# Symmetropy and Self-Organized Criticality

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**Abstract.** Curie symmetry principle (CSP) states that the effects may occasionally have the same or a higher symmetry than the causes, and is often a powerful constraint for setting a bound on the symmetries of the causes or the effects. In this paper, the validity of CSP in a complex system is investigated. The concept of "symmetropy" that can measure the amount of symmetry and entropy of an object is introduced, and the symmetropies evaluating the causes and the effects in a cellular-automaton system with self-organized criticality are examined. During sub-critical states, the symmetropies satisfy CSP, but not always at the criticality (the edge of chaos). Our results show that symmetry breaking can occur without an anisotropy interaction between elements in a cellular-automaton model and without a special tuning parameter in bifurcation process.

### 1. Introduction

Curie symmetry principle (CSP) is an aspect of the causality relationship between the symmetry of the cause and the resultant effect (CURIE, 1894), and allows us to predict possible properties and to forbid impossible ones in physical processes. The CSP is composed of three parts: 1. *If certain causes yield the known effects, the symmetry elements of the causes should be contained in the generated effects.* 2. *If the known effects manifest certain dissymmetry, this latter should be contained in the causes which have generated those effects.* 3. *The converse to these two previous propositions is not true, at least in practical; i.e., the effects may have higher symmetry than the causes, which generate these effects.* JAEGER (1920) restated CSP as follow: *the effect may occasionally have the same or a higher symmetry than the causes, but the last cannot have a higher symmetry than the effects produced* (see also CURIE, 1923; SHUBNIKOV, 1956; COTTON, 1963; NAKAMURA and NAGAHAMA, 2000). For example, when the mechanically isotropic medium is subjected to an orthorhombic stress field, the intersection between the causes (symmetries of the medium and the applied field) will show orthorhombic symmetry (JAEGER, 1920). According to CSP, the symmetry of the resultant effect must be at least of orthorhombic symmetry. In

a macroscopically isotropic system, the symmetry of the effect actually shows an orthorhombic shape under the linear constitutive relation such as Hooke's law. This is consistent with CSP.

However, other researches (BOULIGAND, 1985; STEWART, 1990; STEWART and GOLUBITSKY, 1992; NAKAMURA and NAGAHAMA, 2000) refused to invoke CSP for all fields undoubtedly. For example, when a drop of milk hits the surface of a bowl filled with the same liquid, the shape of a splash looks like a crown called milk crown (STEWART and GOLUBITSKY, 1992). This shows that the effect (milk crown) selects a lower symmetry than the intersection between the causes (drop, bowl and milk). This is inconsistent with CSP, and shows symmetry breaking. Such researches (BOULIGAND, 1985; STEWART, 1990; STEWART and GOLUBITSKY, 1992; NAKAMURA and NAGAHAMA, 2000) lead us to check the validity of CSP in each given system, but no one has checked it in a complex system. This paper is devoted to examine whether CSP holds in a complex system.

It is necessary to evaluate the symmetry of the cause and effect. Without doubt, a very good tool to deal with perfect symmetries is provided by mathematical group theory (SATTINGER, 1978). However, there is no perfect symmetry in nature (e.g., CURIE, 1894; EIGEN and WINKLER, 1975). Thus, it is natural to analyze symmetry properties in terms of a continuous scale rather than in terms of "yes or no" (NAGY, 1996), although there is no general theory that explains the occurrence of continuous symmetries. WHYTE (1949a, b) discussed on "unitary principle" stating that "asymmetry tends to disappear and this tendency is realized in isolable process". Against this principle, DINGLE (1949) has described that "the principle that nothing can be created or destroyed would have been disastrous to science, for it would have prohibited the conception of entropy, yet conservation laws are of the greatest value in restricted field". Moreover, independently from the above studies (DINGLE, 1949; WHYTE, 1949a, b), CAILLOIS (1973) submitted the tendency to dissymmetry connected with negentropy. NAGY (1996) discussed that the topic of (dis)symmetry measures may be connected the information science via the concept of entropy (the measure of disorder and uncertainty). They (DINGLE, 1949; WHYTE, 1949a, b; CAILLOIS, 1973; NAGY, 1996) proposed that the concept of symmetry is related with that of entropy.

