

A Dynamical System Applied to Foreign Currency Exchange Determination

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(Received March 7, 2002; Accepted October 31, 2003)

Keywords: Exchange Rate, Correlation Dimension, Multiple Regression, Principal Component Analysis

Abstract. A problem of finding the number of variables describing monthly JPY/US\$ exchange rate is treated on the basis of fractal theory and principal component analysis. First, we calculated the correlation dimension of the exchange rate according to the idea that the time series of the exchange rate is composed of chaos. The correlation dimension proved to be a value of around 1.4, hence two variables are at most required. Secondly, two principal components were extracted by the multiple regression out of six economic variables. Lastly, we drew comparison the combination method based on the fractal theory and the principal component analysis.

1. Introduction

The exchange rate is a relative price of two currencies. It shows an interaction of supply and demand factors for two currencies in the market that determines the exchange rate at which they trade. This issue has been studied extensively in economic paper and widely discussed among investors, government officials, academicians, traders, etc. There are still no definitive answers (ROSENBERG, 1996; STIGLITZ, 1993).

According to CROSS (1998), there are several approaches to determine exchange rate. The first approach is the Purchasing Power Parity (PPP) Approach. This theory claims that in the long run, exchange rates will adjust to equalize the relative purchasing power of currencies.

The second approach is the Balance of Payments and Internal-External Balance Approach. The approach postulates that the exchange rate changes are determined by an international difference in prices or change in prices and tradable items.

The third approach is the Monetary Approach, which is based on the proposition that exchange rates are established through the process of balancing the total supply of the

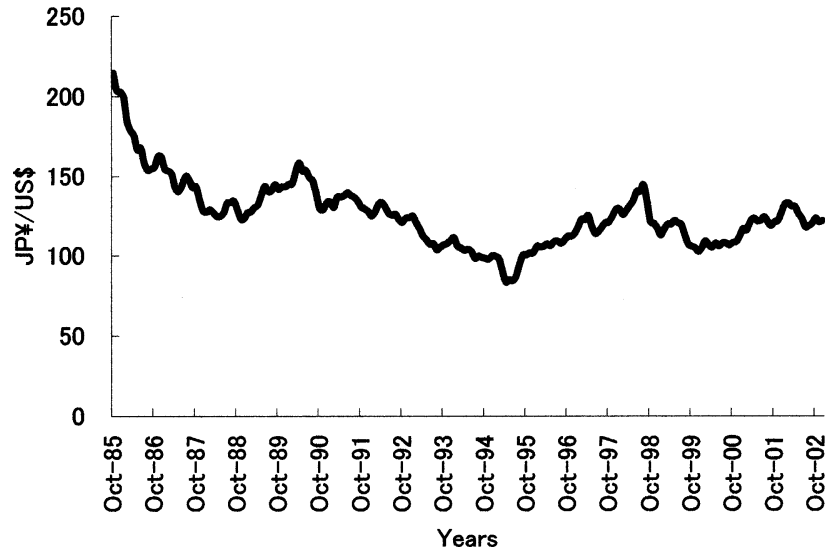


Fig. 1. Time series of JPY/US\$ exchange rate.

national money and the total demand for it, in each nation.

The fourth approach is the Portfolio Balance Approach, which takes a shorter-term view of the exchange rate and broadens the scope from the supply and supply conditions for money to that which takes the demand and supply conditions for other financial assets as well into account.

Although these approaches noted above and are some of the most general and familiar ones, focus on differentials in real interest rates, they have not proved to be adequate for making reliable explanation about exchange rates. In several past studies (THE ECONOMIC PLANNING AGENCY, 1995; HAMADA, 1996; MORIYAMA, 1999), these models did not give us sufficient demonstration.

Our purpose is to propose a new model different from any of these. We adopt interest rates of bank deposit, because the above approaches and models seem to show an importance of the difference between Japanese and American interest rates. FUKAO (2000) obtained a useful result for explaining the exchange rate by the use of the real difference interest rates. Our model based on correlation demission and the principal component analysis is considered to give the most useful result among past studies until now.

2. Definition of Variables

The time series of the exchange rate and its autocorrelation are denoted by $\{x_t\}$, where t is the time measured by month, for example $t = 1985.10$ to 2002.12 . (Fig. 1)

We define the following coordinates with delay time, which is an m dimensional vector.

