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Virtual Polyhedra Created by Computer Aided Drawings on Mixing of Three Basic Forms with Cubic Symmetry

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Abstract. The new procedure for creation of virtual polyhedra by means of computer aided drawings are presented. The essentials of procedure are based on the mixing of three basic forms such as hexahedron (cube), octahedron, and dodecahedron. The three basic forms for combinations are chosen from seven basic forms with cubic symmetry. The combinations as compounds of three forms are classified by 35-sets of "*p-q* diagram" on which various polyhedral forms are characterized by relative mixing rates, p and q. The 35-sets of *p-q* diagrams are shown in Appendix with the polyhedron and its expanded figure drawn at a set of specific values (p,q).

1. Introduction

"Polyhedra" as crystal habits are one of the important and interesting subject in the study field of crystallography and mineralogy, in addition to mathematics (HITOTUMATU, 1983). On the other hand, independent from physical subjects, "polyhedra" have fascinated artists and designers, in addition to physical scientists. In the present day, interesting sites concerning to "polyhedra" are found on "Internet" (http://georgehart.com/; http://www.geocities.com/SoHo/Exhibit/5901/index.html; http://members.nbci.com/_XMCM/steffenweber/POLYHEDRA/p_00.html).

The creation of virtual polyhedra shown in the present study has an origin on the study of morphology for mineral crystals such as quartz and pyrite. The variety of crystal forms observed in natural quartz crystals was studied by the computer simulation in the procedure based on the *p-q* diagram in which the variety of quartz crystal forms are completely classified by two factors, $p = gr(r)/gr(R)$ and $q = gr(m)/gr(R)$. Three terms, $gr(r)$, $gr(m)$ and $gr(R)$, mean the growth rates perpendicular to principal crystal faces, r, m and R, in quartz (IWASAKI, H. and IWASAKI, F., 1993, 1995). The pyrite crystals show more variety in their crystal forms rather than those of quartz (SUNAGAWA, 1957). The various crystal forms of natural pyrite were realized by the computer procedure to make the compound of three basic crystal forms, hexahedron (cube), octahedron and dihexahedron (pyrohedron) (IWASAKI, H. and IWASAKI, F., 1999).

In the present study, the virtual polyhedra as compounds consisting of three forms chosen from seven basic forms with cubic symmetry are presented on the procedure of computer aided drawings. The essentials of procedure are based on the mixing of three forms with cubic symmetry. The procedure of computer drawings creates virtual polyhedra up to 72 faces with much variety in their forms. Totally 35-sets of "*p*-*q* diagram" have obtained as the results of combinations of three forms chosen from seven basic forms.

2. The Computer Program for Drawings of "Virtual Polyhedra"

The original program for drawings is based on the "Sakurai program" (Sakurai, 1988). At first, the Sakurai program was applied to study the morphology of natural and synthetic quartz crystals (IWASAKI, H. and IWASAKI, F., 1993, 1995; IWASAKI, H. *et al*., 1998) and pyrite crystals (IWASAKI, H. and IWASAKI, F., 1999). After that, the original program was improved to be operated on Windows-98/95 (Japanese edition) (NORO *et al*., 1999). The present study of virtual polyhedra was made on the improved program. The improved drawing program for virtual polyhedra has the function to display both the *p*-*q* diagram and the form of polyhedron at the coordinates, *p* and *q*, on the computer screen. The variety of polyhedral forms is displayed automatically and successively on the screen by the scanning of (*p*,*q*) on *p*-*q* diagram. The additional function can offer the printed papers of drawn polyhedron and its expanded two-dimensional figure from which the polyhedral model can be made of the paper by the hand manipulation. The computer program for drawings of virtual polyhedra is opened on the Internet (http://www.fsinet.or.jp/~noro/polyhedra.htm).

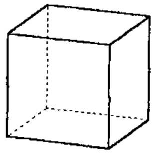
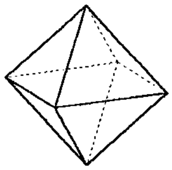
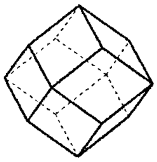
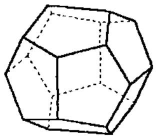
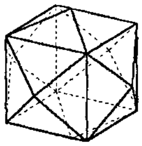
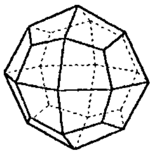
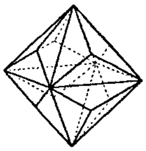
3. The Number of Combinations as the Compound of Two Forms and of Three Forms Chosen from Seven Basic Forms

The seven basic forms used for the making of compounds are shown in Table 1. The 35-sets of combinations of three forms chosen from seven basic forms are classified by the nominal number of total faces on polyhedra;

26-faces;	[o+a+e_+][o+a+d]	2-sets
30-faces;	[d+a+e_+]	1-set
32-faces;	[d+o+e_+]	1-set
38-faces;	[o+a+j][o+a+e][o+a+n]	3-sets
42-faces;	[d+a+e][e+a+e_+][n+a+e_+][d+a+j][d+a+n][j+a+e_+]	6-sets
44-faces;	[d+o+j][d+o+n][o+n+e_+][o+e_++j][o+e+e_+][e+d+o]	6-sets
48-faces;	[d+n+e_+][d+e+e_+][d+e_++j]	3-sets
54-faces;	[n+a+e][j+a+n][j+a+e]	3-sets
56-faces;	[e+o+j][o+n+j][e+o+n]	3-sets
60-faces;	[d+j+e][e+n+e_+][d+n+e][d+n+j][e_++e+j][e_++n+j]	6-sets
72-faces;	[e+n+j]	1-sets.

The polyhedral forms in these combinations are governed by mixing rates characterized by a set (*p*,*q*), as explained in Section 5. These 35-sets of combinations shown above contain the 21-sets of basic combinations of two forms;

Table 1. Basic forms with cubic symmetry.

Name of forms	Symbol	Number of faces	Specific faces	Form
Hexahedron(Cube)	a	6	{100}	
Octahedron	o	8	{111}	
Dodecahedron	d	12	{110}	
Dihexahedron (Pyrohedron)	e_+	12	{210}	
Tetrahexahedron	e	24	{102}	
Trapezohedron	n	24	{112}	
Trisoctahedron	j	24	{221}	

14-faces; [o+a]	1-set
18-faces; [d+a][e_++a]	2-sets
20-faces; [d+o][e_++o]	2-sets
24-faces; [d+e_+]	1-set
30-faces; [n+a][j+a][e+a]	3-sets
32-faces; [n+o][j+o][e+o]	3-sets
36-faces; [d+n][d+j][d+e][j+e_+][n+e_+][e+e_+]	6-sets
48-faces; [j+n][j+e][e+n]	3-sets.

Therefore, 35-sets of *p-q* diagrams contain these 21 sets of basic combinations as three branches in each *p-q* diagram. The basic combinations of two forms make also polyhedra and their forms are governed by mixing rates of two basic forms.

4. The Specific Character of the Combination: [e+e_+]

The forms,[e] and [e_+], are under the mutuality as the holohedral form [e] with 24-faces and the hemihedral form [e_+] with 12-faces. Therefore, in the mixing of these two forms, 12-faces of the holohedral form are held in common with the faces of hemihedral form. Consequently, the number of total faces in the mixing of two forms decreases into 24 from 36. Therefore, the total number of faces in six types of combinations with [e] and [e_+] is modified as follows;

[e+e_+]; 36-faces to 24-faces,
[a+e+e_+]; 42-faces to 30-faces,
[o+e+e_+]; 44-faces to 32-faces,
[d+e+e_+]; 48-faces to 36-faces,
[n+e+e_+]; 60-faces to 48-faces,
[j+e+e_+]; 60-faces to 48-faces.

