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Pixel-Filling by Using Fibonacci Spiral

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Abstract. By examining the pixel-filling that was imitated in the row of a sunflower seeds and a pineapple ramenta, the possibility of the image expression used by the non-regular interval and pixel distribution was demonstrated. In the application of the pixel by Fibonacci spiral, it showed the possibility of transmitting information at high speed and prohibiting the moiré fringe.

1. Introduction

It is well known that the seeds of the sunflower are lined up making a spiral shape. For instance, the surface of a sunflower is filled with its seeds as a result of the seeds growing and making disk-like shape (Fig. 1(a)). The numbers of spiral lines are different for counter-clockwise (ccw) or clockwise (cw), and in this case, they are 89 for ccw and 144 for cw as Fibonacci number**) indicates. On the other hand, ramenta of a pineapple grow making a column-shape unlike seeds of a sunflower. In this case, the number of spiral is 8/13 (Fig. 1(b)). The sunflower seeds and pineapple ramenta are a few examples of phenomenon that are resulted by living things filling the space as they grow. It suggests that how each living thing fills spaces is closely related to how each life is formed, morphogenesis.

We discuss the space-filling theory by focusing on the growth of sunflower seeds and pineapple ramenta. The space-filling curve theory was first introduced by PEANO (1890). He described that space-filling curves can be explained by certain algorithm. Since then, numerous studies including application of the theory have been performed to gain further understanding. In most of cases, the space-filling curves show equal intervals between a certain point and point due to alogrithm (SAGAN, 1994). However, the intervals between points are not only equal intervals in sunflowers. These space-filling curves with unequal intervals have been studied (MATHAI and DAVIS, 1974) and discussed by AZUKAWA and YUZAWA (1990) and TAKAKI *et al.* (2003) in their recent papers.

**A natural number satisfy recursive rule $F_n = F_{n-1} + F_{n-2}$ ($F_1 = F_2 = 1$) as 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584 ···.

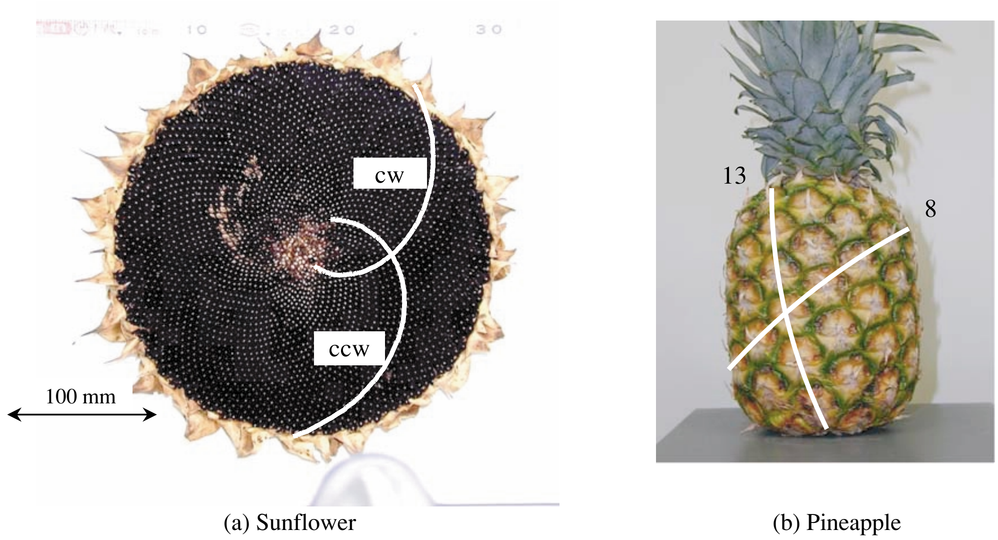


Fig. 1. Row of the sunflower seeds and pineapple ramenta.

