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Verification of Air Temperature Variation with Form of Potential

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Abstract. An upward tendency of air temperature has been pointed out by many researches and it can be also suggested with statistical tests. However, these tests are not enough to elucidate a system of the air temperature variation. We obtained conditions on the variation for their time averaged potentials were constructed with histograms on time series data. We propose a form on the system that gives a physical interpretation with the potentials. We analyzed time series data of air temperature variation at Nagoya. We can observe that a new peak of the histogram appears on higher temperature as time passes, and we can assume the same time variation of form on the time-averaged potentials. This variation can be explained by a hypothesis that the temperature variation system has a physical mechanism in accordance with a delay convention on one-dimensional cut off of wedge catastrophe.

1. Introduction

In recent years, an upward tendency of air temperature has been pointed out and many researches have reported concerning with global warming (IPCC, 1995). However, little is known about these problems, for example, whether the cause is artificial or natural.

Time series of year-averaged air temperature at Nagoya (at 35°10′ N, 136°58′ E; Fig. 1) seem to be upward tendency (Fig. 2). We can use some statistical tests to verify a tendency of the time series. One is a test on significance of a regression coefficient (inclination of a regression line). The coefficient calculated by the least square method is larger than zero and a null hypothesis that the coefficient is zero is rejected with significant level as 0.05 (Appendix A). Then, we can suggest that the coefficient is not zero and the temperature changes with statistical significance.

We also used Welch's test that verifies difference of mean values of two populations. The populations are values on year-averaged temperature from 1900 to 1949 and from 1950 to 1999. A null hypothesis that the mean values of the populations are equal is rejected with a significant level as 0.05 (SHIMIZU and TAKADA, 2000; Appendix B). In other words, we can suggest that the mean values of the year-averaged temperature have changed by

Fig. 1. Location of Nagoya (shown as a cross mark).

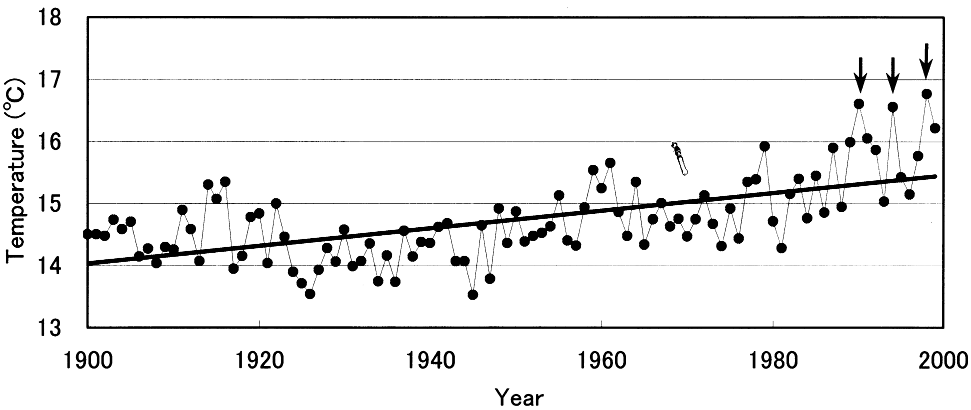


Fig. 2. Time series of year-averaged temperature at Nagoya from 1900 to 1999 and its regression line calculated by least-squared method.

Welch's test. This result is consistent with the test on significance of the regression coefficient.

We were able to show upward tendency of an air temperature variation with these statistical tests. However, these statistical tests cannot give the physical meaning on a