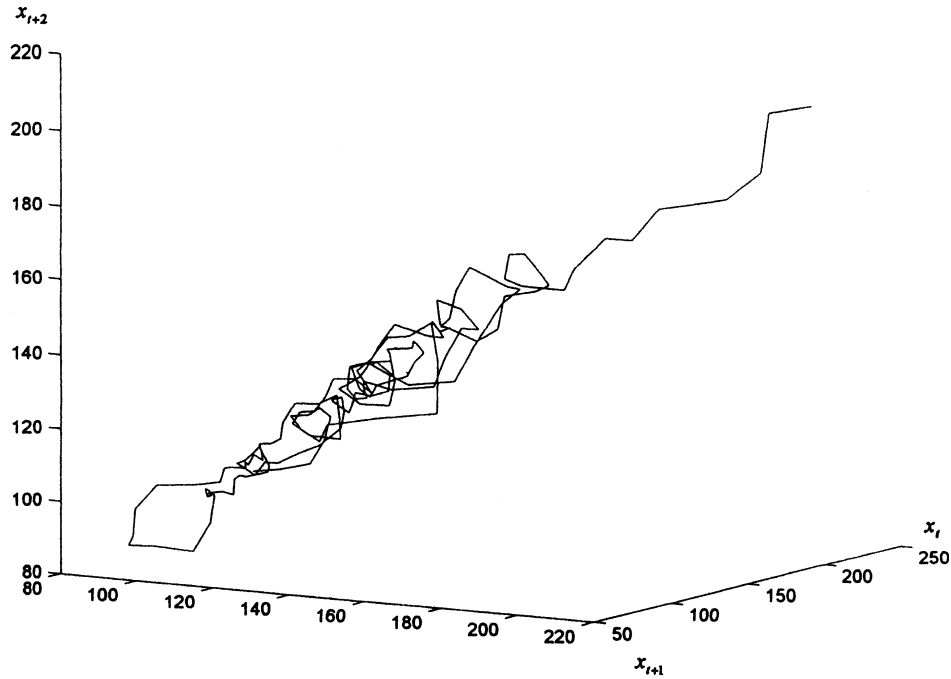


Fig. 2. Plotting of the coordinates with delay time (x_t, x_{t+1}, x_{t+2}) in the embedding space with dimension 3.

$$x_t = (x_t, x_{t+1} \cdots x_{t+m-1}) \quad (1)$$

The trajectory of this coordinate with delay time is embedded in a phase space with m dimensions. Assuming that the dynamical system on the n dimensional manifold generates the time series, we can expect that:

- 1) This trajectory has an attractor, provided the time series is observed over a very long time.
- 2) The transformation from the time series to the attractor is embedding according to Takens's theorem under the condition $m \geq 2n + 1$ (Appendix A). The attractor and the manifold are an isomorphism.

We assume that the exchange rate varies as a dynamical system. This assumption is supported by past researchers (GOODWIN, 1992; KOZIMA, 1994; THE ECONOMIC PLANNING AGENCY, 1995; TAKAYASU, 1999; MANDELBRIT, 1999).

In order to extract attractors for the exchange rate, we plotted the coordinates with delay time (x_t, x_{t+1}, x_{t+2}) in the embedding space with dimension 3. As a result, we could show convergence into a certain manifold i.e. a periodic loop on this manifold from an initial condition, that might be thought of as the equilibrium space for the dynamical system (PEITGEN, 1992) (Fig. 2). The time series of the exchange rate seems not to have randomness but chaos.

3. Estimation of Number of Variables

The phase space in which we have embedded the coordinates given by Eq. (1) can be delimited within spheres whose radius is denoted by ε . The number of the spheres with radius ε is denoted by $n(\varepsilon)$. Let the probability that the embedded points are contained in that sphere with center x_j be denoted by P_j . Then, the correlation dimension of the attractor with delay coordinates is defined as follows:

$$D_2 = \lim_{\varepsilon \rightarrow 0} \frac{\log \sum_{i=1}^{n(\varepsilon)} P_i^2}{\log \varepsilon} = \lim_{\varepsilon \rightarrow 0} \frac{\log C(\varepsilon)}{\log \varepsilon}, \quad (2)$$

where $C(\varepsilon)$ is the correlation integral (Appendix A).

