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Analysis of Spiral Curves in Traditional Cultures

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Abstract. A method is proposed to characterize and classify shapes of plane spirals, and it is applied to some spiral patterns from historical monuments and vortices observed in an experiment. The method is based on a relation between the length variable along a curve and the radius of its local curvature. Examples treated in this paper seem to be classified into four types, i.e. those of the Archimedean spiral, the logarithmic spiral, the elliptic vortex and the hyperbolic spiral.

1. Introduction

Spiral patterns are seen in many cultures in the world from ancient ages. They give us a strong impression and remind us of energy of the nature. Human beings have been familiar to natural phenomena and natural objects with spiral motions or spiral shapes, such as swirling water flows, swirling winds, winding stems of vines and winding snakes. It is easy to imagine that powerfulness of these phenomena and objects gave people a motivation to design spiral shapes in monuments, patterns of cloths and crafts after spiral shapes observed in their daily lives. Therefore, it would be reasonable to expect that spiral patterns in different cultures have the same geometrical properties, or at least are classified into a small number of common types. This is the motivation of the present work. Since the present study is not supported by an accurate measuring method and does not include enough number of examples, it is presented as a Forum paper in order to express the basic idea of the authors.

In general, the simplest index characterizing curves is the radius of curvature. The curvature itself indicates how strongly the line is curved, and it changes if the curve is expanded or contracted without changing its shape. On the other hand, when we mention on a character of a curve, we do not observe its local parts but its total shape. Since the curvature of a curve, except a circle, varies with positions on the curve, it would be reasonable to assume that the character of total shape of a curve is related to how the curvature changes along the curve. This is the basic idea of this work. Then, the character of spiral shapes would be concerned to the relations between local curvature and the length along the spiral. In other words, the character is given by expressing the radius of curvature

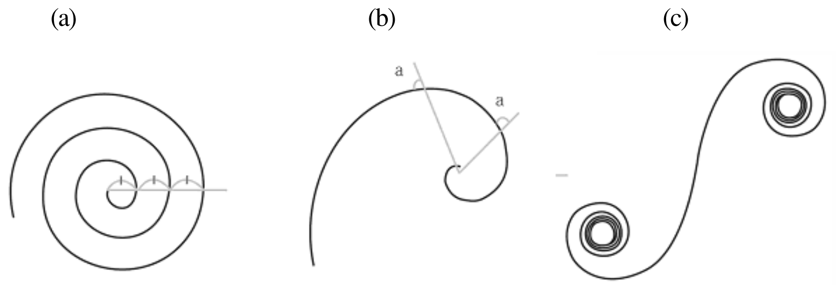

Fig. 1. Three basic spirals. (a) The Archimedean, (b) the logarithmic and (c) the Comu's spirals.

as a function of the length variable measured along the spiral curve. This method is explained in the next section, and results of its application to three geometrical spirals are shown. In Sec. 3 results for some spirals from archaeological monuments and crafts are shown, along with those for visualized vortex patterns obtained by the present authors. In the last section a proposition is made to classify spirals into four types.

2. Method of Analysis

It is assumed, as discussed above, that a spiral pattern is characterized by the variation of its curvature along the spiral curve. Although this idea is convincing, it is difficult to say that it is the best way of characterizing spiral curves. The present authors have assumed it, because the length and the curvature of spirals are easily measured.

Now, let a spiral curve is expressed by a distribution of the radius of curvature $R(l)$ as a function of the length along the curve l. In this work a center of a spiral curve was fixed by naked eyes, and the curve was divided into small arcs with 30 degree around the center. The radius of curvature was measured by fitting a curvature ruler (a plastic plate available at stationary shops), and the length was obtained by accumulating the lengths of the arcs (average distances of the arcs from the center multiplied by $\pi/6$). The origin of the length coordinate is adjusted later to compare several spiral curves.

In order to compare types of spirals with different sizes, their images were expanded or compressed to have common horizontal sizes, i.e. they were normalized, so that the l – R relations are compared in the same diagram.

Validity of the present method was examined by applying it to the three representative spiral curves (Fig. 1), the Archimedean, the logarithmic and Cornu's spirals (LOCKWOOD, 1961). The asymptotic forms of $R(l)$ for $l \to \infty$ for the Archimedean and the logarithmic spirals are $R(l) \sim al^{1/2}$ and $R(l) \sim al$, respectively, where a is a constant. The Cornu's spiral is similar to the logarithmic spiral, but the radius of curvature grows suddenly at a certain distance from the center of spiral.