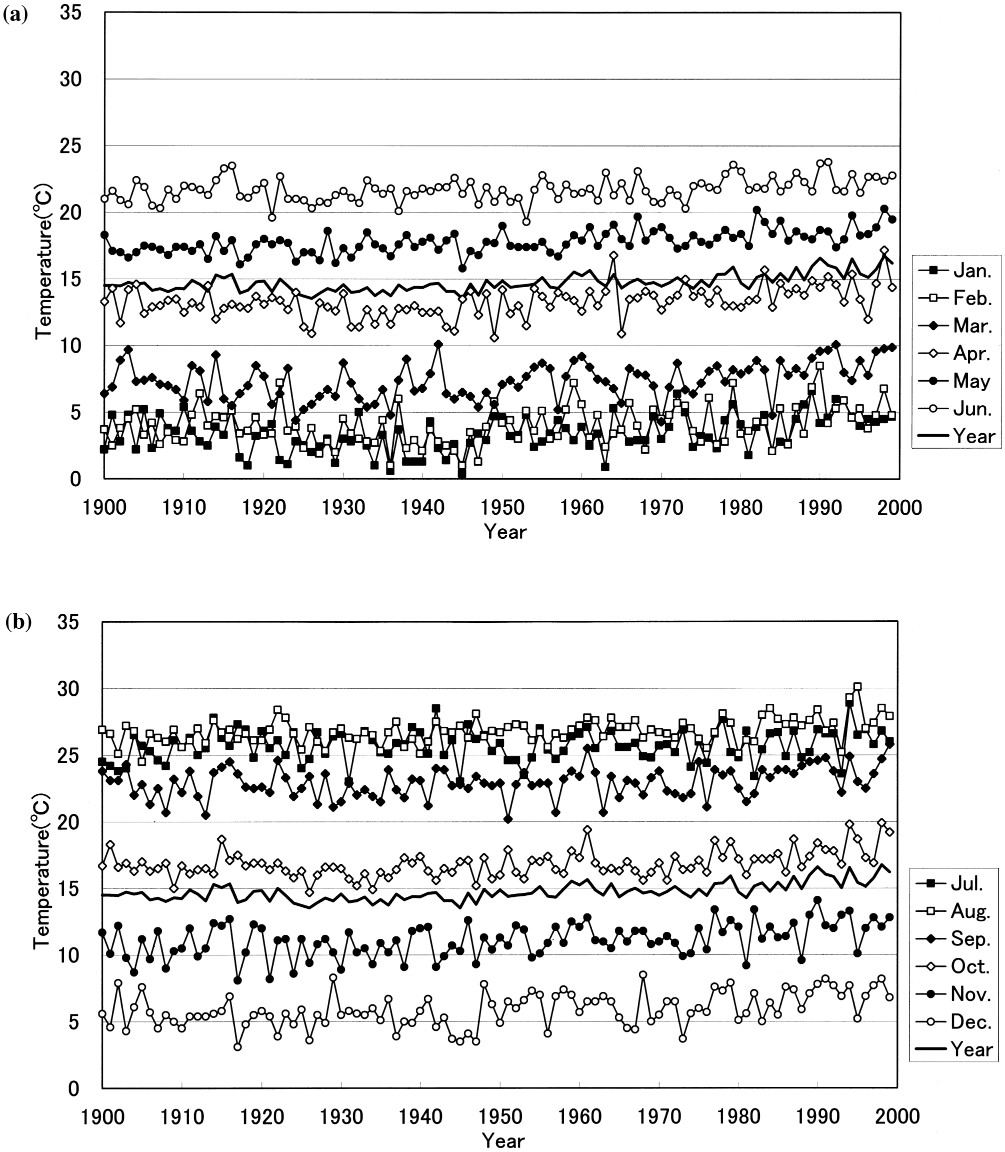
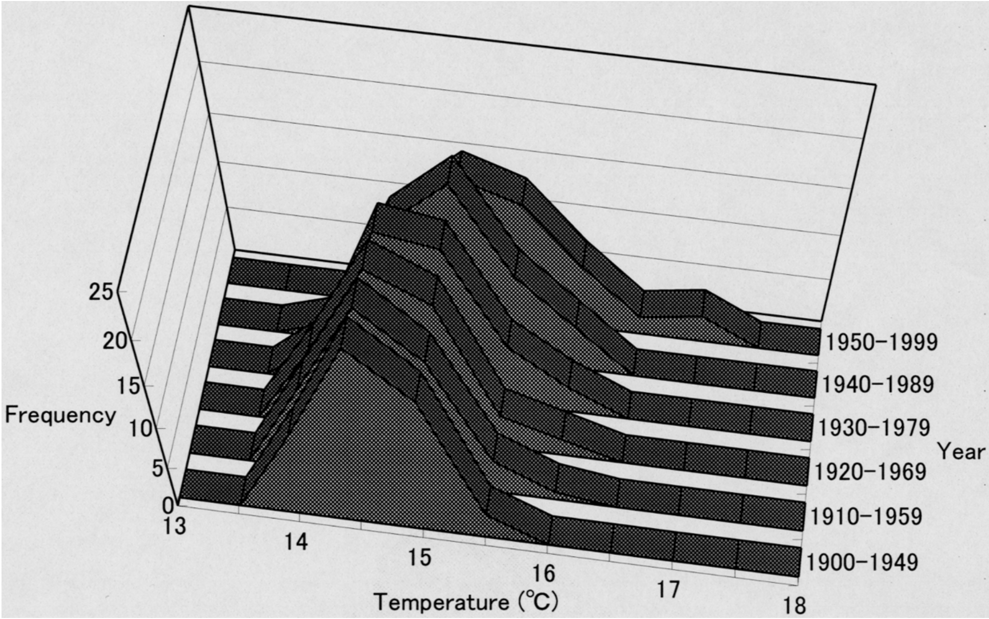


Fig. 3. Time series of month-averaged temperature at Nagoya from 1900 to 1999 (a: January to June, b: July to December).

system of air temperature that controls the phenomena. Purposes of this study is to propose verification of the temperature variation with a theory that describes correspondence between "form of a histogram" and "form of the potential", and to advance a new interpretation for physical explanation of the air temperature variation.



(a)

Fig. 4. Time variation of histogram of the year-averaged temperature (a: An extent of histogram is 50 years, b: An extent of histogram is 20 years).

2. Form of Histogram of Air Temperature

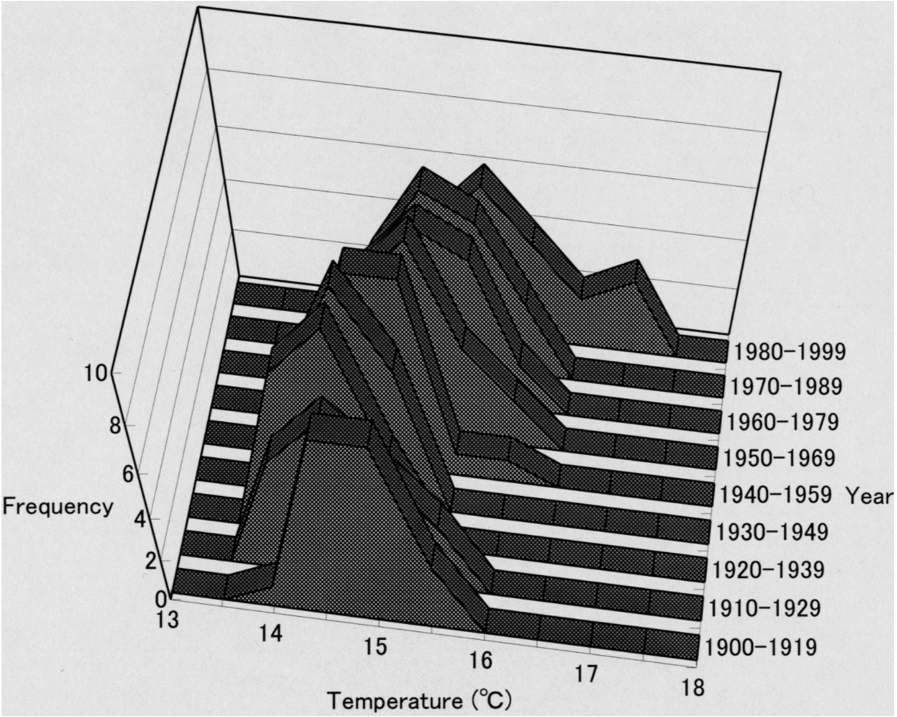
We construct a theory of air temperature variation with a process that generates time series of air temperature. Our theory is based on the stochastic process theory. The time series are composed with a view of the stochastic analysis.