"Symmetropy" mathematically relates between symmetry and entropy through information theory, and measures the amount of symmetry and entropy of a given pattern or shape (YODOGAWA, 1982; NANJO *et al.*, 2000). It is consistent with the previous proposition (DINGLE, 1949; WHYTE, 1949a, b; CAILLOIS, 1973; NAGY, 1996). Symmetropy has been developed as a new objective physical measure of visual symmetry in perception and psychophysics (YODOGAWA, 1982), and applied to spatial distributions of acoustic emissions generated by microfracturings during creep before ultimate whole fracture of a rock specimen (NANJO *et al.*, 2000). However, no one has applied it to spatial patterns emerged from a complex system. The concept of the symmetropy for the use to evaluate the symmetry of the cause and effect is introduced in this paper.

Cellular-automaton systems have been studied for a long time as models of complex systems (in discrete time) in physics, chemistry and biology (e.g., BOULIGAND, 1985; GUNJI, 1990; WALDROP, 1992). Complexity measured by mutual information between two cells to study correlation in systems attains its maximum at the phase transition between periodic and chaotic behavior of cellular-automaton systems (WALDROP, 1992). The

system at the edge of chaos is equal to that of dynamical critical state, where a scale-free behavior emerges. Bak and coworkers (e.g., BAK *et al.*, 1987; BAK, 1996) introduced the concept of self-organized criticality (SOC) in the sand-pile cellular-automaton model. SOC provides a simple mechanism with possible applications to earthquakes, galaxy, turbulence, biological evolution, economic fluctuation and so on. In the present paper, a sand-pile cellular-automaton model is used as a prototype one of a complex system.

Here we first introduce the concept of the symmetropy that measures the amount of symmetry and entropy for spatial patterns in a complex system. Next, a brief explanation of a sand-pile cellular-automaton model is given as a prototype one of a complex system. Moreover, the question of whether CSP holds is examined in the model, with the estimation of the symmetropies evaluating the symmetry of the causes and the effects. Finally, symmetry breaking, continuous symmetry and symmetropy are discussed. The contents of this paper are based on our extended abstract for "Intersections of Art and Science", the 5th Interdisciplinary Symmetry: Art and Science", the Quarterly of the International Society for the Interdisciplinary Study of Symmetry (NANJO *et al.*, 2001).

## 2. Symmetropy

Symmetropy S is defined, utilizing that the two-dimensional discrete Walsh functions are divided into the four types of symmetry: vertically symmetry, horizontally symmetry, centrosymmetry, and double symmetry (Fig. 1). Patterns used in this paper are restricted to square grid, each consisting of  $M \times M$  square cells, where  $M = 2^q$  (q is a positive integer). The two-dimensional discrete Walsh transform of a pattern is given by

$$a_{m,n} = \frac{1}{M^2} \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} x_{i,j} W_{m,n}(i,j),$$
(1)

where  $m, n = 0, 1, 2, ..., M - 1, x_{i,j}$  is the value of gray level of a pattern in the *j*-th row cell in the *i*-th column,  $W_{m,n}(i, j)$  is the value (1 or -1) of the (m,n)-th order of the twodimensional discrete Walsh function in the *j*-th row cell in the *i*-th column (Fig. 1), and  $a_{m,n}$ is the two-dimensional Walsh spectrum. If there are just two gray levels, say, "black" and "white", we usually represent  $x_{i,j}$  by 1 and 0.