Results of the curvature measurements are shown in Figs. 2(a), (b) and (c), where the measurement of Cornu's spiral was made only for its half part. In spite of a rather rough method of measurement these results show agreements with expected behavior. In fact the

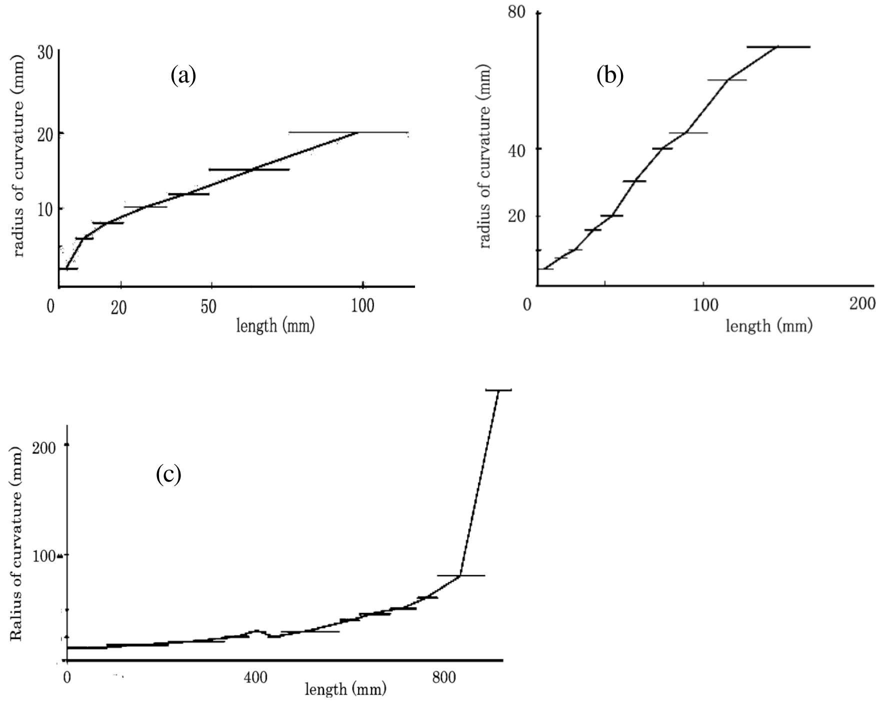

Fig. 2. The measured $R(l)$ curves for (a) the Archimedean, (b) the logarithmic and (c) the Comu's (hyperbolic) spirals. The horizontal bars show the lengths of arcs used in the measurement.

curves in Figs. 2(a) and (b) are nearly a parabola and a straight line, while that in Fig. 2(c) rises suddenly at a certain value of l. The spiral type with this kind of $R(l)$ curve is called here a hyperbolic type.

3. Applications to Vortex in Water Flow and Spirals in Traditional Cultures

The present method to characterize spiral curves was applied first to visualized vortices obtained experimentally, because the vortex in the water flow is considered to be one of common types of spiral patterns observed in the nature. The experimental setup is shown in Fig. 3. It was composed of a water tank of acrylic resin, $260 \times 300 \times 1200$ mm, a motor with variable rotation speed (Zeromax, assembled by Miki Pulley Co.) and a small truck running above the tank pulled by the motor. The truck carried a circular cylinder and a video camera (DCR-HC88, Sony Co.). The main part of the cylinder was held vertically in the water and driven with the truck. The vortices produced behind the cylinder were visualized by injecting a dye from the cylinder surface. The dye pattern was recorded by the video camera.

Figure 4(a) is one of the visualized vortices (one member in the Karman vortex street)

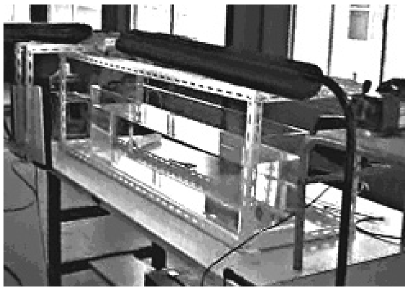

Fig. 3. Experimental setup for visualizing water vortex.