2.1. Transformation of the time series data

We use time series data of air temperature from 1900 to 1999 at Nagoya meteorological observatory (possessed by Japan meteorological agency). The form of histogram is important for stochastic analysis. It is difficult to extract characters of the time series with form of the histogram as described in the followings because some daily and yearly periodicities are admitted. Therefore, the time series are transformed to the time series of year-averaged temperature (Fig. 2) and the time series of month-averaged temperature on every year (Figs. 3a and 3b);

$$\{x\text{-th month in 1900, } x\text{-th month in 1901, ..., } x\text{-th month in 1999}\}\ (x = 1, 2, ..., 12).$$

The extent of averages for the month-averaged temperature is one month (from 28 days to



(b)

Fig. 4. (continued).

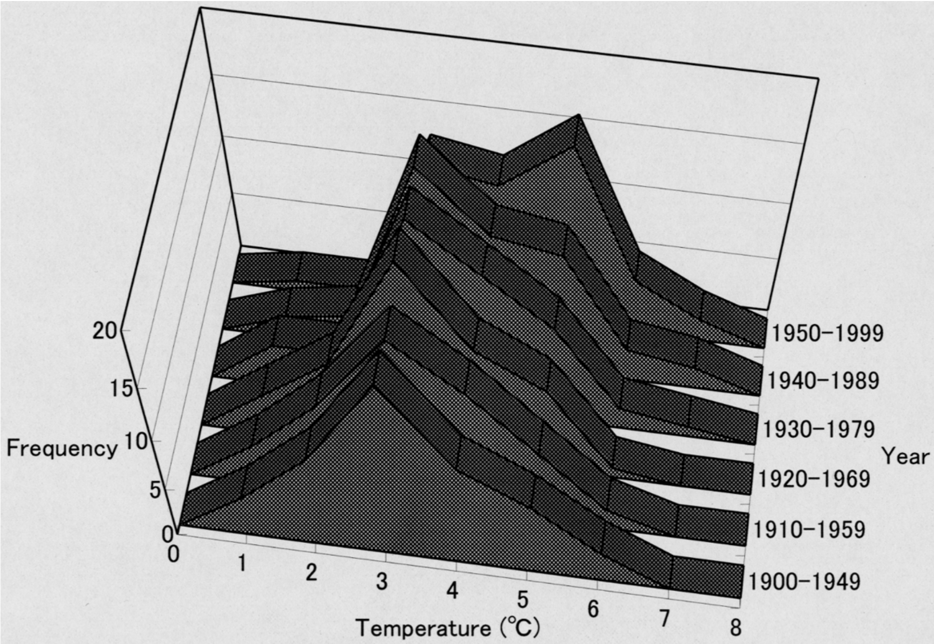
31 days). The extent is determined for simplify of calculations, and we should notice that physical significance of the extent have been not given.

2.2. Histogram of air temperature and the time variation

Histogram is a graph that shows classes of values of the time series on a horizontal axis and each frequency on a vertical axis. A time variation of the year-averaged temperature (Fig. 2) is shown as Figs. 4a and 4b with 3-dimensional histogram where the *x*-, *y*- and *z*-axis mean the class of temperature, the extent of the histogram and frequency. The class interval of the histogram is 0.5 degrees and the extent of histogram is 50 years on Fig. 4a and 20 years on Fig. 4b. We can clarify the time variation of the histogram with use of the *y*-axis of 3-dimensional graph.

We can admit a common type of a character on the time variation of a form of the histogram in Figs. 4a and 4b. There is one peak of the histograms and it is especially similar to a normal distribution except for 1950–1999 on Fig. 4a and 1980–1999 on Fig. 4b.

The peak gradually moves to higher temperature for many histograms. Also, we should notice that a new peak of histograms appears at higher temperature in the extent of



(a)

Fig. 5. Time variation of histogram of the month-averaged temperature (a: on January, b: on July).

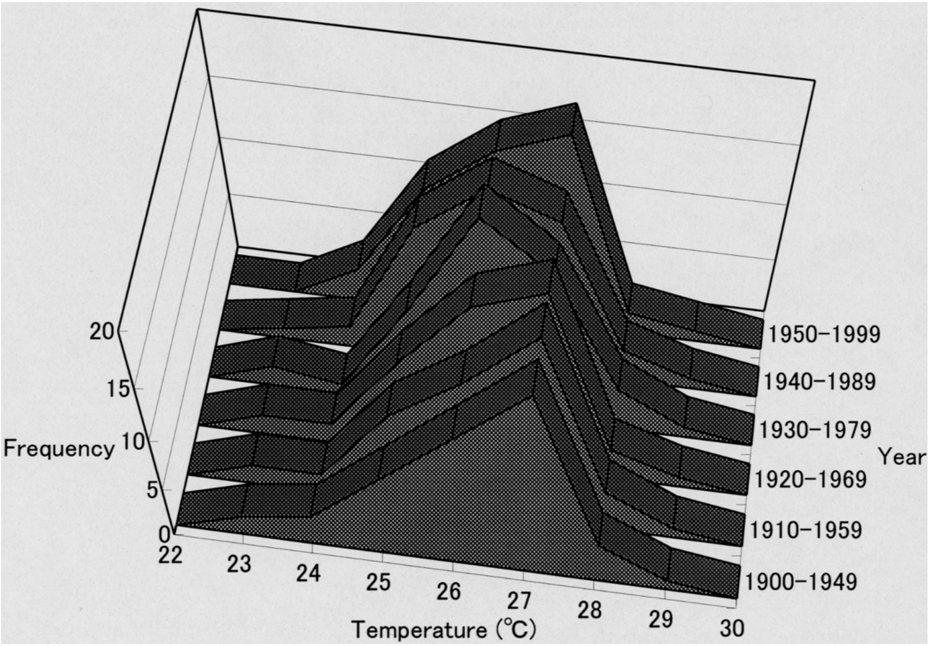
1950–1999 on Fig. 4a and 1980–1999 on Fig. 4b. The same type of the time variation of the histograms is admitted on each month-averaged temperature at intervals of 1 degree. Figure 5a is the histogram of January and it has a typical type of the time variation. However, this variation is not admitted only in the month-averaged temperature on July (Fig. 5b).

