the protostoma either closes or becomes the anus. From cladistic viewpoint it is very important to divide the tree into two; however from topologic viewpoint this means no difference.

However on the deuterostomic side one can (?) observe a different, more complicated history, and *this* is the way leading to us. Based on the embryogenesis of Amphioxus, on observations of recent lower vermin, and on common sense a lest improbable story is reconstructed, which, following KRETZOI (1964), goes in the steps 2–5 on the right side of Fig. 1.

The starting point is a "sphere minus a simple shaft", Stage 2. Then the worm takes U-shape (for any reason, Stage 3). Then the mouth and anus gets close to each other, which is not optimal in the light of the Le Chatelier-Brown Law; nevertheless it is not uncommon (see Phoronoidea, Entoprocta and Echinodermata).

Then, it seems, the animal reintegrated, preserving the U-shape of the channel (4). *Still the topology is unchanged.*

Both the neighbouring positions of the opening and the U-shape of the channel have disadvantages. (For the latter: the flow in the channel is difficult at the turning point.) It is profitable to get a new opening near the turning point (Stage 5). The final result is a sphere minus an Y-shaft; a new topologic class.

Because of the original flow pattern, *without serious reasons the new opening would be an anus*, so still protostomy. To get deuterostomy the animal must have changed its orientation too in this stage.

At this point one branch ceases to be operative, and can be utilized for a new function. From recent higher deuterostomic animals one may guess that the superfluous branch was going to house either the developing neural system (spinal cord) or the developing endoskeleton (chorda dorsalis).

It seems that (some) Deuterostomatae have developed a topology more complicated than that of Protostomatae. A more complicated topology gives more possibilities to build up a versatile body. Is this more complicated topology (and so enhanced potentiality) behind our evolutionary success? Katachi against Symmetry?!

7. Our Family Album

Our Deuterostoma family album is Fig. 2. It starts with Sagitta hexaptera for Chaetognata. We do not see either a chorda dorsalis, or a spinal cord. For any case the neural system starts from the neighbourhood of the mouth, but it is natural to get a neural center at the apex.

Our second kin is extinct *Stylophora*, with expressed lack of a bilateral symmetry. Recent discussions suggest that it was not an Echinodermata proper but rather something between that and Tunicata. In this case the opening at the apex is a mouth, the opposite one is the anus, the tail may have contained a chorda dorsalis, and the thickened part beside the anus housed a brain.

Next comes *Appendicularia sicula*, a Tunicata. There is a well developed chorda dorsalis in the tail, and parallelly on its dorsal side runs a neural fiber analogous to the spinal cord.

Glossobalanus minutus has a notochord in the glans at the apical end, not seen here, it is sometimes regarded as the analogon of the chorda dorsalis.

Rhabdopleura Normani's individuals form a colony connected via the stolon; in each member from the mouth *two* channels start, the enteral one (through the stomach to the anus) and the stomochord (which may be the analogon of the dorsal chord).

The album ends with Amphioxus (*Branchiostoma lanceolatum*) stands for Vertebratae. No need to bore the reader here with further kinfolk as Cyclostomata, Pisces, and c, showing no important difference from us at all. Amphioxus will be discussed later in some details, so now observe only the segmented muscles, the many repeated gills, the homogeneous spinal cord and the lack of any mentionable head at the apical end. (The head will develop in later stages of evolution.)