In general, it is difficult to obtain the theoretical limit by the use of Eq. (2) because of the resolution limit of measurement. For instance, the resolution limit of the amount of dealings of a certain exchange rate is 0.05 Yen as a lower bound. Practical method would be to fit a straight line to data points of $\log C(\varepsilon)$ and $\log \varepsilon$, whose slope gives a correlation dimension of the attractor. The correlation dimension obtained in this way is denoted by $D_2(m)$, where m is the dimension of the coordinate. To be more precise, we follow the process below.

- 1) We integrate $C(\varepsilon)$ (called a correlation integration).
- 2) Select a ε -range where it is possible to fit a line to the data of $\log \varepsilon$ versus $\log C(\varepsilon)$.
- 3) Estimate the gradient of this line $D_2(m)$, called a regression coefficient.
- 4) The obtained $D_2(m)$ must be constant for dimension $m \geq 2n + 1$ from the theorem (see Appendix A). This value gives the correlation dimension D_2 .

In the process 2) above we selected the ε -range with a maximum value of determination coefficients R^2 of the linear regression. The determination coefficients R^2 is a slope in relationship $\log(\varepsilon)$ and $\log C(\varepsilon)$. Although the regression coefficients are often determined in the range $-4 \leq \log C(\varepsilon) \leq -2$ empirically (SANO and SAWADA, 1985), the present method is considered better. The variable intervals within 50 continuous ε -points were examined.

The value of m was changed within $1 \leq m \leq 10$. The fractal dimension of the attractor was calculated from the increase in the dimension of embedding space. Based on the theorem (Appendix A), $D_2(m)$ is saturated. We can accurately estimate the correlation dimension of the attractor as the partial manifold in higher dimensional embedding space. If the time series were generated by a dynamical system on a 2-dimensional compact manifold, we could at least estimate the correlation dimension of it in 5-dimensional embedding space.

Figure 3 shows a relationship between dimensions of the embedding space and the attractor on which a dynamical system generates time series of the exchange rate. $D_2(m)$ converged to a value 1.39 for $m > 3$. That is, 2 variables at most are enough to describe the time series (Fig. 3). We later identify two variables based on the principal components analysis (YOSHIMORI *et al.*, 1999a).

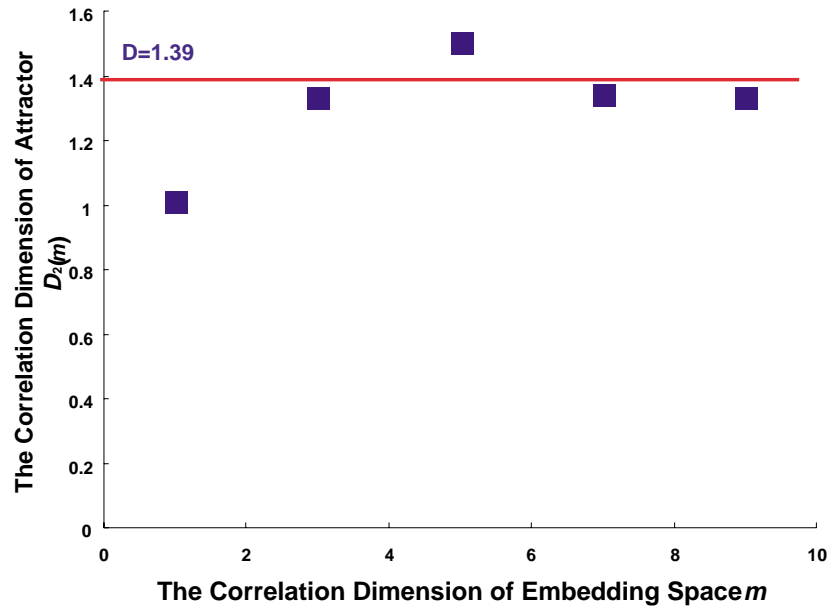


Fig. 3. Relationship between dimensions of the embedding space m and the attractor $D_2(m)$. converges to a value 1.39 with increasing m .

4. Method of Analysis of Economic Variables

In this section, we testify to search correlations between the interest rate, the exchange rate, etc. Many economists consider the interdependence between interest rate and exchange rate. Here, the exchange rate will be described by the multiple regression formula (MRF).

4.1. Used data

In our previous research (MATSUGI *et al.*, 2001; YOSHIMORI *et al.*, 2001), it was concluded that the interest rate, especially the difference between Japanese and American long-term interest rates monthly data, was not significant as economic variables describing the exchange rate. In some quarterly models (FUKAO, 2000), interest rates are estimated to be significant as an exchange rate determinant.