3. Theory on Correspondence between Form of Probability Density Function and Potential

If we can obtain a form of a time-averaged potential on the mathematical model that describes the process of air temperature variations, the following points are expected.

- We can assume a system that describes air temperature variation as a dynamical equilibrium and a structural steady state.
- We will compose a mathematical model as a statistical dynamical equation that is obtained as continuous limit formula on discrete time series data (TAKADA *et al.*, 1999).

We will contribute a theory on the potential for a system of the air temperature variation in this chapter. The stochastic process of air temperature variations is assumed to be a Markov process. The verification of this assumption will be described in Chapter 4.



(b)

Fig. 5. (continued).

3.1. Basic equations on Markov processes

We assume a random process $X(t)$ as a continuous process on state space Ω that satisfies conditions of $X(t_0) = x_0(\in \Omega)$ and $X(t) = x(\in \Omega)$.

Assumption 1 $X(t)$ is one of Markov processes that are determined by $t(>t_0)$ and $X(t_0)$.

At the condition of $X(t_0) = x_0$, the conditional probability $P(x|x_0, t)$ in the case of $X(t) = x$ can be expressed by the following Chapman-Kolmogorov equation (GOEL and RICHTER-DYN, 1978).

$$P(x|x_0, t_1 + t_2) = \int_\Omega P(x|z, t_2) P(z|x_0, t_1) dz. \tag{1}$$

We can obtain Eq. (2) by the Taylor expansion of Eq. (1).

$$\frac{\partial P(x|x_0, t)}{\partial t} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial x^n} \left\{ M_n(x) P(x|x_0, t) \right\} \tag{2}$$

where $M_n(x)$ is a moment of transition probability for degree n (natural numbers):

$$M_n(x) = \lim_{\tau \to 0} \frac{1}{\tau} \int_{\Omega} (z - x)^n P(z|x,\tau) dz. \tag{3}$$

Equation (2) is a partial differential equation that describes variations of the process statistically. Equation (2) is solved under the following initial condition.

$$\lim_{t \to 0} P(x|x_0,t) = \delta(x - x_0) \quad (\delta \text{ is Dirac's delta function}).$$

$M_n(x)$ satisfies either condition 1 or 2 (PAWULA, 1967).

Condition 1: $\begin{cases} M_n(x) \neq 0 & (n \leq 2) \\ M_n(x) = 0 & (n \geq 3). \end{cases}$

Condition 2: $M_n(x) \neq 0$ (n is even number).

In the case of condition 2, the process must be an anomalous process that diffuses rapidly far in short time. It is conflict with regular processes. Then we can give the following assumption for $M_n(x)$ as a regular diffusion process.

Assumption 2 $\begin{cases} M_n(x) \neq 0 & (n \leq 2) \\ M_n(x) = 0 & (n \geq 3). \end{cases}$

3.2. Fokker-Plank equation

The solution of Eq. (2) can be obtained with the permutation of variables:

$$dz = \frac{dx}{\beta(x)}, \quad a(z) = \frac{\alpha(x(z))}{\beta(x(z))} \ \text{ s.t. } \ \alpha(x) = M_1(x) - \frac{1}{4}\frac{\partial M_2(x)}{\partial x}, \quad \beta(x)^2 = M_2(x).$$

Equation (2) goes over into Eq. (4) with Stratonovich's rule (STRATONOVICH, 1963).

$$\frac{\partial g(z|z_0,t)}{\partial t} = -\frac{\partial}{\partial z}\left\{\hat{a}(z) g(z|z_0,t)\right\} + \frac{1}{2}\frac{\partial^2}{\partial z^2} g(z|z_0,t) \equiv -\frac{\partial}{\partial z} J(z|x,t) \tag{4}$$

where $g(z|z_0, t)$ is a probability density function that is transformed from $P(x|x_0, t)$, z_0 is the initial condition corresponding to x_0, $J(z|x, t)$ is flow of the probability and its stationary value $J(z|x, \infty) \equiv J(\infty)$ is a constant. A stationary solution of Eq. (4) can be obtained strictly with a stationary probability density function $g(z)$ (HARKEN, 1975):

$$g(z) = \left[C - 2J(\infty)\int_{\Omega} \exp\left\{-2\int_{\Omega} \hat{a}(\xi)d\xi\right\}dz\right]\exp\left\{2\int_{\Omega} \hat{a}(\xi)d\xi\right\} \tag{5}$$

where C is a normalization factor defined as

$$\int_{-\infty}^{\infty} g(\xi)d\xi = 1.$$

Equation (5) is simplified as Eq. (6) on a natural boundary condition such as $J(\pm\infty) = 0$.

$$g(z) = C\exp\left\{2\int_{\Omega} \hat{a}(\xi)d\xi\right\}. \tag{6}$$

We can expect that the stationary probability density function $g(z)$ can be obtained with the normalized histogram of the time series data.