Symmetric component  $P_k$  (k = 1, 2, 3, 4) quantifying the four types of symmetry is given by

$$P_{k} = \sum_{m,n} (a_{m,n})^{2} / K,$$
(2)

where

$$K = \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} (a_{m,n})^2 - (a_{0,0})^2$$



Fig. 1. The first 16 of the two-dimensional discrete Walsh functions  $W_{m,n}$  (M = 4) of the (m,n)-th order. Black cells and white ones represent 1 and -1, respectively. The function  $W_{m,n}$  is vertically symmetric when m = even and n = odd, horizontally symmetric when m = odd and n = even, centrosymmetric when m = odd and n = odd, and doubly symmetric when m = even and n = even.  $W_{0,0}$  is the exception.

and

$$\sum_{k=1}^4 P_k = 1.$$

Vertically symmetric component  $P_1$  is given when m = even and n = odd. Horizontally symmetric one  $P_2$  is given when m = odd and n = even. Centrosymmetric one  $P_3$  is given when m = odd and n = odd. Doubly symmetric one  $P_4$  is given when m = even and n = even. The sum is taken over all ordered pairs (m, n) for  $0 \le m, n \le M - 1$  where  $a_{0,0}$  is excepted.



Fig. 2. Examples of spatially random distributions of black cells. The patterns consist of  $64 \times 64$  cells, with having 2048 (a), 1024 (b), and 512 (c) black cells. Black and white cells have the values of 1 and 0, respectively, for the estimation of the symmetropy  $S_{random}$ . The symmetropies of these patterns  $S_{random}$  are 2.00 bits.

When the entropy function in information theory is applied to  $P_k$  (k = 1, 2, 3, 4), S is given by

$$S = -\sum_{k=1}^{4} P_k \log_2 P_k.$$
 (3)

S ranging from zero to two bits is explained as follows: A pattern can be considered as a zero-memory information consisting of the four types of symmetry, each occurring with a probability whose value equals the corresponding  $P_k$  (k = 1, 2, 3, 4). Here, an information source is called a zero-memory source if successive symbols emitted from the source are statistically independent. S means the entropy of such an information source, and can be considered as a quantitative and objective measure of symmetry. A pattern has higher symmetric (less anisotropic) in the symmetry sense and more disorder in the entropy sense with increasing S.

In an application example, the symmetropies of spatially random distributions  $S_{random}$  are estimated. Examples of these patterns of grids consisting of 64 × 64 cells with the different ratio of the number of black cells to that of white cells are shown in Fig. 2. The estimated symmetropies  $S_{random}$  are 2.00 bits irrespective of the ratio.

According to JAEGER (1920), CSP states that the effect may occasionally have the same or a higher symmetry than the causes. CSP is represented in symmetropy: the symmetropy evaluating the symmetries of the cause  $S_{\text{cause}}$  is equal to or smaller than that of the effect  $S_{\text{effect}}$ , of the form,

$$S_{\text{cause}} \le S_{\text{effect}}.$$
 (4)

When observed symmetropies regarded as  $S_{\text{cause}}$  and  $S_{\text{effect}}$  in a given system are consistent with Eq. (4), CSP is recognized to hold in its system. It is examined whether Eq. (4) holds in a sand-pile cellular-automaton system or not.

#### 3. Sand-Pile Cellular-Automaton Model

The concept of self-organized criticality (SOC) was proposed by Bak and coworkers (e.g., BAK *et al.*, 1987; BAK, 1996) as an explaination for the behavior of the sand-pile cellular-automaton model they developed. In this model, there is a square grid of cells and at each time step a particle is dropped into a randomly selected cell. When a cell accumulates four particles they are redistributed to the four adjacent cells or in the case of edge cells, lost from the grid. Redistributions can lead to further instabilities with "avalanche" of particles lost from the edge of the grid. Because of this "avalanche" behavior, this is called a "sand-pile" model. It has been argued that earthquakes, landslides, forest fires, and species extinctions are examples of SOC in nature. In addition, wars and stock market crashes have been associated with this behavior.