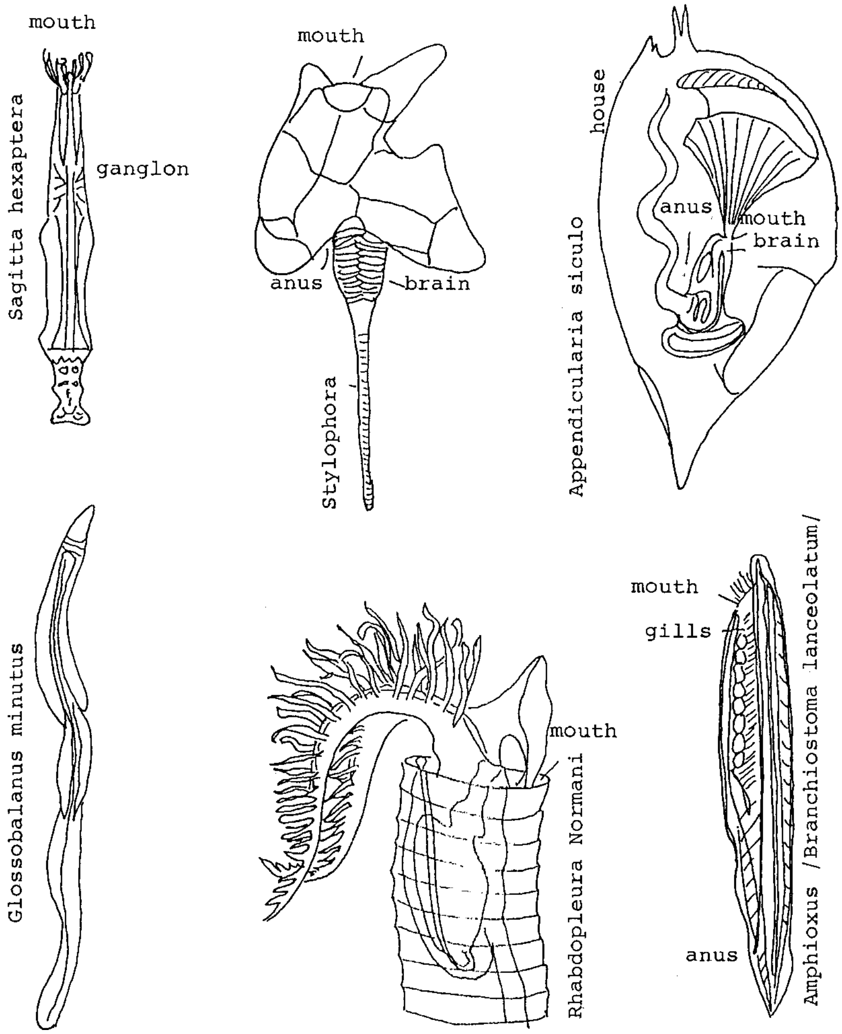

Fig. 2. The Deuterostoma family album shows six stages of shape evolution by the position of the mouth-anus line and the neural system arrangement till the appearance of the spinal cord at the segmented Vertebrates (here Amphioxus represent this stage). Details are given in the text.

Something special must have happened on the Vertebrata branch of the evolution, between the recent Amphioxus (Acrania) and the common ancestor of Acrania and Hemichordata or Tunicata or both, anyway well in the Precambrian. We do not know this common ancestor from fossils just as *any* ancestor of Acrania is unknown (if not Pikaia in the Burgess shale), so there is still room for speculation. (With moderate imaginative power one can then bridge the gap between Acrania and Cyclostomata.)

It is possible that one critical step creating "anomalies"may have been a *unification*. It is easy to visualize an evolutionary path leading from a colony of primitive Deuterostomatae (analogons of Enteropneusta) to an integrated organism whose parts started thereafter to specialise. The idea had originally been suggested more than a century ago by Hatschek, but was opposed by such relevant people as HAECKEL (1878) and Spencer (COLLINS, 1895).

According to Spencer an animal may be of

1) **Primary**, having no separable parts. *Examples: Protozoa.*

2) **Secondary**, having separable primaries. *Examples: lower Metazoa, as Platyhelminthes; Mollusca.*

3) **Tertiary**, having separable secondaries. *Example: Annelida, whose segments more or less resemble individual lower worms.*

A further, **quartiary** step would be the integration of tertiaries into a single organism. *A candidate is an anthill of very integrated insects,* as was philosophically suggested by MARAIS (1972) for termites.

Now the degree of integration and autonomy.

A) **Confederation**. Mainly independent parts with interactions. *Examples: sponges* (separated cells survive but for a different kind of life, until reestablishing the secondary organism); *an earthworm* (can be cut into two and parts may live for a while to regrow the lost parts).

B) **Federation**. Identifiable parts, but with some vital functions located at the level of the whole organism. A separated part can survive for a while but with restricted functions; it cannot regrow the whole organism and cannot live its independent life. *Examples: an insect* (can survive decapitation for a short time, but cannot become a complete insect anymore); *an anthill of termites* (from which a termite can be separated for a short time but will die without reproduction etc.).

C) **Unity**. No possibility for survival of the separate parts for any time. *Example: majority of higher animals* (whose amputated fingers do not survive at all).

The third aspect is the origins of the parts. Again we can formulate in a triad: two extremal cases and one between them:

i) **Symbiosis** of originally independent animals.

ii) **Transitional** cases.

iii) **Self-generation** of new segments.

Here it is difficult to give examples because we should know the *prehistory* of the organism, but for i) we mention the Eucariotes of endosimbionta origin(CAVALIER-SMITH, 1987), while Case iii) is suggested by the *ontogeny* of the tapeworm or the Pogonophora (ancestry unknown).

Amphioxus seems to be tertiary, so 3), because of repeated (metameric) pattern of many fundamental organs (similarly to Annelids) as **Respiration**, **Circulation**, **Secretion**, **Reproduction**, **Sensors** and **Neural system**. Here we discuss only the last one; for the others see LUKÁCS (1993b). Amphioxus' neural system is as follows. Central part: spinal chord in segmental arrangement + giant fibers connecting the segments. Dilatation at the apical end ("brain"). Dominantly gray matter for local functioning. Periphery: segmentally arranged neurons to muscles.

But if Amphioxus is tertiary, then probably all Vertebrata are, we too. In this case our position in the 3 × 3 classification is 3Ciii). And then *our* story is as follows. We lost our last continuous symmetry ca. 800 Ma ago (before Ediacara). Later (perhaps not much later) we gave up the discrete symmetries too, except left-right mirror symmetry which was quite persistent. Then our ancestors (not yet found in rocks) started to form themselves into self-generated colonies, which is again a kind of discrete symmetry (repeated parts) slightly connected in the way of such near kins as extinct Graphtolitha and recent Tunicata. We do not know when was this stage, but from relatives we may guess that not after Cambrian, ca. 550 Ma. Then started an integration of the parts. Our ancestors went through the "confederation" stage of present Amphioxus, but still no fossil records are known. However in the Ordovician (500 Ma) Agnatha "fishes" appear, and they are in the Unity stage without doubts.

Afterwards still important but only gradual changes happened in our form.

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