3.3. Stochastic dynamical equation

We also consider general processes that satisfy $M_n(x) = 0$ for ($n \geq 3$) and we assume a stochastic dynamical equation (SDE) as the following equation that is corresponding to a probability density function $P(x|x_0, t)$. We define an SDE where a random variable depends on the sum of a function $\hat{a}(z)$ and fluctuating force.

$$\frac{dz(t)}{dt} = \hat{a}(z) + F(t) \tag{7}$$

where $F(t)$ is the standardized fluctuating force generated with a Gauss type stochastic process. The time averages of $F(t)$ and $F(t)F(t + \tau)$ are defined as Eq. (8):

$$\begin{aligned} \langle F(t)\rangle &= 0 \\ \langle F(t)F(t+\tau)\rangle &= \delta(t) \end{aligned} \tag{8}$$

where the angle brackets $\langle\bullet\rangle$ denote on the average over time and τ is lag-time ($0 \leq \tau \leq N$, where N is length of data). Otherwise, we can consider a set of zero points of the potential function as equilibrium space. We can give a potential function by the space integral of the function $\hat{a}(z)$ on Eq. (7):

$$V(z) = -\int^{z} a(\xi)d\xi. \tag{9}$$

Here, we can rewrite Eq. (6) by the substitution of Eq. (9).

$$g(z) = C\exp\{-2V(z)\}. \tag{10}$$

Therefore, the following proposition is obtained by differentiating both sides of Eq. (10). (TAKADA, 2001)

Proposition 1 The stationary points of a potential function $V(z)$ and a stationary probability density function $g(z)$ are corresponding.

In Sec. 4, we will describe an application of Proposition 1 for time series of the air temperature variation.

4. Verification for Our Theory

If we verify that the data of air temperature variation satisfies Assumptions 1 and 2, we can apply the theory and the Proposition 1 in Sec. 3. Then, we can expect an interpretation for a form on the potential of the air temperature variation. We propose the following methods to verify them. We tried a type of verification for the data.

4.1. Verification for randomness of the time series

The air temperature variation has been researched by a deterministic theory on meteorology. In this paper, we verified randomness on the time series of month-averaged temperature with run tests (Appendix C). We could suggest that each time series of month-averaged temperature was random except for March, May and December. Therefore, randomness of each time series was verified. Hence, methods of statistical tests could be applied for time series of air temperature.

4.2. Verification for Markov processes

We put a sample correlation function of N_k-pair samples from a binominal population as r where the correlation function of the population is ρ. The N_k-pair samples are composed of $\{x_1, \ldots, x_{N_k}\}$ and $\{x_{1+k}, \ldots, x_{N_k+k}\}$ where $N_k = N - k$ and $1 \le k \le N/2 = 50$ (k is lag-time). We can use a z-transformation of the function r. The function $\tanh^{-1} r$ can be regarded as a standard distribution where its average is $\tanh^{-1}\rho$ and its standard distribution is $1/\sqrt{N_k - 2}$ when N_k is greater than 50 (YOSHIDA and YOSHIDA, 1958). This value of the standard distribution corresponds to the significance level of the correlation coefficients.

Here, we can verify significance of sample auto-correlation coefficients $r_{xx}(k)$ as follows. A null hypothesis H_0 is that one can admit the significant auto-correlation on the time series data statistically. If the following equation holds, H_0 is accepted with a significance level 0.05:

$$\left|\tanh^{-1} r_{xx}(k)\right| > \frac{2}{\sqrt{N_k - 2}} \tag{11}$$

where the reference value $2/\sqrt{N_k - 2}$ is two-times of a speculated standard distribution. The auto-correlation coefficient of discrete time series $\{x_i\}$ is defined as Eq. (12).

