Letter *Forma*, **20**, 39–46, 2005

Experiments for Stick-Slip Motions in One-dimensional Mass-Spring Systems

Takayuki HIGUCHI, Hideaki KURIHARA and Osamu HIRAYAMA*

Institute of Symbiotic Science and Technology, Tokyo University of Agriculture and Technology, Koganei-shi, Tokyo 184-8588, Japan
**E-mail address: hrym@cc.tuat.ac.jp*

(Received July 16, 2005; Accepted September 10, 2005)

Keywords: BK Model, Stick-Slip Motion, Size-Frequency Relationship, Power Law

Abstract. Experiments based on the BK model for stick-slip motions in one-dimensional mass-spring systems were conducted using ten blocks. The movements of each block at every 1/30 second were recorded by video pictures and the state of each block at every time step is classified into stick or slip. It was found that the power law holds between the size of slip event and its frequency. It was also clarified that the ratio of coupling spring constant to the driving spring constant larger, the smaller the power becomes.

1. Introduction

Stick-slip motion has been well known since old days and it has been used as a model for earthquake and avalanche (DURAN, 2000). Especially BK model (BURRIDGE and KNOPOFF, 1967) is famous as an explanation model for earthquakes. The schematic for BK model is shown in Fig. 1.

Many blocks connected to each other with coupling springs are placed on the rest floor, and with driving springs these blocks are connected also to the ceiling that moves with constant velocity. In this model the blocks and coupling springs are regarded as a lithosphere of one plate, the ceiling as the asthenosphere of the same plate and the floor as the lithosphere of another plate. Stick-slip motions occur between the blocks and the floor and slip events correspond to earthquakes. Many researches based on this model have been conducted and some knowledge related to the earthquakes was obtained. For example, CARLSON and LANGER (1989) and CARLSON *et al.* (1991) conducted numerical simulations using BK model and found a power law between the size or energy of slip events and its frequency (Gutenberg-Richter law) holds and HUANG and TURCOTTE (1992) clarified that the larger the asymmetry of the system is, the more chaotic the behavior of the system becomes by the use of two block BK model. However, in spite of these successes in the simulation no experiments have been reported for BK model. One reason is supposed that the experiments related to the macroscopic friction behavior are difficult to obtain some clear results in general and another is that the number of blocks used in the experiments are restricted in the experiments. However, we think it is a valuable trial to examine the actual

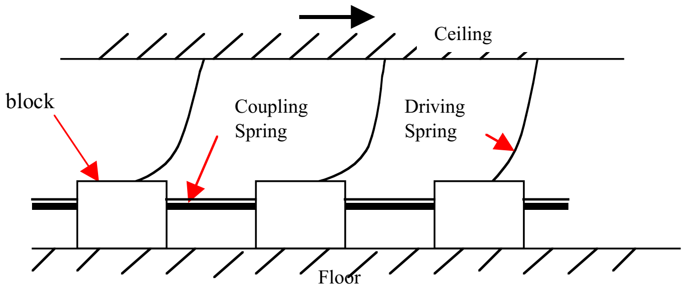


Fig. 1. Schematic of BK model.

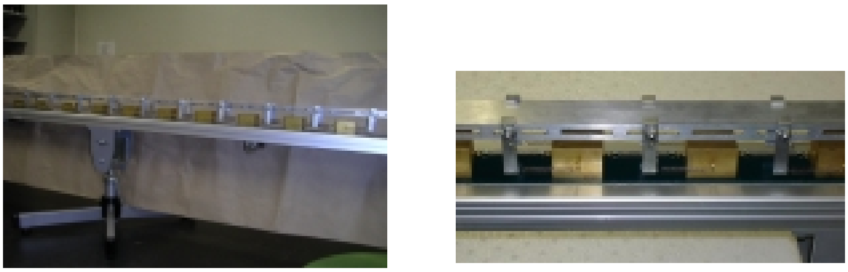


Fig. 2. Experimental devices.

behavior of the BK model system consisted of a small number of blocks, especially whether the Gutenberg-Richter's law holds or not. The purpose of this thesis is to report the methods of the concrete procedure and the results of these experiments.

2. Experimental Apparatus and Method

Ten blocks are arranged one-dimensionally and connected to each other with coupling springs and are placed on a belt conveyor that moves with constant velocity. These blocks are made of brass and their sizes are 39 mm × 30 mm × 30 mm. Each block is connected also to the rest ceiling with driving springs according to the configuration of BK model. Driving springs are connected to the side surfaces of each block in order to prevent unnecessary movement in perpendicular direction to that of belt conveyer velocity. Ten LED are fixed on top of each block in order to show the block positions clearly. The photographs of the experimental devices are shown in Fig. 2.