However, *p-q* diagrams shown in Appendix are classified by the nominal number of total faces.

From the crystallographic view point, the mixing [e+e_+] is not realistic. However, in the present work, the mixing [e+e_+] with various mixing rates is positively adopted as the variety of forms.

5. The Data Needed for Computer Drawings of Virtual Polyhedra and Structures of *p-q* Diagram

As the basic input data for drawings, "lattice constants" and angles between "crystal axes" are respectively chosen as "5" (to be given arbitralily) and "90°", in common with three forms as the compositions of mixing (IWASAKI, H. and IWASAKI, F., 1999).

The following relation is needed to construct the *p-q* diagram for the mixing of three forms,

$$d(hkl) = do(hkl) + gr(hkl)t. \tag{1}$$

In the above relation, $d(hkl)$ means the distance measured from the center of basic form to each face (hkl). $do(hkl)$ means the initial size which is measured from the center of form to each face (hkl) and is usually given as the fixed value such as 1 or smaller values. The term $gr(hkl)$ means "growth rate" with the direction perpendicular to face (hkl). t means "growth time" in the case of crystal growth (IWASAKI, H. and IWASAKI, F., 1993, 1995, 1999). In the drawings of polyhedra, as the first step, the condition "$do(hkl) << gr(hkl)t$" must be kept in the execution of simulation. Therefore, "t" is usually chosen as 100 or larger values. For the form with cubic symmetry such as the cube [a], $d(hkl)$, $do(hkl)$ and $gr(hkl)$ can be presented by d(a), do(a) and gr(a) because the face (hkl) is regarded as equivalent. Others are presented on the same way such as d(o), do(o) $\cdots$.

To explain the procedure for the construction of p-q diagram, the combination [o+a+e_+] which is the case of pyrite, is adopted as an example (IWASAKI, H. and IWASAKI, F., 1999). In the combination [o+a+e_+], p and q are defined as growth rate ratios;

$$p = gr(e_+)/gr(a), \quad q = gr(o)/gr(a), \tag{2}$$

where $gr(e_+)$, gr(o) and gr(a) have the meanings of the growth rates with direction perpendicular to the faces in each basic form [a], [o] and [e_+]. The forms of virtual polyhedra are determined by the relative values of growth rate ratios of faces. Therefore, the condition $gr(a) = 1$ is usually adopted. The data structure for compound or combination [o+a+e_+] can be made as shown in Table 2 where the case, $p = 1.1$ and $q = 1.2$, is presented. In the other sets of combinations, the same way is applicable to define the growth rate ratios, p and q.

Table 2. An example of data structure for compound [o+a+e_+]; $p = gr(e_+)/gr(a) = 1.1$ and $q = gr(o)/gr(a) = 1.2$.

No.	face (hkl)	do	gr(hkl)	
1	1 0 0	1	1	
2	0 1 0	1	1	
3	0 0 1	1	1	
4	-1 0 0	1	1	gr(a)
5	0 -1 0	1	1	
6	0 0 -1	1	1	
7	1 1 1	1	1.2	
8	1 -1 1	1	1.2	
9	-1 1 1	1	1.2	
10	-1 -1 1	1	1.2	
11	1 1 -1	1	1.2	gr(o)
12	1 -1 -1	1	1.2	
13	-1 1 -1	1	1.2	
14	-1 -1 -1	1	1.2	
15	2 1 0	1	1.1	
16	2 -1 0	1	1.1	
17	-2 1 0	1	1.1	
18	-2 -1 0	1	1.1	
19	1 0 2	1	1.1	
20	1 0 -2	1	1.1	$gr(e_+)$
21	-1 0 2	1	1.1	
22	-1 0 -2	1	1.1	
23	0 2 1	1	1.1	
24	0 2 -1	1	1.1	
25	0 -2 1	1	1.1	
26	0 -2 -1	1	1.1	

6. “*p-q* Diagram” for 35-Sets of Compounds

The structures and patterns of 35-sets of “*p-q* diagram” are shown as *pq*-1 to *pq*-35 in Appendix. In addition, the polyhedral form drawn at a set of specific values (p,q) and its expanded figure are shown. The *p-q* diagrams are always composed of 7-areas. In the case [o+a+e_+] shown as *pq*-1, the 7-areas are distinguishable as [o], [a], [e_+], [o+a], [o+e_+], [a+e_+] and [o+a+e_+]. In the areas, [o], [a], [e_+], the polyhedron in each area shows the basic form. Polyhedra in the areas, [o+a], [o+e_+], [a+e_+], are composed of the mixing of two basic forms and they have faces, 14, 20, and 18, respectively. The polyhedra in the areas, [a+e_+] and [o+a], are governed respectively by only p and only q. In the area [o+e_+], the polyhedral forms apparently depend both on p and q but show same forms at $p = q$. In the central area [o+a+e_+], the polyhedra with 26 faces show the variety of forms governed by both p and q.

The following sets of *p-q* diagrams have same patterns;

pq-1: [o+a+e_+] = *pq*-6: [o+a+e], *pq*-10: [n+a+e_+] = *pq*-23: [n+a+e]
pq-11: [j+a+e_+] = *pq*-25: [j+a+e], *pq*-34: [e_++n+j] = *pq*-35: [e+n+j].

However, the patterns, *pq*-3: [d+a+e_+] and *pq*-8: [d+a+e], are not same but show small different values of the coordinates, B. Furthermore, in the following 5-sets of *p-q* diagrams with [e+e_+] branch,

pq-9: [e+a+e_+], *pq*-18: [o+e+e_+], *pq*-21: [d+e+e_+],
pq-33: [e_++e+j], *pq*-30: [e+n+e_+],

the drawn polyhedra at $p = q = 1$ show an anomaly on their shapes. These facts suggest that the phenomena are relating to the characters of basic forms, e and e_+ and partly the character of form, d, in connection with e and e_+. The anomaly of shapes at $p = q = 1$ may be due to the confusion on calculating process relating to mixing with same weight of holohedral and hemihedral forms.

7. Specific Values Characterizing the Mixing of Two Basic Forms

The essentials in the creation of polyhedra are to mix the three basic forms with cubic symmetry. However, as shown in Section 3, the mixings or combinations such as [o+a], [d+a] ... exist in *p-q* diagrams. The boundary at which the mixing of [o] and [a] begins, is characterized by the specific values of “growth rate ratio; $gr(o)/gr(a)$ The specific values at boundaries for 21-sets of basic combinations were evaluated under the simulation process on 35-sets of *p-q* diagrams. The evaluated results are shown in Table 3.

On the one hand, historically, Wells analyzed the mixing of forms, [o] and [a], from the view point of thermodynamics for the morphology in crystal growth. Wells evaluated the values of surface free energy ratios of mixed cubic crystal at the boundaries, [o] – [o+a] and [o+a] – [a], respectively, as 0.58 and 1.73 (WELLS, 1946). These values agree with the values shown in Table 3. This fact means that the surface free energy ratios and surface

Table 3. Mixing of forms A and B characterized by “growth rate ratio”, *gr*(A)/*gr*(B).