We first examine whether the interest rates of both countries are effective for analysis of the exchange rate. The list of data used to analyze relationship between interest rate and exchange rate is given in Table 1. In our monthly model we analyzed relations between the exchange rate and economic variable with back word and forward time lags. In previous research, it was shown that exchange rate at a time was expressed by economic variables before that time. But, we also believe that the exchange rate at a time is related to economic variables after that time.

Table 1. Used data of interest rates.

Number	Interest rate
#1	Short-term nominal interest rate in USA
#2	Long-term nominal interest rate in USA
#3	Short-term nominal interest rate in Japan
#4	Long-term nominal interest rate in Japan
#5	Difference between the short-term nominal interest rate in USA and Japan
#6	Difference between the long-term nominal interest rate in USA and Japan
#1'	Short-term real Interest rate in USA
#2'	Long-term real Interest rate in USA
#3'	Short-term real Interest rate in Japan
#4'	Long-term real Interest rate in Japan
#5'	Difference between the short-term real interest rate in USA and Japan
#6'	Difference between the long-term real interest rate in USA and Japan

The short-term interest rates in Japan and USA are those of 6-month Government Bond.

The long-term interest rates in Japan are 10-year Government Bond, and the long-term interest rates in USA are 30-year Government Bond.

Nominal interest rates are given interest rates in market.

Real interest rates are given the nominal interest rates minus the rates of inflation.

Here, the term “lag” is defined to show a correlation between interest rate at a time and exchange rate before that time. The term “forward” is for interest rate at a time and exchange rate after that time. We calculated correlation functions between the interest rates given in Table 1 and the exchange rates. Lag and forward when maximum values of these correlation functions are given in Table 2. We found that the Japanese short-term interest rate has the highest correlation coefficient with the exchange rate. The next significant correlation is with the US long-term interest rate. From this result, we use in the following the Japanese short-term interest rate and the American long-term interest rate as economic variables.

Secondly, we searched economic variables that seemed to depend on exchange rate. All economic variables from several papers and books (AMANO, 1978; THE ECONOMIC PLANNING AGENCY, 1995; ROSENBERG, 1996; RONALD and OONO, 1998; MORIYAMA, 1999; INSTITUTE FOR INTERNATIONAL MONETARY AFFAIRS, 2001) were examined such as money supply, trade balance, index of wholesale prices, foreign reserves, industrial production, interest rates, current account, direct investment, portfolio investment and unemployment rates. We also considered the economic variables of business conditions, the macro index, and share prices. These variable are listed in Table 3.

Thirdly, we calculated correlation functions between the exchange rate and each economic variable in Table 3 like relationship between the exchange rate and each interest rate. At first, we employ the moving averages method so that the time series smooth out. We testify the moving average of several periods and found out the moving averages of seven month for each correlation function. Next, we calculated correlation functions between the exchange rate and each economic variable with seven-month moving averages.

Table 2. Maximum values of the correlations between the exchange rates and interest rates. The lag intervals (+) and the forward intervals (–) of economic variables before (after) the exchange rate.

Interest rate	Correlation	Lag and forward
#1	0.48	+3
#2	0.50	+8
#3	0.61	+7
#4	0.47	–1
#5	–0.38	+2
#6	–0.09	+3
#1'	0.50	+0
#2'	0.53	+3
#3'	0.68	+3
#4'	0.49	+3
#5'	–0.37	+1
#6'	–0.10	+6

Table 3. The used date of economic variables (macro index). They are from financial economic statistics monthly report, financial statistics, Business Statistics and finacia economic Survey of Current.

	Business condition	Current account	Trade balance	Index of wholesale price
Japan	Diffusion index (Leading index)	(Seasonal adjustment)	—	Wholesale price
USA	Diffusion index (Leading index)	—		Producer price index
Japan	Foreign reserves (Seasonal adjustment)	Industrial production (Seasonal adjustment)	Share price Nikkei Dow average	Interest rate The short-term real interest rate in Japan
USA	(Seasonal adjustment)	(Seasonal adjustment)	Dow-Jones averages	The long-term real interest rate in USA
Japan	Money supply M3	Direct investment	Portfolio investment	Unemployment rate
USA	M3	—	—	

The time intervals with maximum correlation are given in Table 4 (YOSHIMORI *et al.*, 1999b). We neglected economic variables without a significant level of correlation coefficients with the exchange rate (see Table 5). For example, share prices in Japan, direct investment in Japan, and portfolio investment in Japan were neglected.