At first, all block intervals are set up equal to natural length of coupling springs. Next, the belt conveyer is switched on and begins to move with constant velocity. The video camera is set up in front of the system and the position of each block at every 1/30 second is recorded on video tapes.

Table 1. Parameter values used in the experiments.

Number	k_c (N/m)	k_d (N/m)	α	Maximum friction coefficient
Case 1	438	296	1.48	0.33–0.43
Case 2	438	296	1.48	0.33–0.43 (0.83 for Block 5)
Case 3	438	296	1.48	0.33–0.43 (0.83 for Block 2)
Case 4	438	296	1.48	0.33–0.43 (0.83 for Block 7)
Case 5	148	296	0.50	0.33–0.43 (0.83 for Block 5)
Case 6	148	296	0.50	0.33–0.43 (0.83 for Block 8)
Case 7	148	296	0.50	0.33–0.43 (0.83 for Block 3)
Case 8	148	296	0.50	0.33–0.43

The parameter values in the experiment are set up as follows. Mass of blocks m and driving spring constant k_d are fixed as $m = 0.300$ kg, $k_d = 296$ N/m, respectively. Coupling spring constant k_c was varied as 438 N/m and 148 N/m, namely, the spring constant ratio $\alpha = k_c/k_d$ as 1.48 and 0.50.

The belt conveyor velocity v_f used in the experiment was determined as follows.

The preliminary experiment was conducted for some values of v_f, and it was clarified that it was difficult for the blocks that once had fallen into the slip state to recover the stick state in case of large belt conveyor velocity. Then the minimum velocity for the conveyor to be required to move, $v_f = 1.7$ mm/s, was adopted for the experiment.

The maximum friction coefficient of each block was determined by the following method. First, one block is placed on a rest belt conveyor. One end of this block is connected to the fixed wall with a spring balance. Then the belt conveyor is set to move with very small velocity and the block begins to move with the same velocity as that of the conveyor. The spring balance has been elongated during the stick period of the block. These movements are recorded by the video camera, and the maximum value that the indicator of the spring balance points can be read from the video picture and is regarded as a maximum friction force between the block and the conveyor. These measurements were conducted multiple times for each block and the average values were used to determine the maximum friction coefficient of the block.

It should be noted that the maximum friction coefficients depend on the combination of the block and the contact part of the belt conveyor. As a result they distribute within certain ranges, though the contact surfaces of all blocks had been polished with same kinds of sandpaper. These values are within 0.33–0.43. We also set up another experimental condition so that the contact surface of one block had been polished with a finer sand paper in order to make its friction coefficient larger than others to make a trigger of slip events. These experimental conditions are summarized in Table 1.

3. Results of Experiments

3.1. Variation of position and classification into stick or slip for each block

The positions of each block at every 1/30 second were derived from the video-pictures by the use of the software "Dip-Motion". The variations of the positions of block 1–10 in