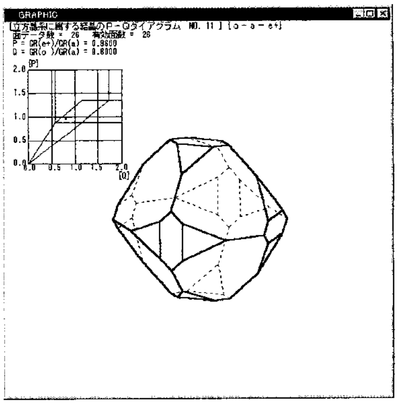
No	Form A	Value of gr(A)/gr(B) (0, 1, 2; A / A+B (Mixed))	(A+B (Mixed) / B)	Form B
1	o	0.58	1.73	a
2	d	0.72	1.42	a
3	n	0.82	1.63	a
4	d	0.82	1.23	o
5	j	0.96	1.16	o
6	j	0.65	1.67	a
7	j	0.81	1.09	n
8	d	0.86	1.15	n
9	d	0.85	1.07	j
10	n	0.94	1.41	o
11	d	0.79	1.07	e
12	d	0.79	1.19	e_+
13	j	0.75	1.24	e
14	j	0.75	1.24	e_+
15	e_+	0.83	1.1	n
16	e	0.83	1.1	n
17	e_+	0.77	1.55	o
18	e	0.77	1.55	o
19	e_+	0.89	1.35	a
20	e	0.89	1.35	a
21	e_+	0.8	1.0	e

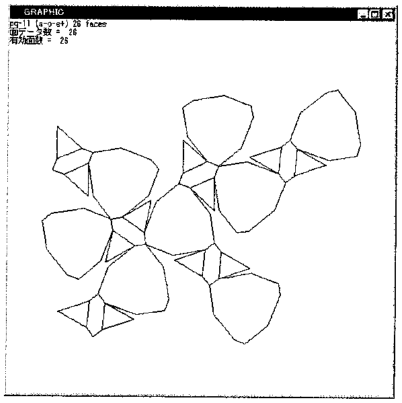
growth rate ratios are equivalent.

The forms, [e] and [e_+], show normal characters on the mixing with [j], [n], [o] and [a], as shown in Table 3. However, in the mixing with [d+e] and [d+e_+], specific values at the boundaries, [d+e] – [e] and [d+e_+] – [e_+], show different characters. This fact may be relating to the small difference between patterns, *pq*-3 and *pq*-8, as pointed out in Section 6.

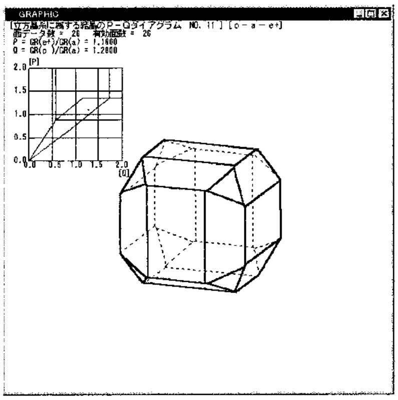
Table 4. Polyhedra and their expanded figures in *pq*-1: [o+a+e_+].

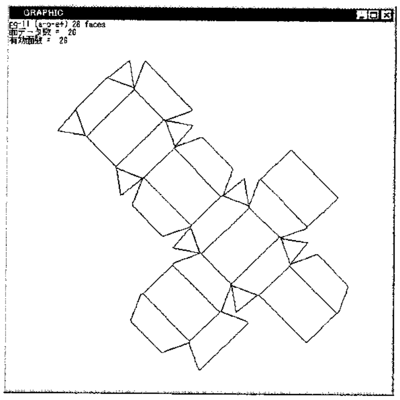
p=0.96, q=0.8



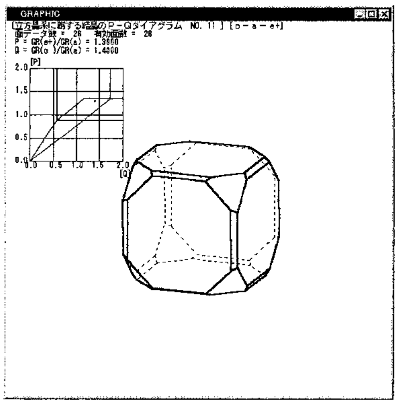


p=1.10, q=1.28





p=1.30, q=1.40



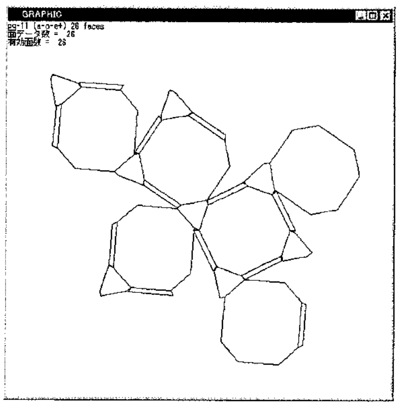
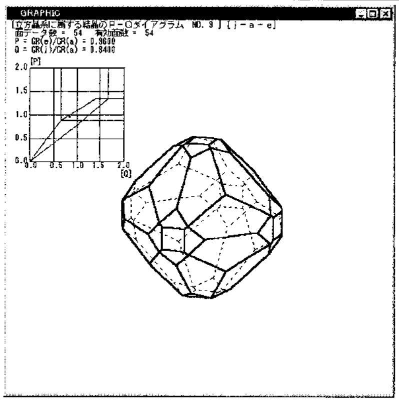
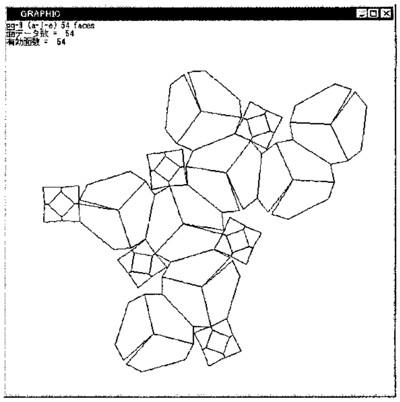


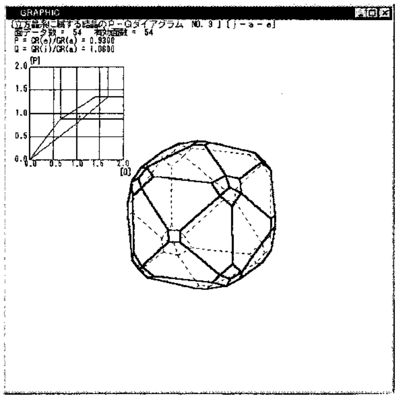
Table 5. Polyhedra and their expanded figures in *pq*-25: [j+a+e].