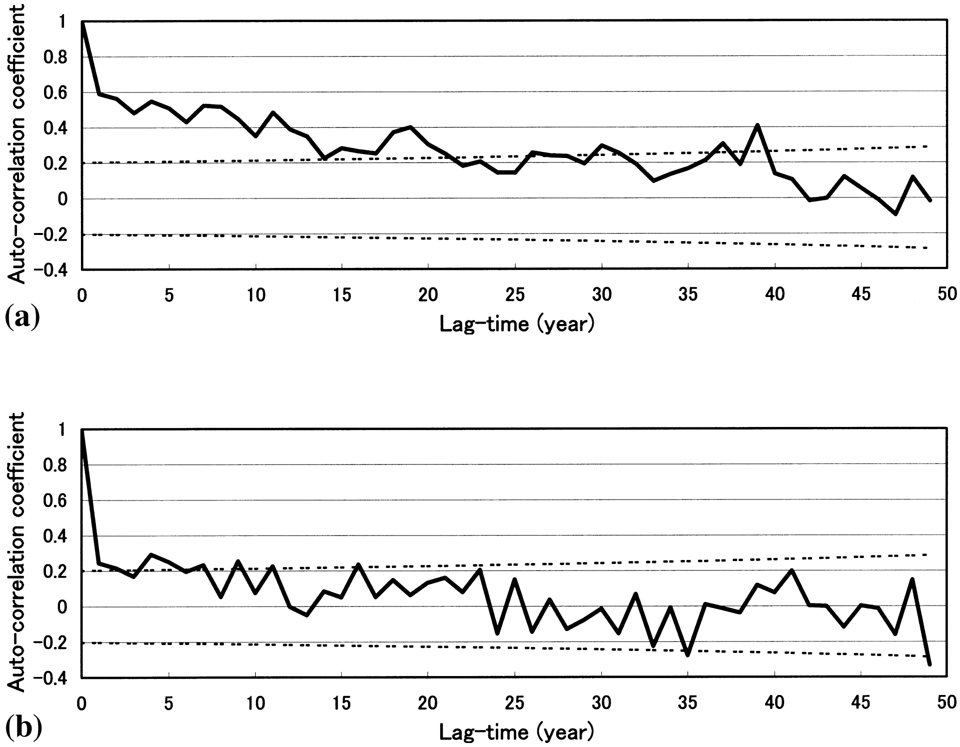


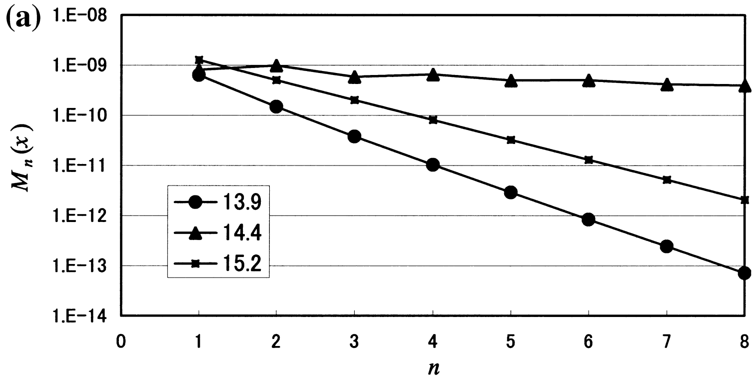
Fig. 6. Auto-correlation coefficient of the time series (a: the year-averaged temperature, b: the month-averaged temperature on January).

$$r_{xx}(k) = \frac{\sum_{i=1}^{N-k}(x_i - \bar{x}_k')(x_{i+k} - \bar{x}_k'')}{\sqrt{\sum_{i=1}^{N-k}(x_i - \bar{x}_k')^2}\sqrt{\sum_{i=1}^{N-k}(x_{i+k} - \bar{x}_k'')^2}} \tag{12}$$

where

$$\bar{x}_k' = \sum_{i=1}^{N-k} x_i / (N_s - k), \quad \bar{x}_k'' = \sum_{i=k+1}^{N-k} x_i / (N_s - k).$$

The relation between $\tanh^{-1} r_{xx}(k)$ and k is shown as Fig. 6a for the time series of year-averaged temperature and Fig. 6b for the time series of month-averaged temperature on



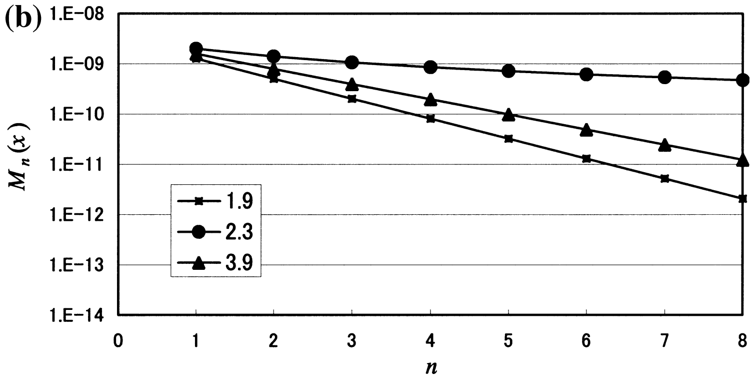


Fig. 7. Numerical integration of moment of transition probability (a: the year-averaged temperature, b: the month-averaged temperature on January).

January as a typical representation where supremum and inferior limit of reference values are shown as dashed lines. The absolute values $|\tanh^{-1} r_{xx}(k)|$ were smaller than each reference value except for small k ($k \leq 13$, $18 \leq k \leq 20$, $k = 37$, $k = 39$) on the year-averaged temperature. The absolute values of the sample auto-correlation coefficients were almost smaller than the reference values for all time series of month-averaged temperature. Then we suggest that H_0 was rejected on the month-averaged temperature, but not rejected on the year-averaged temperature for small k. We have considered that the regularity of the year-averaged temperature is caused by the additive operation. We can extract the randomness on the air temperature variations with effect of the transformation as mentioned in Sec. 2.

There are significant auto-correlation coefficients for small k because the extent of span for the average is too long. We assume that variations of the air temperature almost satisfy Assumption 1. We will discuss on the time-averaged potential of the year-averaged temperature for a purpose of the suggestion to their mathematical model.

4.3. Verification for regular diffusion processes

Assumption 2 is verified by numerical integrations of the moment of transition probability on a differentiated description:

$$M_n(x) = \frac{1}{\tau} \sum_{z \in \Omega} (z - x)^n P(z|x, \tau) \tag{13}$$

where $P(z|x, \tau) = \#\{X_i = x \cap X_{i+\tau} = z\}/\#\{X_i = x\}$, τ is a time step of data (here, treated as 1 for simplification), Ω is state space that x can be given and the character # means "the number of". If $M_n(x)$ is sufficiently small for $n \geq 3$ than $n \leq 2$, Assumption 2 can be verified. To show the all case of $M_n(x)$ is complex because $M_n(x)$ can be defined for all values of x. Relations between n and $M_n(x)$ are shown as Fig. 7a for typical x of the year-averaged temperature and shown as Fig. 7b of the month-average temperature on January. We could admit that $M_n(x)$ were simply decrease as an increment of n and $M_n(x)$ ($n \geq 3$) were sufficiently small than $M_2(x)$. The same results were admitted for each month-averaged temperature. Then we can suggest that Assumption 2 is appropriate for the time series data.

5. Discussion

We analyzed the time series of air temperature variation with views of

a) a dynamical steady state of the system

b) a structural steady state

and described as a stochastic process. This kind of research with use of stochastic analysis has been not reported on meteorology.

5.1. Analysis of air temperature variation

We have shown an upward tendency of air temperature with statistical tests in Sec. 1. These tests were reasonable to show a general tendency, but these were insufficient to analyze detail of the time variations. We have shown the time variations of a form among histograms in Sec. 2. We could find a time variation of the histogram that a new peak of histogram appears at higher temperature. We can also suggest that this analysis of the time variation on the form of histograms is meaningful for the analysis of time series.

5.2. Stochastic dynamical equation of air temperature variation

We have shown that the form of a histogram and the potential is corresponding under the assumptions. We examined the verification of these assumptions for time series of air temperature variations and discussed suitability on this theory. As a result, we could consider that the process that reproduces the series of month-averaged temperature satisfies the assumptions. But, on the series of year-averaged temperature, the auto-correlation coefficients exceed significant level with the range of small lag-time. The significantly large correlation in small lag-time might be produced with an operation of year-average.