4.2. Multiple regression analysis

We use interest rates and short term and long term interest rates for both countries to determine their dependences on the exchange rate. Here, in order to compare 12 kinds of

Table 4. The lag intervals (+) and the forward intervals (–) of economic variables before (after) the exchange rate.

	Business condition	Money supply	Trade balance	Index of wholesale price
Japan	+4	–4	—	+4
USA	–4	–6	–4	–4
	Foreign reserves	Industrial production	Share price	Interest rate
Japan	+3	–8	+9	+3
USA	–9	+5	–6	+3
	Current account	Direct investment	Portfolio investment	Unemployment rate
Japan	–10	+3	+5	+7
USA	—	—	—	+9

Unit: Month.

time series of the interest rates with the interest rates difference shown in Table 1, we calculated the correlation coefficients between the exchange rate and interest rates. The biggest value of correction was that between the interest rates for short-term real interest rates in Japan, and the long-term real interest rate in US. In the following, we use the time series of the short-term interest rates in Japan and the long-term real interest rate in US.

We got 15 kinds of economic variables in Table 5 and looked for the multiple regression formula (MRF) for possible combinations of the economic variables; ${}_{15}C_2$ combinations. Here, we selected the significant multiple regression formulae with F-test* and counted the number of times that each economic variable was used in the significant MRF. It has been shown by YOSHIMORI *et al.* (1999b) that Table 6 shows the fraction where each economic variable was appears in MRF. Among these fractions these exceed 40% Japanese Wholesale Price Index (X_1), Japanese Gold and Foreign Exchange Reserves (X_2), Japanese Short-term Interest Rate (X_3), Japanese Current Balance, (X_4) United States of America Money Supply (X_5), United States of America Index of Industrial Production (X_6). These six economic variables were considered to be useful to explain the exchange rate.

In general, the multiple regression analysis is used to study the relationship a dependent variable and several independent variables. We employed the multiple regression analysis (GREEN, 2000) for the exchange rate with economic variables. First, we try to get the coefficient of determination in six economic variables. Secondly, we eliminate next one economic variable that had the smallest t-value and derived the coefficient of determination in five economic variables, and so on. The coefficient of determination is defined by the proportion of the total variation in the independent variable that is explained or accounted for variation in the dependent variable.

*The F-test of whether two samples are drawn from different populations has the same standard deviation, with a significant level.

Table 5. The result of the correlation.

Macro index	Correlation (Japan)	Correlation (US)
Business condition	*-0.28	*-0.82
Money supply	*-0.73	*-0.67
Trade balance	—	*-0.34
Index of wholesale price	*0.90	*0.75
Foreign reserves	*-0.56	*-0.74
Industrial production	*-0.59	*-0.54
Share price	0.13	*-0.41
Interest rate	*0.62	*0.53
Current account	*-0.62	—
Direct investment	-0.07	—
Portfolio investment	0.05	—
Unemployment rate	0.12	0.17

Coefficients between the exchange rate and economic variables. Maximum values are listed in this table among those with various time intervals. We neglect variables with correlation coefficients whose absolute values are less than 0.2, and use economic variables with symbol * for the regression analysis.

Table 6. Results of the analysis for all combinations.

	Business condition	Money supply	Trade balance	Current account
Japan	16%	25%	—	95%
USA	14%	86%	26%	—
	Foreign reserves	Industrial production	Share price	Index of wholesale price
Japan	42%	19%	—	97%
USA	26%	39%	29%	21%
	Interest rate			
Japan	63%			
USA	27%			

To verify the null hypothesis, i.e. A multiple regression formula cannot estimate the exchange rate, F-test was employed with a significance regression and residual (KAWAGUCHI, 1973). The multiple regression formula is valid only if the null hypothesis is rejected statistically.

4.3. Principal component analysis

In tradition, the standard econometric analysis aim at getting a higher value of the coefficient of determination by adding economic variables so that they pass the T-testing*. Therefore, this method ignores the number of the correlation dimension. We employ the

*The T-test is a statistical tool used to determine whether a significant difference exists between the means of two distributions.

Table 7. Results of multiple regression analysis based on the six economic variables.