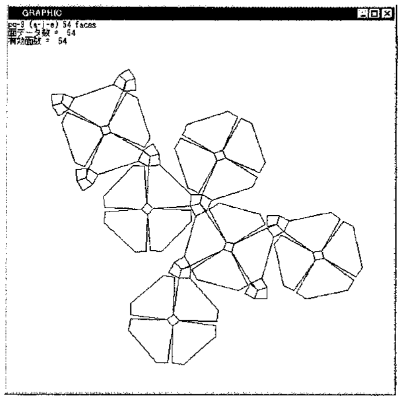
p=0.96, q=0.84



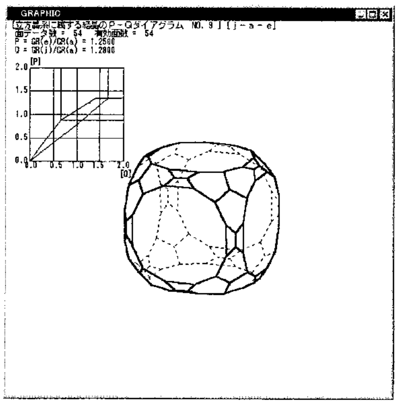


p=0.93, q=1.06





p=1.25, q=1.28



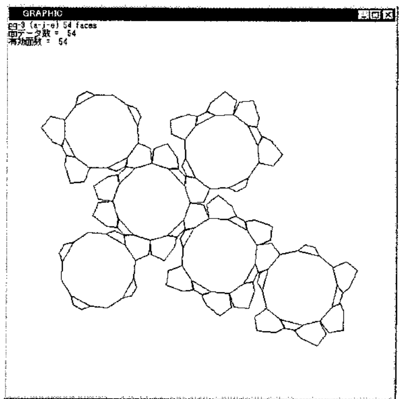
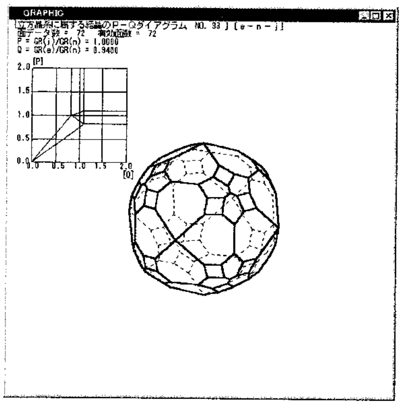
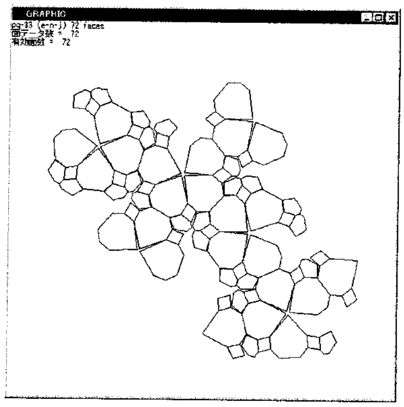


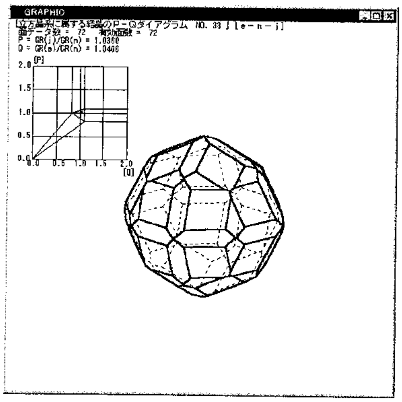
Table 6. Polyhedra and their expanded figures in *pq*-35: [e+n+j].

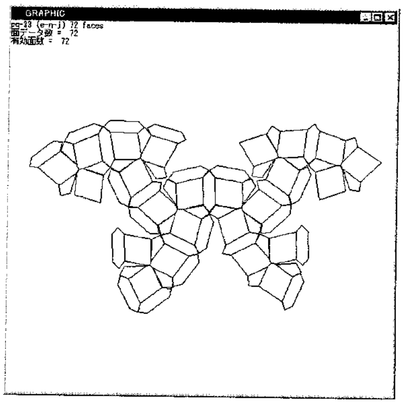
p=1.0, q=0.94



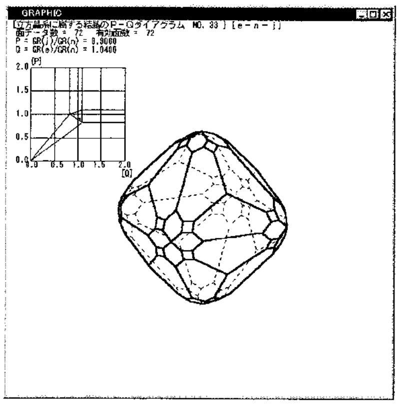


p=1.03, q=1.04





p=0.9, q=1.04



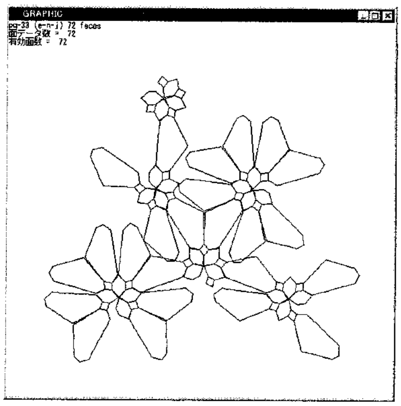


Table 7. Variations of polyhedral forms as a function of “time” in the case, *pq*-32: [d+n+j].

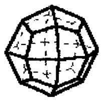
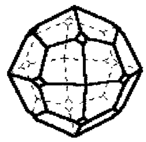
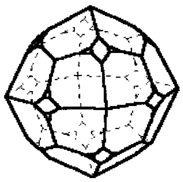
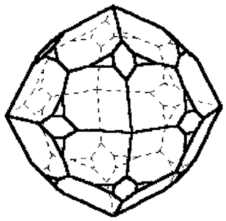
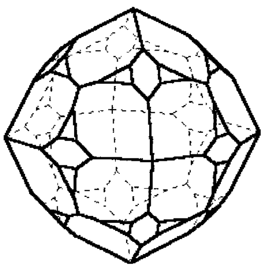
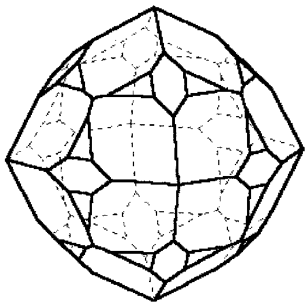
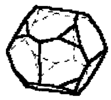
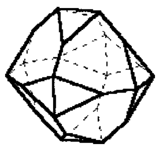
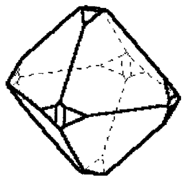
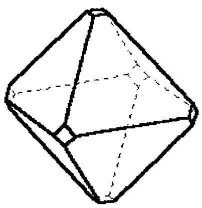
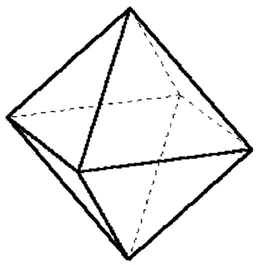
do(d)=1.2, do(n)=0.5, do(j)=1; gr(d)=gr(n)=gr(j)=1	
Time t=4 [n]	Time t=6 [n+d]
	
Time t=8 [n+d+j]	Time t=10 [n+d+j]
	
Time t=12 [n+d+j]	Time t=14 [n+d+j]
	

Table 8. Variations of polyhedral forms as a function of “time” in the case, *pq*-1: [o+a+e_+].

do(o)=1.0, do(a)=1.0, do(e+)=0.5; gr(o)=0.5, gr(a)=1.2, gr(e_+)=1.5	
Time t=0.25 [e_+]	Time t=0.35 [o+e_+]
	
Time t=0.8 [o+e_+]	Time t=1.05 [o+a+e_+]
	
Time t=1.25 [o+a]	Time t=2.5 [o]
	

8. Examples of Virtual Polyhedra and Their Expanded Figures

Three sets of *p*-*q* diagrams, *pq*-1: [o+a+e_+], *pq*-25: [j+a+e], and *pq*-35: [e+n+j], are chosen as examples to show virtual polyhedra and their expanded figures at the selected values (*p*,*q*) in the central area in each *p*-*q* diagram. Those are shown in Tables 4, 5, and 6. Numbers of their faces are, respectively, 26, 54, and 72. It should be noticed that the shapes of polyhedra and corresponding expanded figures vary remarkably with the small changes of values (*p*,*q*).