The variation of a form of a potential function is assumed to be obedient to a delay convention on one-dimensional cut off of wedge catastrophe (Fig. 8) on differential

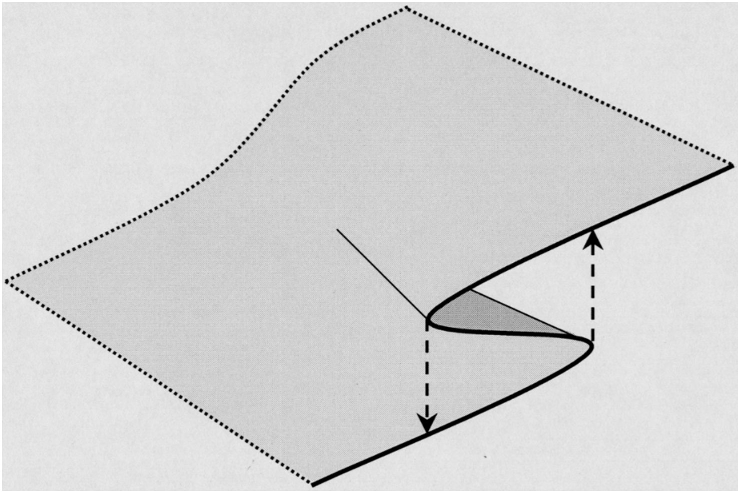


Fig. 8. One-dimensional cut off of wedge catastrophe.

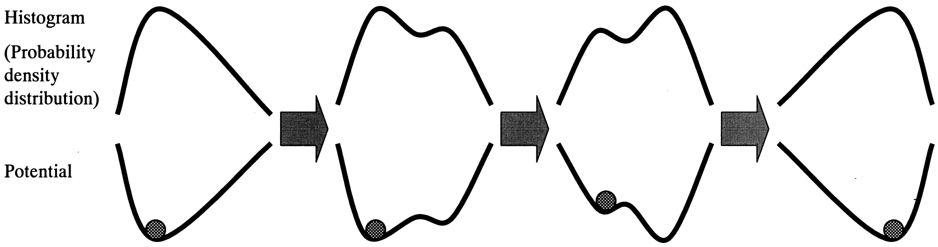


Fig. 9. Hypothesis of time variation of form of time-averaged potential (horizontal axis is temperature).

geometry (POSTON and STEWART, 1978). If we assume the same type of convention on the time-averaged potential, we can build up the following hypothesis on the physical system that describes air temperature variation (Fig. 9).

Hypothesis 1 A stational point on the time-averaged potential moves in accordance with a delay convention on catastrophe theory.

Moreover, we can give a physical explanation of time series with satisfaction of those assumptions, for example, appearance the stable equilibrium point on high temperature in 1990s (downward arrows on Fig. 2). We suggest that our proposed description and modeling of the system are meaningful.

6. Conclusion

1) Form of the histogram of air temperature has been changed with statistical significant. We have shown time variations of a form of the potential for the time series.

We could find that the variation of time series is not monotonous increment but a spatial type of variation that a new peak appears on higher temperature with the view of form of histograms. The time variations of the histograms seem to be meaningful for analysis of time series. However, this result is just a description of tendency on the temperature variation at one measurement point of Nagoya, and we cannot say on global warming with this result.

2) We can assume that "form of the histogram" and "form of the potential" are corresponding on the system of the air temperature variation. And we can build up the hypothesis that form of the potential changes in accordance with delay convention on a catastrophe theory. However, the hypothesis is induced with an assumption that the system of the air temperature variation can be described with our mathematical model as the SDE. Its verification is a problem in future.

Appendix A. Statistical Test of Regression Coefficient

A regression line of the time series (year-averaged temperature) is described as $y = \hat{a} + \hat{b}x$ where $\hat{b}$ is a regression coefficient (MATSUMOTO and MIYAHARA, 1990). If Eq. (A1) holds, a null hypothesis $\hat{b} = b_0$ is rejected with a significant level α:

$$\frac{\left|\hat{b} - b_0\right|}{\sqrt{S_E / (N-2)S_{xx}}} > t_{N-2}(1 - \alpha / 2) \tag{A1}$$

where N is the number of data,

$$S_{xx} = \sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2, \quad S_E = \sum_{i=1}^{N}\left\{y_i - \left(\hat{a} + \hat{b}x_i\right)\right\}^2,$$

$$\hat{a} = 1/N\left(\sum_{i=1}^{N} y_i - \hat{b}\sum_{i=1}^{N} x_i\right), \quad \hat{b} = \left[\sum_{i=1}^{N}\left\{x_i - (1/N)\sum_{j=1}^{N} x_j\right\} y_i\right] / S_{xx}$$

and $t_{N-2}(1 - \alpha/2)$ is t-distribution. The value of the left side of Eq. (A1) for $b = 0$ is 7.68 and the value of the right side is 1.98 at $a = 0.05$; therefore the null hypothesis is rejected. We could admit an upward tendency of year-averaged temperature.

Appendix B. Welch's Test

In Sec. 2, we have shown the time variation of form of histograms. In this section, we employ statistical tests of the time variation of air temperature.