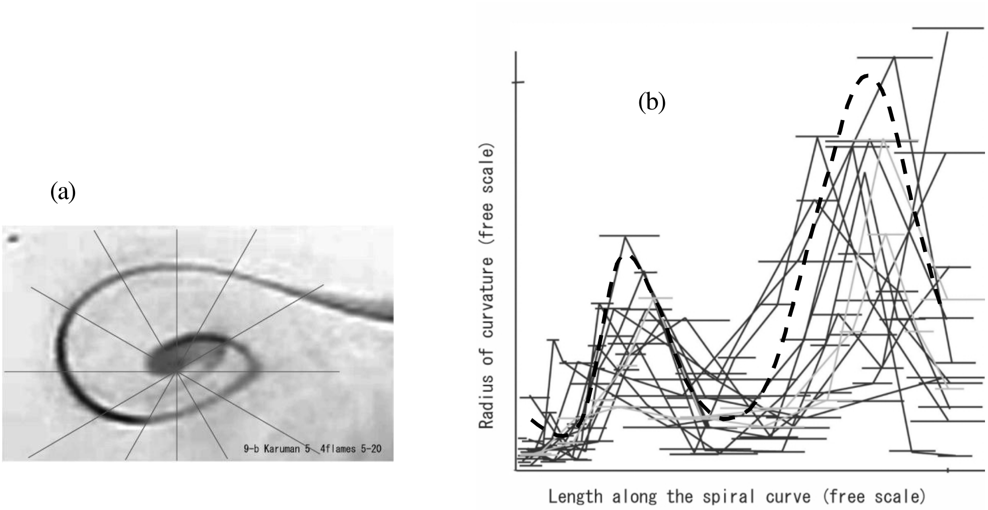

Fig. 4. (a) A visualized vortex, (b) $R(l)$ curves for visualized vortices behind a moving cylinder, where two gray curves are obtained from the vortex shapes visualized by TANEDA (1988). The dashed line indicates a common feature of these curves.

and Fig. 4(b) shows the measured curves of $R(l)$ for ten such vortices. These curves are rather scattered, but seem to have a common feature, i.e. they have two or more humps. The dashed line in this figure is drawn according to the impression of the authors. This feature is easily understood from the visualized vortex (Fig. 4(a)). It has an elliptical shape compressed in the up-and-down direction, hence the curvature is large at the right and left ends and the function $R(l)$ acquires a few humps. This type of spiral is called here an elliptic spiral.

Next, three spiral patterns were chosen from the monuments and crafts produced in ancient ages, as shown in Figs. 5(a), 6(a) and 7(a), and the present method was applied to obtain characteristics of the curves. Results of measurements are shown, respectively in

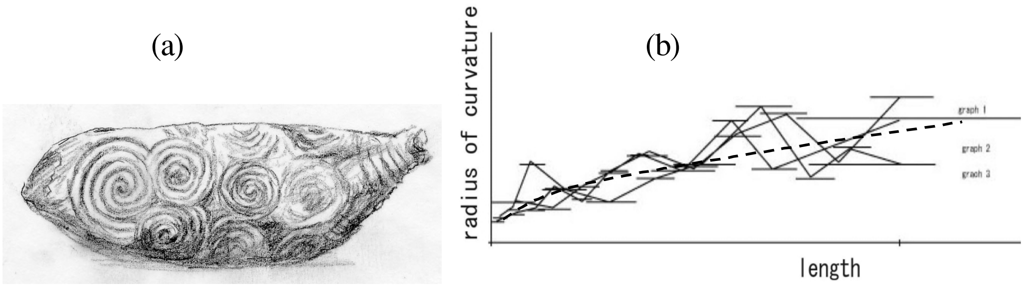

Fig. 5. (a) The stone at the New Grange ruin, Ireland, BC.30c. (sketch by one of the present authors, Takaki). (b) $R(l)$ curves for the three spirals on the stone (two in the upper left, one at the center), which suggest the Archimedean type mixed with the elliptic vortex type.

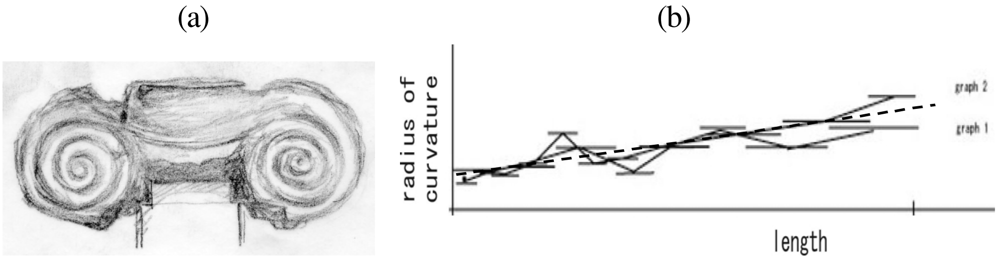

Fig. 6. (a) A capital of Ionic order, BC.7c. (sketch by one of the present authors, Takaki). (b) $R(l)$ curves for the two spirals, which suggest the logarithmic type.