Economic variables	t-values				
	Step 1	Step 2	Step 3	Step 4	Step 5
X_1	0.39	0.39	0.38	0.28	—
X_2	0.28	0.30	0.29	—	—
X_3	0.16	0.25	0.33	0.33	0.49
X_4	0.06	—	—	—	—
X_5	-0.45	-0.54	-0.72	-0.82	-0.84
X_6	-0.12	-0.10	—	—	—
R^2	0.76	0.76	0.76	0.75	0.73

We get a coefficient of determination of 0.76 for this analysis.
 R^2 is a coefficient of determination.

principal component analysis as the method of reducing economic variable. This analysis shows the number of economic variables, which was determined by the estimated value of the correlation of determination. Thus, this method is considered to be useful for the multiple regression analysis to obtain two economic variables. On the other hand, we also paid attention to the multicollinearity* was often occurred in the multiple regression so that it is necessary to orthogonalize those variables through the principal component analysis.

We first calculated elements of the simple correlation matrix using the same six economic variables from X_1 to X_6 . Based on this matrix we calculated six eigen-values as well as six eigen-vectors. The first principal component corresponds to the eigen-vector with the largest eigen-value; the second principal component corresponds to that with the second largest eigen-value, and so on. Moreover, we employed the multiple regression analysis of the exchange rate by the use of PC1 ... PC6. PC defines the principle component; a set of variables defined as a projection which encapsulates the maximum amount of variation in a dataset and is orthogonal to the previous principle component of the same dataset. We can understand that PC1 is composed of those factors affecting Japanese exports, because increasing X_5 and X_6 indicates a US economic boom, while declining X_1 and X_3 indicate exporters selling. In addition, these economic variables also show Japanese recession. This implies that PC1 would have negative effects on the exchange rate through increasing exports from Japan to the United States. Based on Appendix B, we last calculated the regression coefficient and composed of the multiple regression equation.

*The multicollinearity is the degree of correlations between independent variables. It is an important consideration when using the multiple regressions on data that had been collected without the aid of a design of experiment (DOE), a Design of Experiment (DOE) is a structured, organized method for determining the relationship between factors affecting a process and the output of that process. The coefficients cannot change drastically without mulicolinear variables in or out of the multiple regression models.

Table 8. Results of the principal component analysis.

PC	PC1	PC2	PC3	PC4	PC5	PC6
Eigen-V	3.88	1.29	0.50	0.23	0.06	0.04
X_1	-0.41	0.37	-0.49	0.45	0.22	0.46
X_2	0.48	-0.20	-0.02	-0.31	0.27	0.75
X_3	-0.42	-0.24	-0.51	-0.63	-0.32	0.05
X_4	-0.02	-0.85	-0.24	0.44	0.16	-0.09
X_5	0.44	0.20	-0.60	-0.16	0.42	-0.46
X_6	0.48	0.08	-0.31	0.30	-0.76	0.10

PC means principal component and Eigen-V means eigen value.

5. Results

5.1. Results of multiple regression analysis

Results of this analysis is shown in Table 7. From the multiple regression analysis with the six economic variables, we get a coefficient of the determination of 0.76 (this process is called step 1). We deleted one economic variable that had the smallest t-value in this regression for each case. The next step 2 was made for the five variables, for example, of which the variable X_4 had the smallest t-value and was deleted. After continuing this process the multiple regression analysis reached a final result in which the exchange rate could be explained by X_3 (the Japanese short-term interest rate) and X_5 (the US money supply) with a coefficient of determination of 0.73. We verified the result with monthly data of economic variables so that Moriyama Model (MORIYAMA, 1999) employs quarterly data. We got the result of an extremely low value as a coefficient of determination of 0.56 on three economic variables and a dummy variable.

5.2. Results of principal component analysis

The six eigen-vectors obtained in this way are shown in Table 8 with corresponding eigen-values in the first row. Eigen value of the six elements, when multiplied by the square root of the first eigen value 1.97 produces the simple correlation coefficient between PC1 and each X_i , respectively. This implies that PC1 is positively well correlated with X_2 , X_6 and X_5 , while it is negatively well correlated with X_3 and X_1 .

From the regression analysis starting with the six principal components (see Table 9), we get a coefficient of the determination of 0.89. We also deleted one principal component that had the smallest t-value in this regression.

It was found that the regression in step 5 gave an estimated coefficient of determination of 0.84, which is larger than that all cases in Table 9. It was quite remarkable to see that the first principal component PC1 solely determined the exchange.