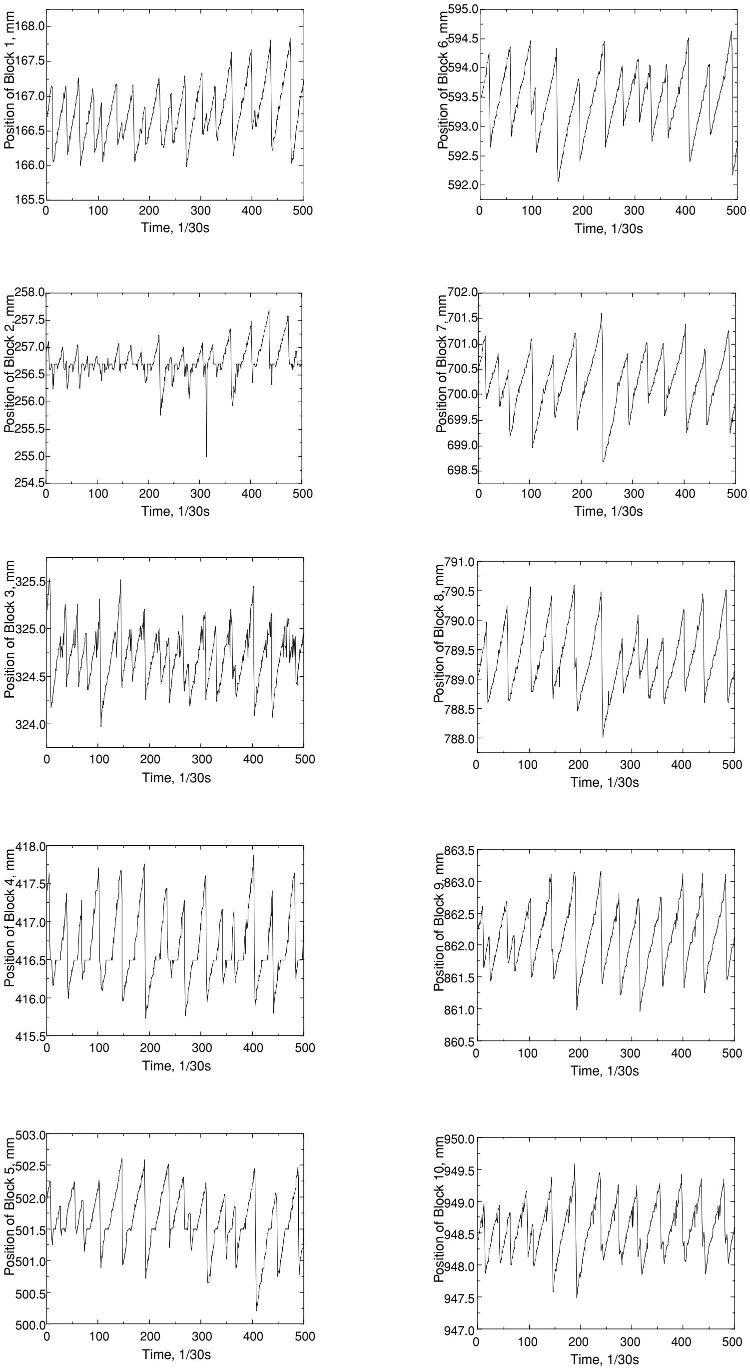
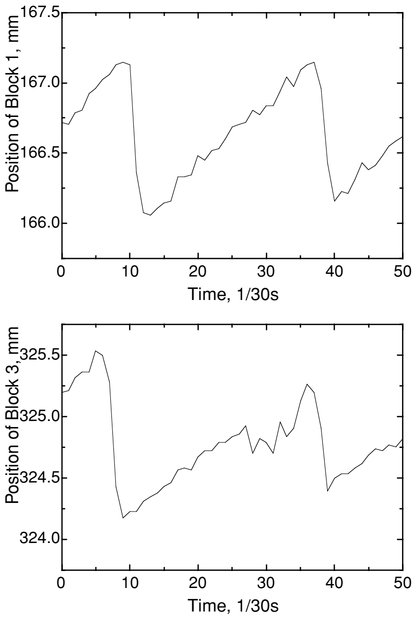


Fig. 3. Variation of position of each block in Case 8.

(a)



(b)

Time Unit	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	Maximum Cluster size	Total number of slipping block
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	1	0	1	0	0	0	0	0	0	1	2
7	0	1	1	1	0	0	0	0	0	0	3	3
8	0	1	1	1	0	0	0	0	0	0	3	3
9	0	1	1	1	1	0	0	0	0	1	4	5
10	0	0	1	0	1	0	0	0	1	1	2	4
11	1	0	1	1	1	0	0	0	1	0	3	5
12	1	1	0	1	1	0	0	0	1	0	2	5
13	1	1	0	1	1	0	0	0	1	1	2	6
14	1	1	0	1	0	0	1	0	0	1	2	5
15	0	1	0	0	0	0	1	0	0	1	1	3
16	0	0	0	0	0	0	1	0	0	1	1	2
17	0	0	0	0	0	1	1	0	0	0	2	2
18	0	0	0	0	0	1	1	1	0	0	3	3
19	0	0	0	0	0	1	0	1	0	0	1	2
20	0	0	0	0	0	1	0	1	0	0	1	2

Fig. 4. (a) Extended variation curve of the positions of block 1 and 3 during 1st–50th time units. (b) Classification map between Stick-state and Slip-state for all blocks at every time unit during 1st–20th time units. Stick-state is denoted as symbol "0" and Slip-state as "1". The number in the second right side column shows the maximum cluster size and the first right side the total number of blocks in Slip-state.

Case 8 in Table 1 are shown in Fig. 3.