Consider a square grid of cells in two dimensions. Each cell is identified by a row and column. Beginning configuration of particles on the grid is established before the simulation is run. At each time step, one particle is added randomly to a cell. When a cell accumulates four particles at a time step, this cell is now unstable and the four particles are redistributed to the four adjacent cells. If one or more of these four adjacent cells accumulate four particles, they are further redistributed to the four adjacent cells. In the possible case that redistribution occurs at an edge cell the redistribution causes particle(s) to be lost from the grid, and the number of particles on the grid is reduced. The redistributions continue until the number of particles at each of all cells is less than four. After the sequence of the redistributions terminates, one particle is added randomly to a cell at the next time step. The



Fig. 3. Temporal evolution of the average number of particles per cell. The average is taken over all  $64 \times 64$  cells. At about  $9.0 \times 10^3$  time step, the average number reaches a constant of about 2.1. Before arriving at the constant, the system is in the sub-critical states. The system is in the self-organized critical states after arriving at the constant.

spatial pattern constituted by the cells participating in the sequence of redistributions occurring at a time step is regarded as the spatial pattern of an "avalanche". If a spatial pattern of avalanches occurring in a certain time span is represented by two gray levels, for example "black" and "white", "black" is given to the cells participating in the sequences of redistributions and "white" is given to others.



Fig. 4. Examples of the spatial patterns of avalanches and of the cells from which avalanches start. Black cells join in avalanches occurring from 2501 to 4500 time steps (a), from 2501 to 5000 time steps (b), from 13001 to 10030 time steps (c) and from 14001 to 14020 time steps (d). (a) and (b) are in the sub-critical states. (c) and (d) are in the critical steady states. Each of the black cells is the cell at which the first redistribution in an avalanche occurs, from 2501 to 4500 time steps during the sub-critical states (e) and from 11001 to 12000 time steps during the critical steady states (f). The estimated symmetropies of these patterns (a, b, c, d, e and f) are 2.00, 2.00, 1.82, 1.78, 2.00 and 2.00 (bits), respectively. Black cells get 1 and others get 0 for symmetry analyses.

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This model of a square grid of  $64 \times 64$  square cells in two dimensions is performed. In this study, the system starts from an initial condition of no particles for all cells. The average number of particles per cell increases with increasing time, and it reaches the constant of about 2.1 at about  $9.0 \times 10^3$  time step (Fig. 3). This is known to correspond to the fact that the system naturally self-organizes through sub-critical states into the critical steady ones at which the noncumulative frequency-area distribution is found to satisfy a power-law (fractal) distributions with a slope near unity. Moreover, the system selforganized into criticality is believed to be equivalent to that at the edge of chaos (WALDROP, 1992). In the present model, the causes are particles dropped at randomly selected cells, and the effects are avalanches.

### 4. Results

The definitions of  $S_{\text{cause}}$  and  $S_{\text{effect}}$  in the present model are given, and the comparison between them is carried out in order to examine Eq. (4).

The symmetropy  $S_{\text{cause}}$  is defined by that of the spatial pattern of particles dropped at randomly selected cells. According to the fact that the symmetropies  $S_{\text{random}}$  defined by those of spatially random distributions are 2.00 bits, the symmetropy  $S_{\text{cause}}$  is 2.00 bits. Moreover, the symmetropy  $S_{\text{effect}}$  is defined by that of the spatial pattern of avalanches. Examples of the spatial patterns are shown in Figs. 4a–d. The estimated symmetropies  $S_{\text{effect}}$  are plotted in Fig. 5.



Fig. 5. The symmetropies  $S_{\text{cause}}$ ,  $S_{\text{effect}}$  and  $S_{\text{f}}$  as a function of time step. Circles represent the estimated symmetropies  $S_{\text{effect}}$ . The horizontal line of 2 bits shows the symmetropies  $S_{\text{cause}}$  and  $S_{\text{f}}$ . The vertically dashed-line indicates  $9.0 \times 10^3$  time step. This figure indicates the system reaches, via sub-critical states, to the self-organized critical states at about  $9.0 \times 10^3$  time step. No avalanche occurs in the period from 0 to  $2 \times 10^3$  time steps.

