Model Analysis for Evolution of Population and Railroad

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Abstract. A model analysis is made for evolution of population distribution and railroad network based upon a mathematical model where the population density distribution is determined so that a quantity standing for the uncomfortableness of residents is minimized. It is assumed that a new interval of the railroad with a constant length is added when the total population increases and the average uncomfortableness exceeds a threshold value. The position of the new interval is chosen so as to minimize the total uncomfortableness. As a result the railroad network develops straight at first and makes branching later and loops are formed in the final stage. The mirror symmetry in the network tends to appear during the evolution.

1. Introduction

Recently, a model analysis is made for the mechanism of formation of population distribution where travel times between two points are given in two-dimensional space (HIRAYAMA *et al.*, 2001). In this analysis two factors are taken into consideration; an uncomfortableness due to access time to other points and that due to high density of population. The total uncomfortableness in the whole space is evaluated by an integration of them and the population distribution is determined so that it is minimized for given total population and railroad. It is concerned to a static case in a sense that the total population and the railroad remain unchanged.

In the present paper this model is extended to evolution of population distribution and railroad network with time. This analysis is expected to elucidate a process of population growth and railroad network formation. Furthermore, this model is considered to be related to the developments of branching systems, such as veins of leaves, blood vessels, nervous systems, because the organisms should have optimum states while developing.

In this model the total population is assumed to increase with a certain rate and a new interval of railroad with a fixed length is added to the network in a way to lower uncomfortableness. This model is based on the idea that the population distribution is determined optimally by the instantaneous railroad network at each time step because the population increases slowly enough. In this sense the evolution occurs in a quasi-static way. Detail of the model is explained in the following sections.

2. Basic Model of Formation of Population Distribution in Static Case

First, we mention briefly how the population distribution is determined when the total population is given (HIRAYAMA *et al.*, 2001).

We set up the following assumptions.

(i) A boundary and a total population N is given in a two-dimensional space, i.e.

$$N = \int \rho(x, y) dS,$$
 (1)

where $\rho(x,y)$ is the population density at the point (x,y) and dS = dxdy.

(ii) Travel time T(P,P') between two arbitrary points P(x,y) and P'(x',y') is given (see Eqs. (6) and (7) below).

(iii) The total uncomfortableness E of the space is expressed by the following integral:

$$E = \iint \psi(x, y) \rho(x, y) dx dy, \tag{2}$$

where $\psi(x,y)$ is a linear combination of an average access time U(x,y) to other regions in the space and the population density, as follows:

$$\psi(x, y) = U(x, y) + b\rho(x, y), \tag{3}$$

where *b* is a constant and $U(x,y) \equiv U(P)$ is given as

$$U(x, y) = \frac{a}{N} \int T(P, P') \rho(P') dS'.$$
(4)

The population distribution ρ is determined so that *E* is minimized under the condition (1). This problem is solved by minimizing $I = E - \lambda N$, where λ is a Lagrange's coefficient. After some manupulations we obtain the integral equation for $\rho(P)$:

$$\rho(P) = \langle \rho \rangle + \frac{a}{2bN} \left[\frac{1}{S} \iint T(P, P') \rho(P') dS' dS - \int T(P, P') \rho(P') dS' \right].$$
(5)

The travel time T(P,P') is given in the absence of the railroad by

$$T(P, P') \equiv T(x, y; x', y') = (|x - x'| + |y - y'|) / v_0,$$
(6)

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Population distribution at t=0 (without railroad)

Fig. 1. Population distribution without railroad at t = 0, where $\varepsilon = E/N = 16.877$, $\rho_{max} = 151.5$, $\rho_{min} = 15.9$.

where v_0 is the walking velocity. In the presence of it the travel time is expressed as

$$T(P, P') = \min(T(P, A) + L(A, B) / v + T(B, P')),$$
(7)

where A and B are the points of stations, L(A,B) is a length along the railroad, and v is the train velocity. The symbol "min" means the minimum value for all possible combinations of stations (A,B) including the case without use of it. In order to solve Eq. (5) we express the two-dimensional space by two-dimensional square grids with uniform size whose area is denoted by ΔS . Then, Eq. (10) is replaced by a set of simultaneous algebraic equations for population densities at the grid points $\rho_1, \rho_2, ..., \rho_{Ng}$, where N_g is the total number of grid points.

An example of the result in case without railroad is shown in Fig. 1, where the space is devided into 21×21 square grids and the value of *N*, *a*, *b* is set up as N = 44100, a = 1, b = 0.04. This figure shows that the population distribution is nearly concentric circles in case without railroad. The ratio of maximum value of population density and minimum one is 8.48 and an average uncomfortableness $\varepsilon \equiv E/N = 16.877$.

3. Model Analysis for Evolutions of Population and Railroad Network

3.1. Procedure of evolution

Evolutions of both population and railroad network is studied through the following procedure.