Figs. 5(b), 6(b) and 7(b). The spirals in the stone of New Grange ruin shown in Fig. 5(a) (PURCE, 1974) is, roughly speaking, looked upon as Archimedean spirals, because a tendency to approach to horizontal lines are recognized for larger values of l in the $R(l)$ curves, as shown by a dashed line in Fig. 5(b). However, they have also a few small humps. Therefore, the possibility cannot be denied that these spirals include elements of elliptic vortex type. On the other hand, the spirals on the capital of Ionic order shown in Fig. 6(a) (SHIMIZU and KAMISAWA, 1999), are looked upon as logarithmic spirals (dashed line in Fig. 6(b)). The spirals in the middle Jomon pottery shown in Fig. 7(a) (SAITO and YOSHIKAWA, 1970) are considered to be of the Archimedean type with mixture of the elliptic vortex type (Fig. 7(b)). Note that the dashed lines in Figs. 5(b), 6(b) and 7(b) are based on the impressions of the authors.

4. Concluding Remarks

The present method to analyze spiral shapes in terms of the curvature-length curves seems to work well to characterize their types. It is noted here that the measurement of

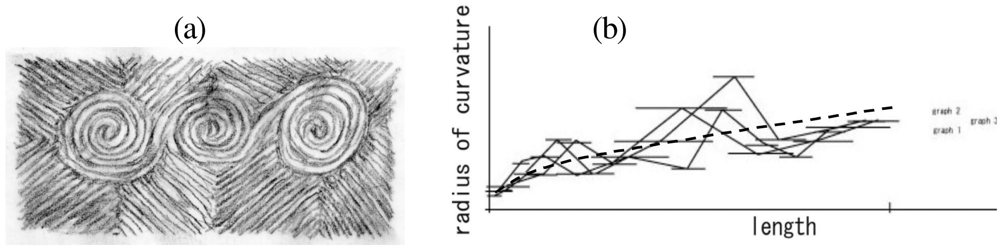

Fig. 7. (a) Pattern on a middle Jomon Pottery, BC.20–30c. (sketch by one of the present authors, Takaki). (b) $R(l)$ curves for the three spirals, which suggest the Archimedean type mixed with the elliptic vortex type.

curvature and length is not influenced by cultural or historical factors, and that judgments on types of spirals are free from prejudice or fixed ideas. In this sense the present method is reliable for classifying spiral shapes. On the other hand, there is a certain degree of ambiguity in identifying a type of each spiral from the curvature-length curve.

The present authors have tried some more examples of spirals from historical and cultural objects, and confirmed that they belong to, or at least lie near to, one of the three types, the Archimedean, the logarithmic and elliptic spiral types. However, there are also spirals which lie between two of these types. No example was found which clearly shows the hyperbolic type among twelve samples tested in this work (only three of them are shown in this paper). It is expected to find some in future. It depends also on the definition of sample. For example, the spirals on the capital (Fig. 6(a)) might be looked upon as a hyperbolic one, if the curve connecting two spirals is taken into account. It means that in this kind of works we must be careful in defining samples.

The present authors have an impression that spirals in historical or traditional objects are classified into four types, the Archimedean type, the logarithmic type, the elliptic spiral type and hyperbolic type. It will be interesting to remark that these types have correspondences to natural objects, which would have caught the minds of ancient people, as shown below.

Archimedean type: winding of a rope or a snake,
logarithmic type: shell,
elliptic vortex type: water flow with vortices,
hyperbolic type: stem or branch of a grass with winding end.

In the same way artists have applied spiral curves to their artworks. For example, Leonardo Da Vinci drew faithful sketches of water flows; one of them (about in 1507) seems to include vortices of logarithmic type or elliptic vortices.

Finally, a comment is given on the four categories assumed in this work. In general, a classification is possible when all members are divided into several groups based on a certain measure, where differences among these groups based on this measure are clearly recognized. In this work the measure is the shape of the $R(l)$ curve. At present it is not quite certain whether the number four is the best choice with respect to this measure. This problem is left for future studies.

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