9. The Variations of Forms and Sizes as a Function of "Time"

In the data used for drawings at the first step, *do*(*hkl*)s in compositional forms are chosen as same value, 1, as shown in Table 2. If the values *do*(*hkl*)s are chosen as different values in each compositional form, the forms of created polyhedra vary dramatically with the "time" during the selected time interval. As an example, the variations of form and size in the compound *pq*-32: [d+n+j] are drawn under the values *do*(n) = 0.5, *do*(j) = 1.0, and *do*(d) = 1.2 within the time interval from t = 4 to t = 14 and are shown in Table 7. The conditions, *gr*(n) = *gr*(j) = *gr*(d) = 1, are kept. The polyhedral forms are varying in the order, [n] at $t = 4$, [n+d] at $t = 6$, and [n+d+j] at $\sim 7 < t$, with the increase of size. As another example, the case *pq*-1; [a+o+e_+] is shown in Table 8. The forms are changing as a function of "time" in the following order; [e_+] $\rightarrow$ [o+e_+] $\rightarrow$ [a+o+e_+] $\rightarrow$ [a+o] $\rightarrow$ [o].

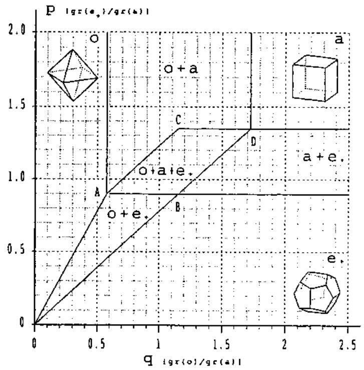
10. Conclusive Summary

1. The present study offers the new procedure for the creation of virtual polyhedra by means of computer aided drawings. The essentials of procedure are based on the mixing of three basic polyhedral forms. The creation of virtual polyhedra with many varieties was realized on the combinations of three forms chosen from seven basic forms with cubic symmetry.
2. The 35-sets of combinations were obtained as the compounds of three forms. In each set, the forms of virtual polyhedra are determined by the relative mixing rates of three basic forms. The characteristic forms of created virtual polyhedra in each set of combination are specified quantitively at the point (*p*,*q*) on the *p*-*q* diagram. This fact suggests the possibility that "a sense of beauty" on polyhedra in the people's heart will be evaluated quantitatively, by looking for the various forms of virtual polyhedra drawn under the numerically controlled data.
3. The present procedure for the computer drawings of virtual polyhedra will be applied as the creative technique in the fields of the pattern-makings in modeling, the formative arts, and the "calculative or computable fine arts".
4. The characteristics of specific patterns drawn by linear lines on *p*-*q* diagrams should be derived from mathematical analysis on the view point of mixing by three basic forms with cubic symmetry.

APPENDIX

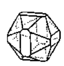
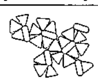
Appendix --- p-q diagram [pq-1 to pq-4]

pq-1 [o-a-e. diagram] (26 faces)

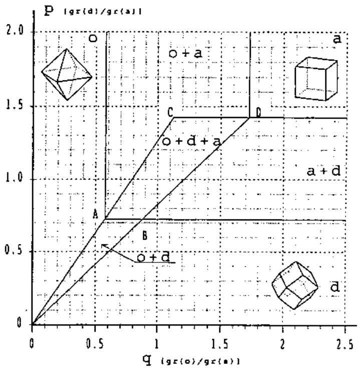


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	1.35	1.35
q	0.58	1.15	1.15	1.73

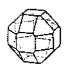
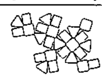
pq-1	Mixed Form	Expanded figure
Faces 26		
p = 1		
q = 1		

pq-2 [o-a-d diagram] (26 faces)

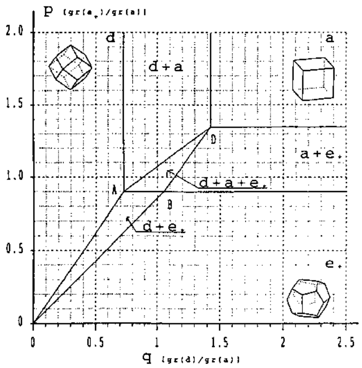


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.72	0.72	1.42	1.42
q	0.58	0.88	1.12	1.73

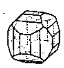
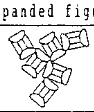
pq-2	Mixed Form	Expanded figure
Faces 26		
p = 1		
q = 1		

pq-3 [d-a-e. diagram] (30 faces)

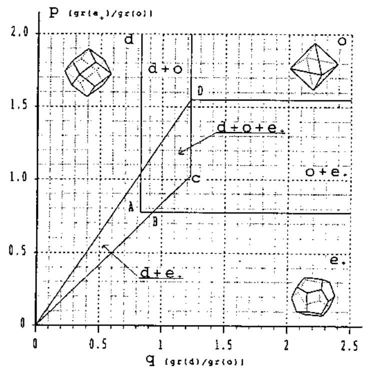


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	--	1.35
q	0.72	1.05	--	1.42

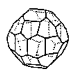
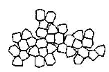
pq-3	Mixed Form	Expanded figure
Faces 30		
p = 1		
q = 1		

pq-4 [d-o-e. diagram] (32 faces)



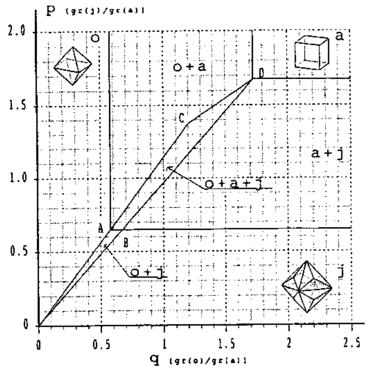
Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.77	0.77	1.025	1.55
q	0.82	0.92	1.23	1.23

pq-4	Mixed Form	Expanded figure
Faces 32		
p = 1		
q = 1		

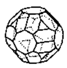
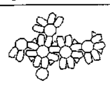
Appendix --- p-q diagram [pq-5 to pq-8]

pq-5 [o-a-j diagram] (38 faces)

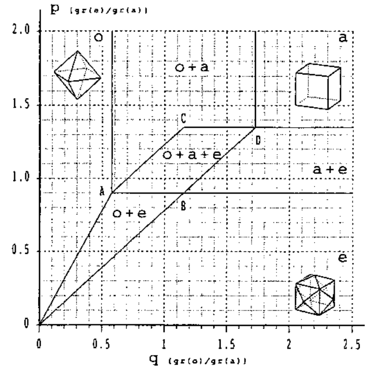


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.65	0.65	1.38	1.67
q	0.58	0.67	1.2	1.73

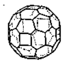
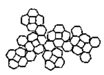
pq-5	Mixed Form	Expanded figure
Faces 38 p = 1 q = 1		

pq-6 [o-a-e diagram] (38 faces)

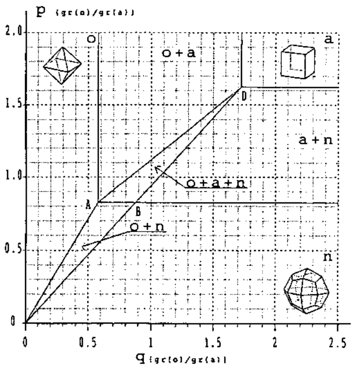


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	1.35	1.35
q	0.58	1.15	1.15	1.73