(i) A square space is devided into 21×21 square grids, and the total population is assumed to increase exponentially from 44100 with doubling time = 1. The time step is fixed to 0.01.

(ii) When the average uncomfortableness $\varepsilon \equiv E/N$ exceeds an certain threshold $\varepsilon_0 = 16.5$, where *E* and *N* are given by Eqs. (2) and (1), a railroad interval with length L = 4 is added, then ε reduces its value. Among plans of addition of an interval we choose the one which leads to the least value of ε . Even if the least value of ε after the addition is larger than ε_0 , we accept this addition and proceed to the next time step.

(iii) For the new railroad, the population distribution is computed.

(iv) Repeat the steps (ii)–(iii) while increasing N until new addition of railroad interval becomes impossible.

3.2. Results

Figure 2 shows the population distribution and the first railroad at t = 0. In this figure and following ones, the circles show the stations and the numbers the order of addition. The position of the first railroad in Fig. 2 is chosen so that it leads to the least value of ε .

This railroad is unchanged until t = 0.11, where ε increases but $\varepsilon < \varepsilon_0$. At $t = 0.12 \varepsilon$ exceeds ε_0 and a new interval is added. Similarly the third one is added at t = 0.32 as shown in Fig. 3. Evolution of the value of ε is shown in Fig. 4. We can find that the railroad develops straight in the first stage (during $t = 0 \sim 0.32$), and then a branching begins at t = 0.63 as shown in Fig. 5.

Figures 6–9 show successive evolution of the population distribution and railroad network. We can see in these figures that the mirror-symmetry (not point symmetry) tends to be realized.



Fig. 2. Population distribution and railroad plan at t = 0, where $\varepsilon = 16.051$, $\rho_{\text{max}} = 140.7$, $\rho_{\text{min}} = 16.6$.



Fig. 3. Population distribution and railroad network at t = 0.32, where $\varepsilon = 15.704$, $\rho_{max} = 171.8$, $\rho_{min} = 49.9$.



Fig. 4. Time evolution of the value of ε .



Fig. 5. Population distribution and railroad network at t = 0.63, where $\varepsilon = 15.984$, $\rho_{max} = 206.4$, $\rho_{min} = 72.7$.



Fig. 6. Population distribution and railroad network at t = 1.15, where $\varepsilon = 16.558$, $\rho_{max} = 261.6$, $\rho_{min} = 179.9$.



Fig. 7. Population distribution and railroad network at t = 1.24, where $\varepsilon = 16.311$, $\rho_{max} = 273.6$, $\rho_{min} = 195.2$.



Fig. 8. Population distribution and railroad network at t = 1.33, where $\varepsilon = 16.427$, $\rho_{max} = 285.9$, $\rho_{min} = 208.2$.



Fig. 9. Population distribution and railroad network at t = 1.36, where $\varepsilon = 16.395$, $\rho_{\text{max}} = 286.1$, $\rho_{\text{min}} = 211.9$.



Fig. 10. Population distribution and railroad network at t = 1.46, where $\varepsilon = 16.466$, $\rho_{\text{max}} = 304.3$, $\rho_{\text{min}} = 249.2$.



Population distribution at t=1.51 (31intervals)

Fig. 11. Population distribution and railroad network at t = 1.51, where $\varepsilon = 16.559$, $\rho_{max} = 311.7$, $\rho_{min} = 259.4$.

Figure 10 shows the population distributions and railroad networks at t = 1.46, where a loop is formed. The ratio of maximum value of population density and minimum one is 303.6/249.2 = 1.218 and $\varepsilon = 16.466$.

We have persued the present procedure till t = 1.51. However after t = 1.48 the effect of addition of new intervals is not so large as to assure $\varepsilon < \varepsilon_0$, as is seen in Fig. 4. Figure 11 shows the situation at t = 1.51, where $\varepsilon = 16.559$. After this interval no room is left for adding a new station.

4. Conclusions and Discussions

From the above analyses following conclusions are derived.

(i) The railroad network extends straight at first and makes branches later. Near the final stage loops are formed.

 (ii) There is a tendency that the mirror symmetry with respect to central axis is kept. Some comments on the above conclusions must be added here. First, these conclusions seem to match to real phenomena, but no data is available now to prove it.

Second, there arises a question whether the above results depend on the conditions of numerical simulations or not, for example, if we adopt the railroad interval with length 5 (not 4), or if we choose other values than a = 1, b = 0.04. These values are determined by trial and error so that the values of population density of all the points are positive (note that the numerical solution of Eq. (5) may give negative values of ρ).

Third, the cost for the construction and maintenance of the railroad is not taken into consideration. It would be an interesting problem to consider it.

Fourth, in the present analysis the railroad is extended according to the total uncomfortableness. However in actual social development, railroad planning is considered to be made owing to local increase of the population. The present analysis could be extended to that more realistic situation according to these comments.

REFERENCES

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