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The symmetropies  $S_{\text{effect}}$  during the sub-critical states are the constant close to two bits irrespective of time spans (Fig. 5). That is, the symmetropies  $S_{\text{cause}}$  and  $S_{\text{effect}}$  are almost consistent with Eq. (4), indicating that CSP almost holds during the sub-critical states. The symmetropies  $S_{\text{effect}}$  during the critical steady states are smaller than or, at most, equal to 2.00 bits (Fig. 5). That is, the symmetropies  $S_{\text{cause}}$  and  $S_{\text{effect}}$  are not necessarily consistent with Eq. (4). This shows that CSP does not always hold during the critical steady states, and symmetry breaking is possible to be observed. Therefore, CSP does not necessarily hold in the present model.

Moreover, the symmetropies  $S_f$  of the spatial patterns of the active cells, at which the first redistribution in an avalanche occurs, are estimated. Examples of the spatial patterns are given in Figs. 4e and 4f. The estimated symmetropies  $S_f$  are exactly 2.00 bits irrespective of time spans and system's states (Fig. 5). Therefore, the symmetry breaking in the present model is due to avalanche dynamics of critical states, not to the arrangement of the first redistribution cells.

#### 5. Discussions

CURIE (1894) submitted CSP based on the discussion as follows: "Certain elements can coexist with certain phenomena, but they are not necessary to them. That which is necessary is that certain ones among these elements shall not exist. It is dissymmetry that creates the phenomenon. When several phenomena are superposed in the same system, the dissymmetries are added together" (see also CURIE, 1923; COTTON, 1963). If the "phenomenon" is regarded as an avalanche of a critical state in the present model, the discussion of CSP seems to satisfy the present model where CSP does not necessarily hold. Here is a contradictory situation.

The situation comes from that CURIE (1894) failed to address system's stability (STEWART, 1990; STEWART and GOLUBITSKY, 1992). If a symmetric state becomes unstable, the system will do something else resultingly. That resultant something else need not be equally symmetric, and therefore symmetry breaking can occur. However, CSP does not fail to the powerful constraint for symmetry argument. CSP can be applied to linear problem under the linear constitutive equation.

STEWART and GOLUBITSKY (1992) discussed how the resultant something else is chosen, and required only "imperfection" for the choice. Nature is never *perfectly* symmetric. Nature's circles always have tiny dents and bumps. There are always tiny fluctuations, such as the thermal vibration of molecules. These tiny imperfections load Nature's dice in favor of one or other of the set of possible effects that the mathematics of perfect symmetry considers to be equally possible. In this study, the estimated symmetropies of  $S_{effect}$  and  $S_f$  derived that symmetry breaking depends not on the arrangement of the first redistribution cells but on avalanche dynamics of critical states. This shows that the rule of the intracellular interaction capable of leading to avalanche dynamics of critical states in the present model is required for the choice. Therefore, the rule is one of the necessities for the occurrence of symmetry breaking.

BOULIGAND (1985) has analyzed the evolution of symmetry in some morphogenetic games. In Lück's model, dissymmetry (anisotropy) is built into the rules of the model at the intracellular level, and symmetry breaking is observed at a higher organization level than

the single cell. We analyzed the symmetropies in the sand-pile cellular-automaton model. In this model, the rules of the model at the intracellular level (four particles are redistributed to the adjacent four cells) are isotropic, and symmetry breaking is possible to be observed at a macroscopic level during the critical steady states. Therefore, symmetry breaking does not always need anisotropic rules of the intracellular interactions. Using a cellular-automaton model of evolution, EIGEN and WINKLER (1975) illustrated that symmetric structures are more efficient than asymmetric ones in exploiting a functional advantage that promotes to reproduce themselves. If the evolution corresponds to the self-organization into the criticality and our results are taken into consideration, it is proposed that anisotropic interactions between elements are not necessarily required in their reproduction.