We also set the multiple regression equation.

$$S_t = 1.58z_1(t) + 0.37z_2(t) \quad (3)$$

Table 9. Coefficients of determination based on the six principal components.

Principal component	t-values				
	Step 1	Step 2	Step 3	Step 4	Step 5
PC1	1.66	1.66	1.67	1.66	1.58
PC2	0.32	0.32	0.33	0.32	—
PC3	0.13	0.13	0.14	—	—
PC4	0.04	0.04	—	—	—
PC5	0.04	—	—	—	—
PC6	0.39	0.39	0.38	0.39	0.37
R ²	0.89	0.89	0.88	0.86	0.84

We get a coefficient of determination of 0.84 from the regression analysis with the two principal components PC1 and PC6.

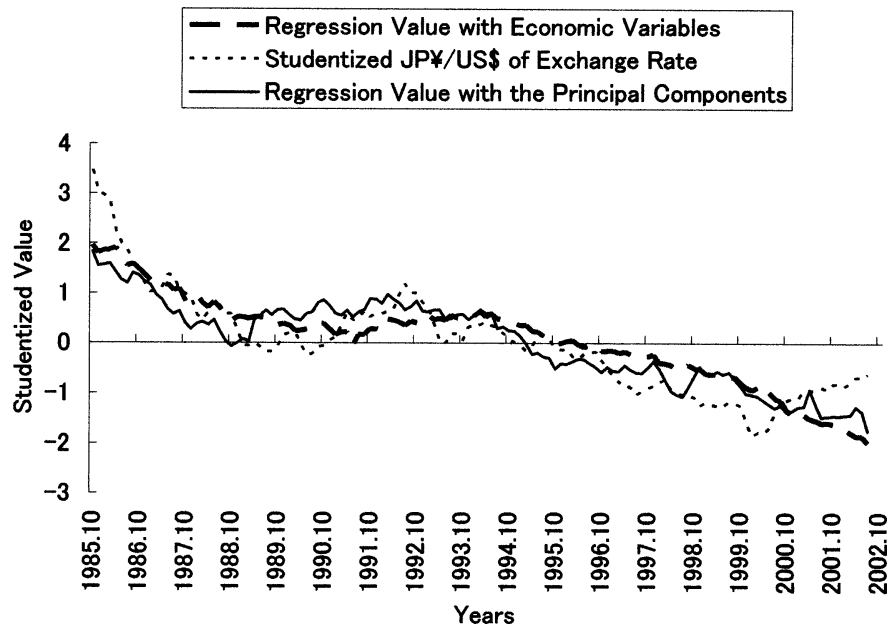


Fig. 4. Comparisons of the time series of the exchange rate with those reproduced by our regression model.

where S_t was a regression value at time and z_1, z_2 are principal components PC1, PC6. Figure 4 showed comparisons of the time series of the exchange rate with the time series reproduced by our multiple regression model.

When we could point out the applicability of fractal analysis to econometric studies, in conclusion, we were satisfied with the estimated coefficient of determination. This value 0.84 is remarkable result compared with other previous studies.

6. Conclusion and Discussion

In this paper our research started by estimating the correlation dimension of the exchange rate for the period 1985.10–2002.12, and came to a result the exchange rate should be explained by at most two variables. We confirmed this result by the principal component analysis. Our regression model consisting of two principal components gets a much better result with the coefficient of determination of 0.84 than the result of other econometric model with using six economic variables.

Discussion is made on this conclusion. First principal component is considered to be connected to behavior of individual dealers in the monthly exchange market. Many dealers may collect information relevant to buying or selling foreign currencies at present and future markets. Their information is statistical data such as economic variables (called fundamentals), or sometimes political statements made by influential persons, etc. The information which dealers get by summarizing a variety of quantifiable information into a single indicator like a weighted average is considered to be the principal component that we discovered.

Secondly, we consider an ensemble of dealers' behavior. Many dealers gather information of others on a personal basis. Buying and selling at the market determine the price of exchange rate through a balance of supply and demand. Thus, personal weighted averages of fundamentals are converted to the market weighted average. We have concluded that the two principal components would be useful by each dealer in dealing with the exchange rate at the market.