From these figures we can see that these blocks oscillate irregularly and their amplitude are about 1–3 mm. Furthermore, we can regard approximately that motions of each block is consisted of major two parts, one is right-up part with a relatively small gradient and another right-down with steep gradient. The decision in which state (stick or slip) a block stay at each time must be made by the criterion that the velocity of the block is equal to that of belt conveyor or not. However, it is difficult to apply this criterion strictly to judge the state since some errors and inaccuracies are generated in data analyses process. For example, the actual length of the system that corresponds to one pixel on the screen at image sensing process was estimated as 0.75 mm in the present case, which brings the large vagueness to the decision of the block velocity at each time unit.

Then we adopted following criterion so that the right-up parts in the position variation curve was regarded as stick and right-down as slip. For example, during 1st–10th time units block 1 is in stick and during 11th–14th in slip (the extended variation curve of the positions of block 1 and 3 are shown in Fig. 4). In actual the average velocity of block 1 during 1st–10th time unit is about 1.38 mm/s, which is smaller than the belt-conveyor velocity 1.7 mm/s. This fact suggests that during 1st–10th time units small slip events actually occur,

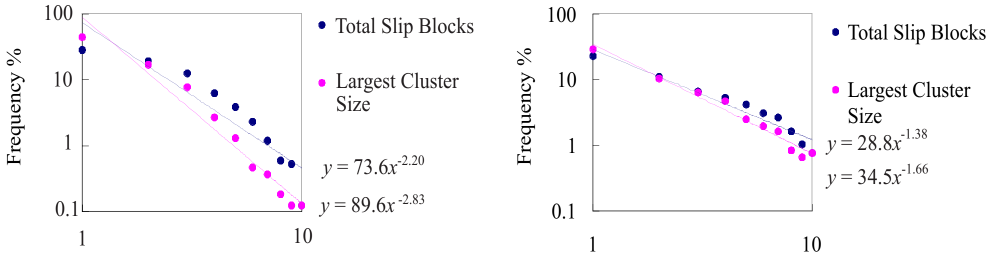


Fig. 5. Relationship between the size of slip event and its frequency in Case 8 and Case 1. Both axes are taken in log-scale.

however we regarded block 1 is in stick at every time unit during this period in order to simplify our analysis. Based on this method the classification map into stick (shown by the symbol “0”) or slip (symbol “1”) for all blocks at every time unit during the whole time period of the experiment were obtained for all cases. One part of this map in Case 8 is shown in Fig. 4(b). Thus we can obtain the pictures of stick-slip motions of the macroscopic system approximately. We should notice that the periods of stick-slip motion of each block are irregular.

3.2. Relationship between the size and the frequency

The size of the slip event was defined in two ways in the present research as follows. One is the total number of slip block at each time and another the maximum cluster size at each time. The numbers in the two right side columns in Fig. 4(b) denote these values respectively. The frequencies corresponding to these sizes of slip events were counted through all the time. The relationships between the size of the slip event S and its frequency $f(S)$ in Case 8 and 1 are shown in Figs. 5(a) and (b), respectively. We can see from these figures that the relationship

$$\log f = a - \delta \log S \tag{1}$$

holds approximately, which means the power law

$$f = cS^{-\delta}. \tag{2}$$

We obtained the value of power δ as 2.20 and 2.83 in Case 8, and 1.38 and 1.66 in Case 1. Similar results were obtained in other cases in Table 1. The relationships between δ and α in all cases in the experiment were plotted in Fig. 6. We can see from this figure that values of δ in Cases 1–4 (the spring constant ratio $\alpha = 1.48$) are clearly smaller than those in Cases 5–8 (the spring constant ratio $\alpha = 0.50$). This result shows that the larger the coupling spring constant is, the larger relative frequency of the large size slip events becomes. We could confirm this relationship between δ and α in the corresponding simulations, which is also shown in Fig. 6.

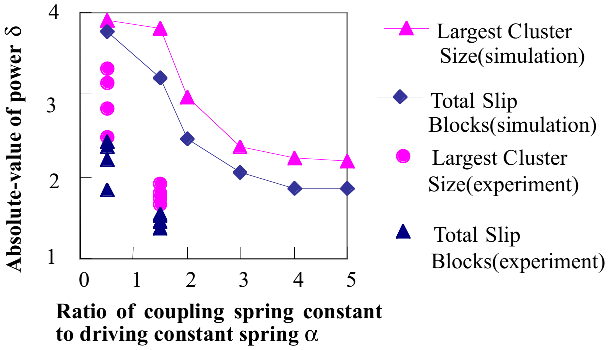


Fig. 6. Spring ratio dependence of the power in the experiments and simulation.