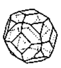
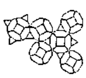
pq-6	Mixed Form	Expanded figure
Faces 38 p = 1 q = 1		

pq-7 [o-a-n diagram] (38 faces)

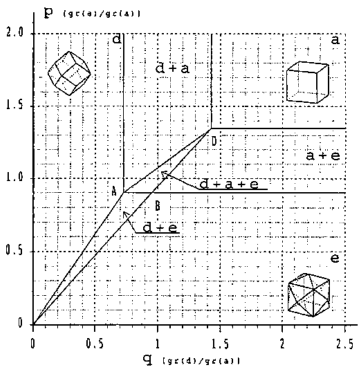


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.82	0.82	--	1.63
q	0.58	0.88	--	1.73

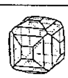
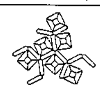
pq-7	Mixed Form	Expanded figure
Faces 38 p = 1 q = 1		

pq-8 [d-a-e diagram] (42 faces)



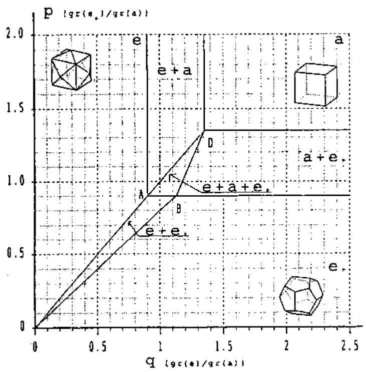
Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	--	1.35
q	0.72	0.95	--	1.42

pq-8	Mixed Form	Expanded figure
Faces 42 p = 1 q = 1		

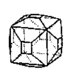
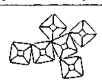
Appendix --- p-q diagram [pq-9 to pq-12]

pq-9 [e-a-e. diagram] (42 faces)

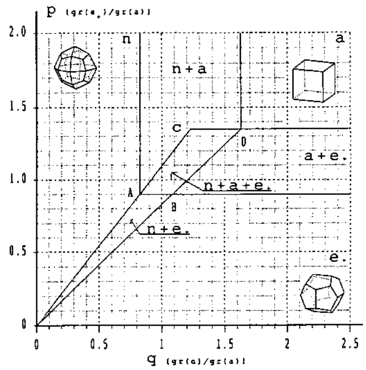


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	--	1.35
q	0.89	1.12	--	1.35

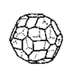
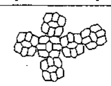
pq-9	Mixed Form	Expanded figure
Faces 30(42)		
p = 1		
q = 0.99		

pq-10 [n-a-e. diagram] (42 faces)

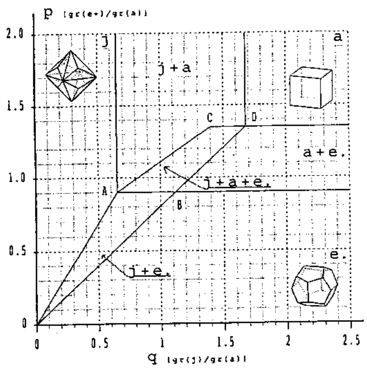


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	1.35	1.35
q	0.82	1.07	1.22	1.63

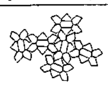
pq-10	Mixed Form	Expanded figure
Faces 42		
p = 1		
q = 1		

pq-11 [j-a-e. diagram] (42 faces)

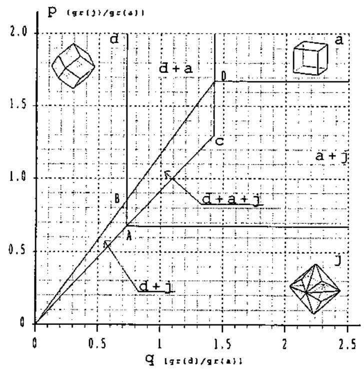


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	1.35	1.35
q	0.65	1.12	1.4	1.67

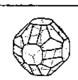
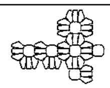
pq-11	Mixed Form	Expanded figure
Faces 42		
p = 1		
q = 1		

pq-12 [d-a-j diagram] (42 faces)



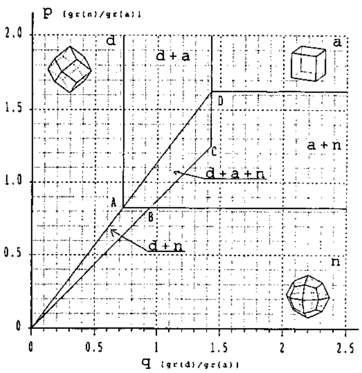
Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.65	0.85	1.31	1.67
q	0.72	0.72	1.42	1.42

pq-12	Mixed Form	Expanded figure
Faces 42		
p = 1		
q = 1		

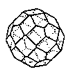
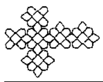
Appendix --- p-q diagram [pq-13 to pq-16]

pq-13 [d-a-n diagram] (42 faces)

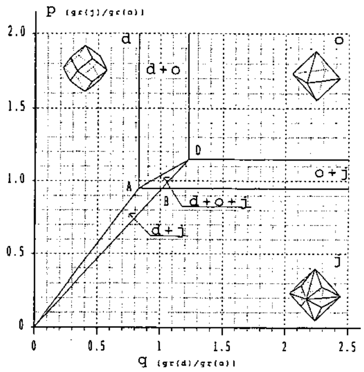


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.82	0.82	1.25	1.63
q	0.72	0.94	1.42	1.42

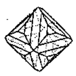
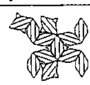
pq-13	Mixed Form	Expanded figure
Faces 42		
p = 1		
q = 1		

pq-14 [d-o-j diagram] (44 faces)

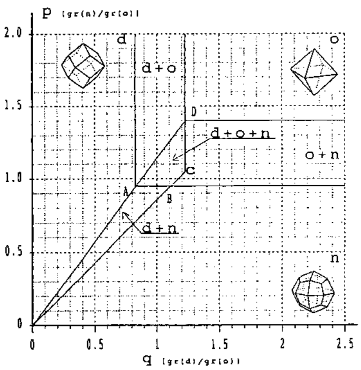


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.96	0.96	--	1.16
q	0.82	1.02	--	1.23

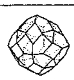
pq-14	Mixed Form	Expanded figure
Faces 44		
p = 1		
q = 1		

pq-15 [d-o-n diagram] (44 faces)

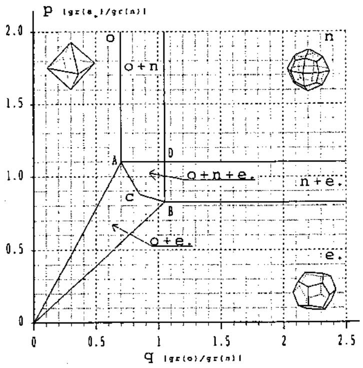


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.94	0.94	1.05	1.42
q	0.82	1.075	1.23	1.23

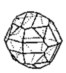
pq-15	Mixed Form	Expanded figure
Faces 44		
p = 1		
q = 1		

pq-16 [o-n-e. diagram] (44 faces)