Finally, many economists are inclined to improve the determination coefficient on the multiple regression through an increase of economic variables, while our principal components verified importance of two variables. When economic variables such as JP¥/US\$ show to make up the dynamical-system, the approach applying for fractal analysis provides us with several advantage. In first advantage, it is more excellent in that fractal analysis can determine the number of independent variables before carrying out the multiple regression. In secondly advantage, we can grasp and analyze economical phenomenon correctly so that we simply performance without purpose of high value of the determination coefficient. It is concluded that fractal analysis is very useful for counting the economic variables in a regression equation. Hence, we need to incorporate the fractal analysis of the numbers associated with the attractors and then use the number of in those numbers econometric analysis.

We would like to thank Dr. Sadao Naniwa (Ritsumeikan University) for his comments on the time series analysis. We are deeply grateful to Dr. Masaru Miyao (Nagoya University) and Mrs. Tomono Yoshimori for their technical helps. We also would like to thank Mr. Christopher Paul Morrone for his English assistance.

Appendix A: Theoretical Grounds for Analysis Method

Theorem (TAKENS, 1981) It is assumed that dimensional compact manifold M is given. A mapping $\Phi_{(\phi, g)}: M \rightarrow \mathbf{R}^{2n+1}$ on the pair (ϕ, g) , i.e.,

$$\Phi_{(\phi, g)} = \left(g(x), g(\phi(x)), \dots, g(\phi^{2n}(x)) \right), \quad (\text{A.1})$$

is an embedding, where ϕ, g are C^2 -function and smooth. One to one correspondence exists between M and $2n + 1$ dimensional Euclidean space. The pair (ϕ, g) by which $\Phi_{(\phi, g)}$ is assumed to be the embedding mapping is dense in the function space (TSUDA, 1999). We can apply this theorem if we consider that ϕ is a diffeomorphic mapping on M , ϕ^t is flow on the dynamical system, and g shows the observed value; therefore Φ on Eq. (A.1) transforms from each point on the manifold M onto each delay coordinate. It is important in the theorem mentioned above that the transformation Φ onto the delay coordinate is embedding. If the transformation Φ is embedding, it is immersion; therefore the fractal dimension of the attractor is preserved (IKEGUCHI and MATOZAKI, 1996). Calculating the fractal dimension of the attractor with delay coordinates, we can analyze the geometrical structure of the manifold M .

The correlation integral is described by the following expression:

$$C(\varepsilon) = \frac{1}{N^2} \sum_{i,j=1}^N \Theta(\varepsilon - |X_i - X_j|), \quad (\text{A.2})$$

where Θ is Heviside's function.

Appendix B: Method of Regression Coefficient

In the multiple regression analysis (MRA), the value of dependent variable Y from a set of K independent variables $\{X_1, \dots, X_k, \dots, X_K\}$. We denote by \mathbf{X} the $N \times (K + 1)$ augmented matrix collecting the data for the independent variables (this matrix is called augmented because the first column is composed only of ones), and by \mathbf{y} the $N \times 1$ vector of observations for the dependent variable (see Matrix 1).

The multiple regression finds a set of partial regression coefficients b_k such that the dependent variable could be approximated as well as possible by a linear combination of the independent variables (with the b_j 's being the weights of the combination). Therefore, a predicted value, denoted \hat{Y} , of the dependent variable is obtained as:

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + \dots + b_k X_k + \dots + b_K X_K. \quad (\text{B.1})$$

The value of the partial coefficients are found using ordinary least squares. It is often convenient to express the MRA equation using matrix notation. In this framework, the predicted values of the dependent variable are collected in a vector denoted $\hat{\mathbf{y}}$ and are obtained using MRA (ISHIMURA, 1987) as:

$$\hat{\mathbf{y}} = \mathbf{X}\mathbf{b} \quad \text{with} \quad \mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}. \quad (\text{B.2})$$

Matrix 1: The structure of the \mathbf{X} and \mathbf{y} matrices

$$\mathbf{X} = \begin{bmatrix} 1\mathbf{x}_{11} & \cdots & \mathbf{x}_{1k} & \cdots & \mathbf{x}_{1K} \\ \vdots & & \vdots & & \vdots \\ 1\mathbf{x}_{n1} & \cdots & \mathbf{x}_{nk} & \cdots & \mathbf{x}_{nK} \\ \vdots & & \vdots & & \vdots \\ 1\mathbf{x}_{N1} & \cdots & \mathbf{x}_{Nk} & \cdots & \mathbf{x}_{NK} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_n \\ \vdots \\ \mathbf{y}_N \end{bmatrix}.$$

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