4. Conclusions and Discussions

The main results of the present researches are summarized as follows. First, the power law between the size of slip event and its frequency was obtained by the experiment based on BK model with ten blocks. Second it was clarified that the power is smaller in the case of large spring constant ratio.

Some comments are added here. First, we mention about the accuracy of the analyses. Since the range of the size of slip event is 10 in the present analysis, it is difficult to determine the fitting curve uniquely. We must admit that other kinds of fitting curves than those for power law are available in the present case. In this sense, we must say the power law is one of possible explanations at the present stage in the research.

The reason that the number of blocks is restricted as ten is as follows. One reason is that the total length of the system must not be too large in order to avoid the large inaccuracy accompanied by reading the display pixel. Another reason is the number of the subjects targeted in the analyses used with “Dip-Motion” is restricted. We think it is difficult to improve the data by increasing the number of the blocks due to these reasons.

However, we think it is possible to improve our data by changing the definition for the size of slip event. Two kinds of the definitions for size of the slip event were adopted in the present research, namely, one as the maximum cluster size and one as total number of blocks in slip. These definitions were adopted in order to simplify the analyses, however, we can adopt the sum of the displacements of the slip blocks as a definition for size of slip event. It is expected that this definition will bring the wider range of the size of the slip event than the present one, which makes results more precise. The analyses based on this definition are planned as a next subject.

Second, we consider about the fact that the behavior of the system is irregular. In the case of microscopic tribological research the experimental condition is strictly controlled so that the contact surface is almost perfectly uniform, and in these cases the stick-slip motion is possible only under the condition that both spring constant and floor velocity are small enough (PERSSON, 2000). On the other hand, in the macroscopic case the contact surfaces are composed of many small surfaces with different maximum friction coefficient. We think that even in these cases the stick-slip motion is capable. We regard this problem as follows: the maximum and sliding friction coefficient depend on the combination of the contact positions between one block and the belt conveyor. In these cases it is supposed that average friction coefficients of this block can be defined as

$$\mu_{\text{ave}} = \sum_i \mu_i f_i \Big/ \sum_i f_i \,,$$

where μ_i and f_i are friction coefficient of the i-th part of the surface and normal force applied to that part, respectively. In the present case the normal forces (block weights) are almost uniformly distributed then above expression reduces to

$$\mu = \sum_i \mu_i \Big/ N \,,$$

where N is a total number of the parts. Thus we can obtain the average friction coefficients of this block at one time. However, we should notice that this average friction coefficient distribute within some range since the contact parts of the conveyor varies with time during the experiment. This situation brings the result that the amplitudes and periods of successive stick-slip motion become irregular, and it is explained the irregularity of the behavior of the system shown in Fig. 3. Furthermore, it is also important to examine whether some critical values for spring constant and floor velocity within which the stick-slip motion is possible exist or not in the macroscopic case such as the present experiments. This problem is also remained as a future subject.

Third, we consider about effects of adopting the method of classifying the state of blocks into stick or slip mentioned in the Subsec. 3.1. We must notice that this method brings a result so that the size of slip event is underestimated than actual value. It is expected that if the more precise method is adopted than the present one, the number of small slip events will increase, and results that support the power law clearer will be obtained. Furthermore, it is also expected the more precise method will bring the larger value for the power δ in the experiments, which will be closer values to those of simulations than the present ones. We think an analysis based on the precise classification method must be conducted, and this subject is planned in near future.

REFERENCES

BURRIDGE, R. and KNOPOFF, L. (1967) Model and theoretical seismicity, *Bull. Seismol. Soc. Am.* **57**, 341–371.

CARLSON, J. M. and LANGER, J. S. (1989) Mechanical model of an earthquake fault, *Phys. Rev. A*, **40**, 6470–6484.

CARLSON, J. M., LANGER, J. S., SHAW, S. A. and TANG, C. (1991) Intrinsic properties of a Burridge-Knopoff model of an earthquake fault, *Phys. Rev. A*, **44**, 884–897.

DURAN, J. (2000) *Sands, Powders, and Grains*, Springer, pp. 138–147.

HUANG, J. and TURCOTTE, D. L. (1992) Chaotic seismic faulting with a mass-spring model and velocity-weakening friction, *PAGEOPH*, **138**, 569–589.

PERSSON, B. N. J. (2000) *Sliding Friction*, Springer, pp. 17–25.