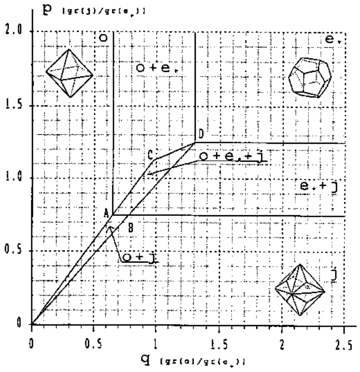
Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.09	0.83	0.875	1.1
q	0.70	1.06	0.85	1.06

pq-16	Mixed Form	Expanded figure
Faces 44		
p = 1		
q = 1		

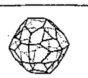
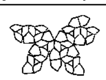
Appendix --- p-q diagram [pq-17 to pq-20]

pq-17 [o-e.-j diagram] (44 faces)

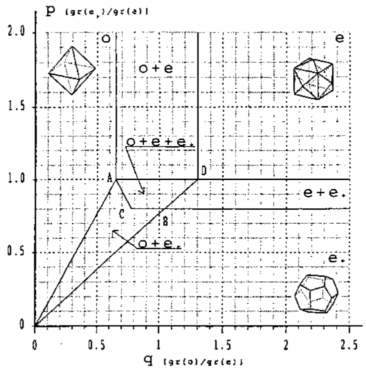


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.75	0.75	1.125	1.24
q	0.64	0.775	0.975	1.29

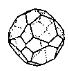
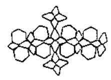
pq-17	Mixed Form	Expanded figure
Faces 44		
p = 1		
q = 1		

pq-18 [o-e-e. diagram] (44 faces)

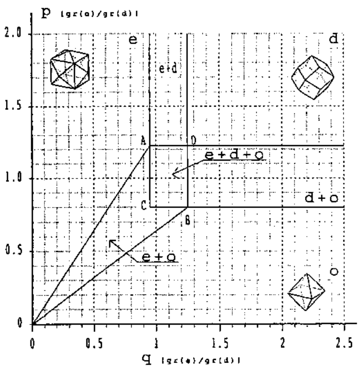


Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.0	0.8	0.8	1.0
q	0.65	1.05	0.775	1.3

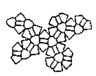
pq-18	Mixed Form	Expanded figure
Faces 32(44)		
p = 0.98		
q = 0.88		

pq-19 [e-d-o diagram] (44 faces)

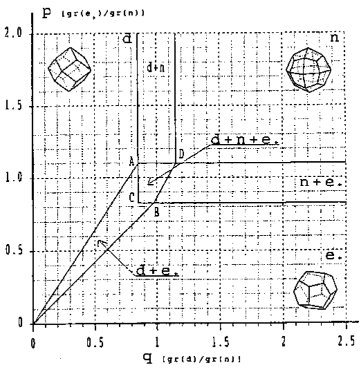


Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.22	0.81	0.81	1.22
q	0.94	1.26	0.94	1.26

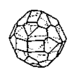
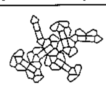
pq-19	Mixed Form	Expanded figure
Faces 44		
p = 1		
q = 1		

pq-20 [d-n-e. diagram] (48 faces)



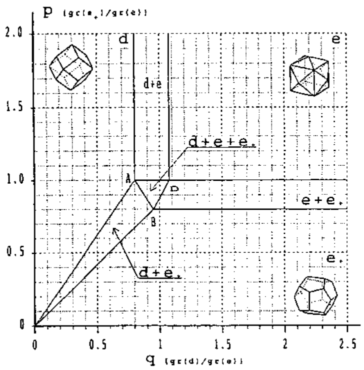
Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.10	0.83	0.83	1.10
q	0.86	0.975	0.86	1.15

pq-20	Mixed Form	Expanded figure
Faces 48		
p = 1		
q = 1		

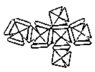
Appendix --- p-q diagram [pq-21 to pq-24]

pq-21 [d-e-e. diagram] (48 faces)

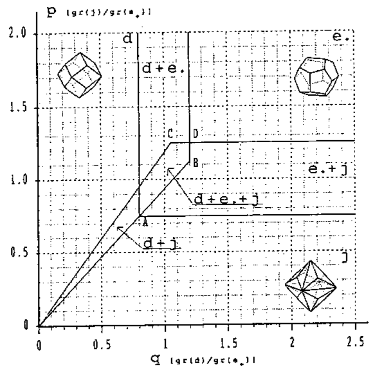


Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.00	0.80	--	1.00
q	0.79	0.95	--	1.07

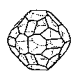
pq-21	Mixed Form	Expanded figure
Faces 36(48)		
p =0.99		
q = 1		

pq-22 [d-e.-j diagram] (48 faces)

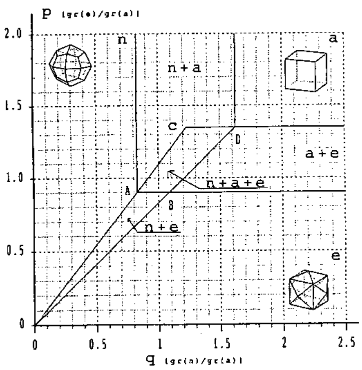


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.75	1.125	1.24	1.24
q	0.79	1.19	1.05	1.19

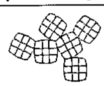
pq-22	Mixed Form	Expanded figure
Faces 48		
p = 1		
q = 1		

pq-23 [n-a-e diagram] (54 faces)

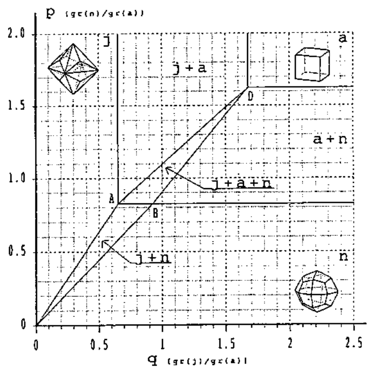


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	1.35	1.35
q	0.82	1.07	1.22	1.63

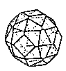
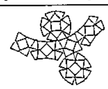
pq-23	Mixed Form	Expanded figure
Faces 54		
p = 1		
q = 1		

pq-24 [j-a-n diagram] (54 faces)



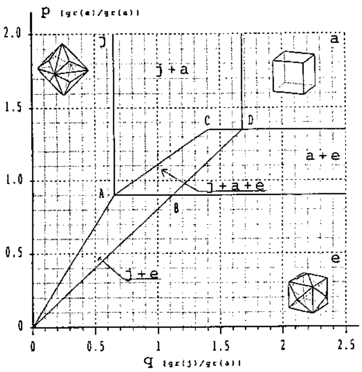
Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.82	0.82	--	1.63
q	0.65	0.92	--	1.67

pq-24	Mixed Form	Expanded figure
Faces 54		
p = 1		
q = 1		

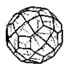
Appendix --- p-q diagram [pq-25 to pq-28]

pq-25 [j-a-e diagram] (54 faces)

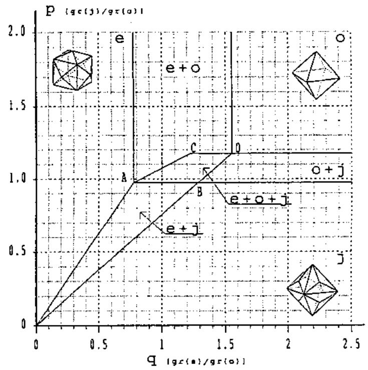


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.89	0.89	1.35	1.35
q	0.65	1.12	1.4	1.67

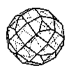
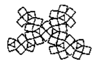
pq-25	Mixed Form	Expanded figure
Faces 54		
p = 1		
q = 1		

pq-26 [e-o-j diagram] (56 faces)