In bifurcation process and mathematical group theory, symmetry breaking regarded as pattern formation is a change in the symmetry group, from a larger one to a smaller one or from the whole to the part. Its change needs to control a parameter. Thus, for a given range of a parameter, we suppose that a physical system written by functional structural equations known as "bifurcation equations" possesses a stable solution invariant under a symmetry group. But as the parameter crosses a critical value with no change of the same equation, new solutions appear which are invariant only under a subgroup (SATTINGER, 1978). In bifurcation process of equilibrium statistical mechanics, symmetry breaking can be reached only by tuning a parameter to the critical values such phase transition points (NICOLIS and PRIGOGINE, 1977). In the present model, the system self-organizes unavoidably into the criticality without the need of a fine-tuning parameter. Therefore, our results are different from symmetry breaking occurred with tuning parameters in bifurcation process.

NICOLIS and PRIGOGINE (1977) or SATTINGER (1978) have discussed a bifurcation process and symmetry breaking with a presupposed functional structural equation. However, as GUNJI (1990) pointed out, the functional structural equation for the autonomous process in biosystem (e.g., the pattern formation in ontogeny of biology) should not be a priori supposed. In the present study, the symmetry breaking is a priori not based on the use of a functional structural equation and the cellular-automaton system self-organized into criticality is similar to GUNJI (1990)'s asynchronous automata system for the autonomous process in biosystem. Therefore, our discussion concerning with symmetry breaking can be applied to the autonomous process in biosystem.

There was a claim that it is natural to analyze symmetry properties by a continuous scale rather than by a discrete feature "symmetric or non-symmetric" (NAGY, 1996). However, no general theory that explains the occurrence of continuous symmetries has been made. In the present research, the symmetropies  $S_{effect}$  take various values during the critical steady states. That is, various symmetries are observed from the viewpoint of the symmetropy. This indicates that symmetry is continuous rather than discrete, and meets the previous claim (NAGY, 1996). The concept of SOC is strongly proposed to be the underlying one for continuous symmetries.

In the 20th century, there is a general tendency to split our culture: art and the humanities versus science and technology. In the case of solving complicated problems, these halves should cooperate from the viewpoints of the western symmetry (e.g., NAGY, 1996) and the Japanese Katachi (form, shape, figure, appearance) (e.g., OGAWA, 1994). Pasteur said as follows: "The universe is a dissymmetric whole. I am led to believe that life, as it is revealed to us, must be a function of the dissymmetry of the universe, or of the

*consequences that involves*" (CURIE, 1923; COTTON, 1963). Referring Pasteur's studies, NAGY (1996) emphasized that the concept of symmetry should be extended from "digital" to "analog", and measures of (dis)symmetry are much more useful for the halves to cooperate. Symmetropy has the possibility for helping to build an interdisciplinary "bridge" between various fields of art and science in the corresponding cultural circles. A concept that can scientifically treat the order lying in a typical Japanese garden (for example a stone garden in Ryoanji temple in Kyoto, Japan) is now required even in physics (OGAWA, 1994). It will be interesting to note that the concept of symmetropy may be this required concept.

#### 6. Conclusions

For the investigation of the validity of CSP in a complex system, the continuous measure of symmery, termed "symmetropy", which can be evaluated for any shape or pattern in a complex system is introduced. A sand-pile cellular-automaton model is used, where the system consisting of  $64 \times 64$  cells self-organized through sub-critical states into critical steady states. During the sub-critical states, the symmetropies of the spatial distributions of the causal particles dropped at randomly selected cells  $S_{cause}$  and those of the resultant avalanches  $S_{effect}$  satisfy CSP, but not always at the critical steady states. Considering the present model, where the rules at the intracellular level are isotropic without the existences of functional equations and tuning parameters, symmetry breaking can occur without an anisotropic interaction of elements in a cellular-automaton model and without a special tuning parameter in bifurcation process.

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