Welch's test is a statistical test that finds difference of averages of two populations $\{a_1, ..., a_{N_p}\}$ and $\{b_1, ..., b_{N_p}\}$ where N_p is the number of data points if variance of the populations are unknown (SOKAL and ROHLF, 1969). Here, $\{a_1, ..., a_{N_p}\}$ is a population as the temperatures from 1900 to 1949, $\{b_1, ..., b_{N_p}\}$ is from 1950 to 1999. They are corresponding to first and last extents on the histogram (Figs. 3a and 3b). If populations $\{a_1, ..., a_{N_p}\}$ and $\{b_1, ..., b_{N_p}\}$ are normal populations, Welch's test could be applied. Goodness tests of the populations to normal distributions are shown as follows. According to Welch's test, if Eq. (B1) holds, we can reject the hypothesis that mean values of two populations are equal with significance level $\alpha = 0.05$.

$$\frac{\left|\bar{a}-\bar{b}\right|}{\sqrt{\left(s_a^2+s_b^2\right)/N_p}} > t_m\left(1-\frac{\alpha}{2}\right) \tag{B1}$$

where $\bar{a}$ and $\bar{b}$ are mean values of populations $\{a_1, ..., a_{N_p}\}$ and $\{b_1, ..., b_{N_p}\}$, $m = \{(s_a^2 + s_b^2)/N_p\}^2/[(s_a^4 + s_b^4)/\{N_p^2(N_p - 1)\}]$,

$$s_a^2 = \sum_{i=1}^{N_p}\left(a_i-\bar{a}\right)^2/\left(N_p-1\right), \quad s_b^2 = \sum_{i=1}^{N_p}\left(b_i-\bar{b}\right)^2/\left(N_p-1\right)$$

and $t_m(1 - \alpha/2)$ is t-distribution. The value of the left side of Eq. (B1) was 7.11 and the right side was 0.717. These values satisfied Eq. (B1). We can suggest that the mean values of each population are significantly different.

We explain verification of the normal distribution here. If the following Eq. (B2) is hold, a null hypothesis $p_i \neq p_i^0$ is rejected with a significant level α

$$\sum_{i=1}^{k}\frac{\left(x_i-N_p p_i^0\right)^2}{N_p p_i^0} \le \chi_{k-1}^2(1-\alpha) \tag{B2}$$

where k is the number of classes of the histogram, $k - 1$ is the degree of freedom, x_i is frequency of the class i, p_i is a probability density function assumed from the population, p_i^0 is the normal distribution whose average and dispersion are equal to the sampling average and sampling dispersion and $\chi_{k-1}^2(1 - \alpha)$ is the chi-square distribution. A class step is set to 1 degree. The values of the left side of Eq. (B2) was 1.51 for $\{a_1, ..., a_{N_p}\}$ and 11.27 for $\{b_1, ..., b_{N_p}\}$ and the values of the right side of Eq. (B2) was 66.5. Then, Eq. (B2) was hold for both populations and the null hypothesis was rejected. We can assume that Welch's test is reasonable.

Appendix C. Run Test

A run test is used for verification of hypothesis "the time series is random". A significant point of verification on the number of run is introduced to the run test. This method is competent for a large number of samples.

Table 1. The statistical value of run test for each month-averaged temperature.

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
–1.49	–0.59	–3.32	–1.51	–3.04	–0.18	–0.18	–1.44	–1.69	–1.47	–0.10	–2.06

A reference value is ±1.96.

A value of each point of time series is compared to a median of the time series, and the value is written as;

a) a symbol "+" for a larger value

b) a symbol "–" for a smaller value

where the continuous symbols + or – are called a run. The number of the continuous the symbol + is presented as m and – is n. Length of the run L is defined as length of the continuous group of the same symbol. If the length of the run is too long, it shows that observed values makes the same type of groups. Otherwise, If the different symbols (+ and –) appears alternatively and the length of the run is too short, it shows that the series have regularity. These series of the observed value are not random series. Then, we think verification of the randomness of the time series with a null hypothesis "the time series is not random". If m and n are larger than 20 at least and the time series are random, a statistical test value on L satisfies Eqs. (C1) and (C2) with a normal approximation (MINOTANI, 2000).

$$E(L) = \frac{2mn}{m+n} + 1 \tag{C1}$$

$$V(L) = \frac{2mn(2mn - m - n)}{(m+n)^2(m+n-1)} \tag{C2}$$

where $E(L)$ is a average, $V(L)$ is a standard deviation of the standard distribution.

The reference value is 1.96 for a significant level. The statistical test value on each time series calculated with Eq. (C1) is shown as Table 1. Then the null hypothesis is accepted except for the time series on March, May and December.

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REFERENCES

GOEL, N. S. and RICHTER-DYN, N. (1978) *Stochastic Models in Biology*, Academic Press, New York.

HARKEN, H. (1975) Cooperactive phenomena in systems far from thermal equilibrium and in nonphysical systems, *Reviews of Modern Physics*, **47**, 67–121.

IPCC (1995) *Greenhouse Gas Inventory, Reporting Instructions*, Intergovernmental Panel on Climate Change.

MATSUMOTO, H. and MIYAHARA, Y. (1990) *Suuri Toukei Nyumon* (*Introduction to Mathematical Statistics*), Gakujutsu Tosho, Tokyo (in Japanese).

MINOTANI, C. (2000) *Black and Scholes Model*, Toyo Keizai Shinposha, Tokyo (in Japanese).
PAWULA, R. F. (1967) Approximation of the linear Boltzmann equation by the Fokker-Plank equation, *Phys. Rev.*, **162**, 196.
POSTON, T. and STEWART, I. N. (1978) *Catastrophe Theory and Its Applications*, Pitman Publishing, London.
SHIMIZU, Y. and TAKADA, H. (2000) Verification of air temperature variation with form of time-averaged potential, *Bulletin of Society for Science on Form*, **15**(1), 42–43 (in Japanese).
STRATONOVICH, R. (1963) *Topics in the Theory of Random Noise*, 1, Gordon and Breach, New York.
TAKADA, H., SHIMIZU, Y., YOSHIMORI, M. and MATSUGI T. (1999) Stochastic model on exchange rate time series, *Bulletin of Society for Science on Form*, **14**(3), 208–209 (in Japanese).
TAKADA, H. (2001) Construction method of mathematical model on time series data with character of Markov process and verification, Ph.D. Thesis, Nagoya University (in preparation).
YOSHIDA, Y. and YOSHIDA, M. (ed.) (1958) *The Number Table*, Baifukan, Tokyo (in Japanese).