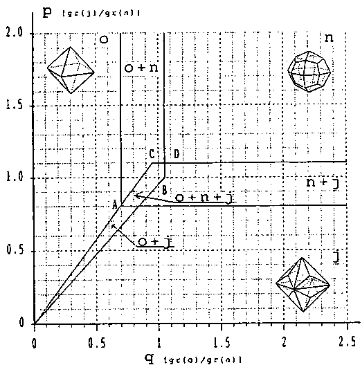


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.96	0.96	1.16	1.16
q	0.77	1.275	1.225	1.55

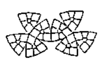
pq-26	Mixed Form	Expanded figure
Faces 56		
p = 1		
q = 1		

pq-27 [o-n-j diagram] (56 faces)

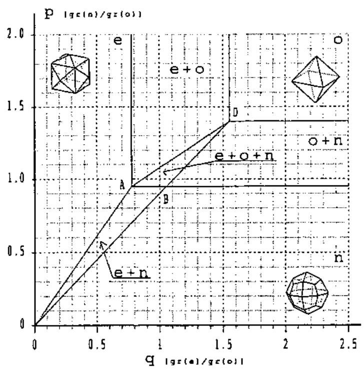


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.81	1.0	1.09	1.09
q	0.7	1.05	0.95	1.06

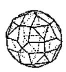
pq-27	Mixed Form	Expanded figure
Faces 56		
p = 1		
q = 1		

pq-28 [e-o-n diagram] (56 faces)



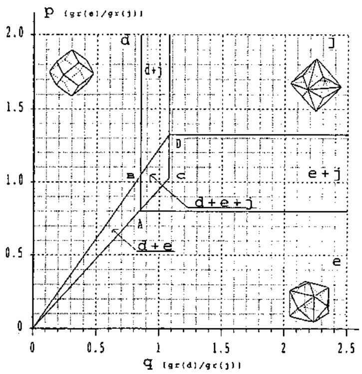
Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.94	0.94	--	1.41
q	0.77	1.05	--	1.55

pq-28	Mixed Form	Expanded figure
Faces 56		
p = 1		
q = 1		

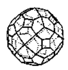
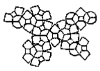
Appendix --- p-q diagram [pq-29 to pq-32]

pq-29 [d-j-e diagram] (60 faces)

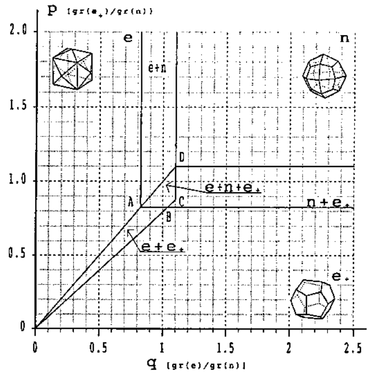


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.81	1.05	1.02	1.32
q	0.85	0.85	1.07	1.07

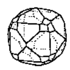
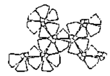
pq-29	Mixed Form	Expanded figure
Faces 60		
p = 1		
q = 1		

pq-30 [e-n-e+ diagram] (60 faces)

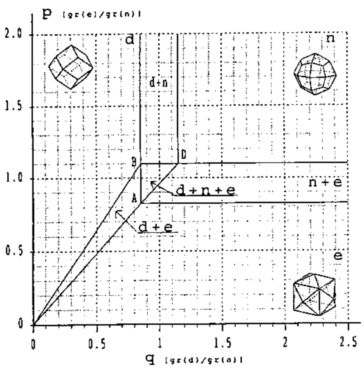


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.83	0.83	0.875	1.1
q	0.83	1.04	1.1	1.1

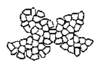
pq-30	Mixed Form	Expanded figure
Faces 48(60)		
p = 0.9		
q = 0.92		

pq-31 [d-n-e diagram] (60 faces)

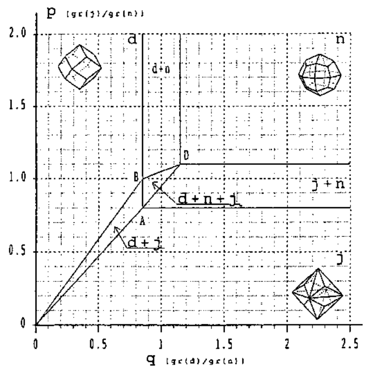


Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.83	1.1	--	1.1
q	0.86	0.86	--	1.15

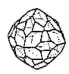
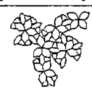
pq-31	Mixed Form	Expanded figure
Faces 60		
p = 1		
q = 1		

pq-32 [d-n-j diagram] (60 faces)



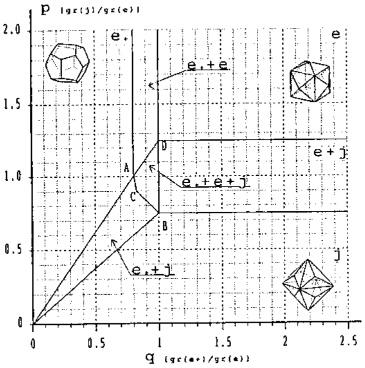
Coordinates of points, A, B, C and D.

	A	B	C	D
p	0.81	1.0	--	1.09
q	0.86	0.86	--	1.15

pq-32	Mixed Form	Expanded figure
Faces 60		
p = 1		
q = 1		

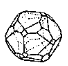
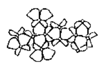
Appendix --- p-q diagram [pq-33 to pq-35]

pq-33 [e₊-e-j diagram] (60 faces)

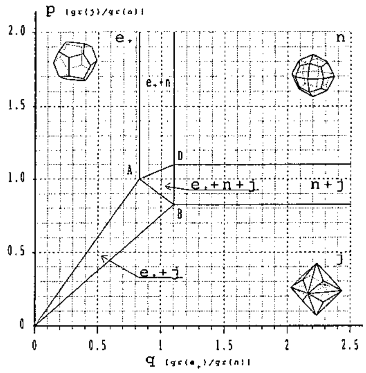


Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.025	0.75	0.9	1.24
q	0.80	1.0	0.825	1.00

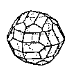
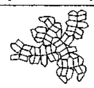
pq-33	Mixed Form	Expanded figure
Faces 48(60)		
p = 1		
q = 0.87		

pq-34 [e₊-n-j diagram] (60 faces)

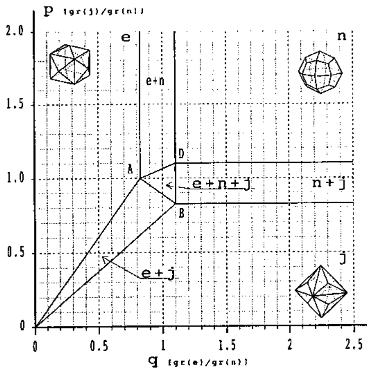


Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.0	0.81	--	1.09
q	0.83	1.1	--	1.1

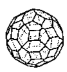
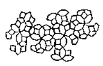
pq-34	Mixed Form	Expanded figure
Faces 60		
p = 1		
q = 1		

pq-35 [e-n-j diagram] (72 faces)



Coordinates of points, A, B, C and D.

	A	B	C	D
p	1.0	0.81	--	1.09
q	0.83	1.1	--	1.1

pq-35	Mixed Form	Expanded figure
Faces 72		
p = 1		
q = 1		

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