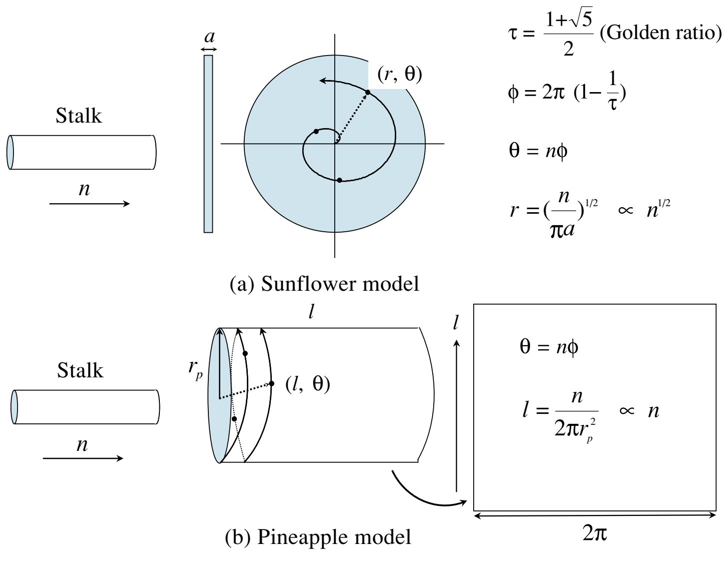
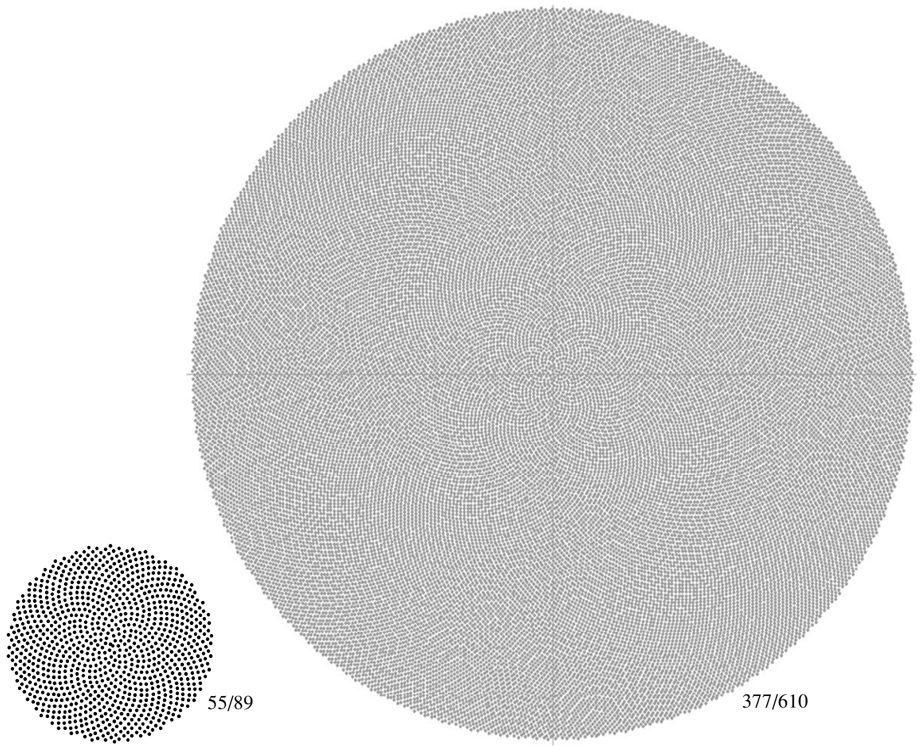


Fig. 2. Draw method of sunflower and pineapple model.

In this paper, the space-filling theory is discussed based on a study and observations of pixels pictures of spiral lines formed by sunflower seeds and pineapple ramenta, Fibonacci spiral.



(a) $n = 1000$ (b) $n = 65536$

Fig. 3. Pixels points are dispersed by sunflower model in plane all over.

2. Drawing Method

In these simulations, replicating the row of sunflower seeds is called "sunflower model", and replicating pineapple ramenta is called "pineapple model". Here after, each point are treated as pixels.

2.1. Sunflower model

When sunflowers grow, their seeds outflow from each stem with the amount of n; n is equal to the inflow amount per time unit. They form a disk-like shape with constant thickness a. Then, a radius, r, of each disk-like shape can be shown as, $r \propto n^{1/2}$. As the inflow amount is n, the drawing points (r, θ) are determined on polar coordinates along with ccw (Fig. 2(a)), where, τ is Golden ratio.

2.2. Pineapple model

When a pineapples grow, their ramenta inflow with the amount of n forming a column-like shapes. The growth length, l, of each column can be shown as $l \propto n$. Thus, the drawing

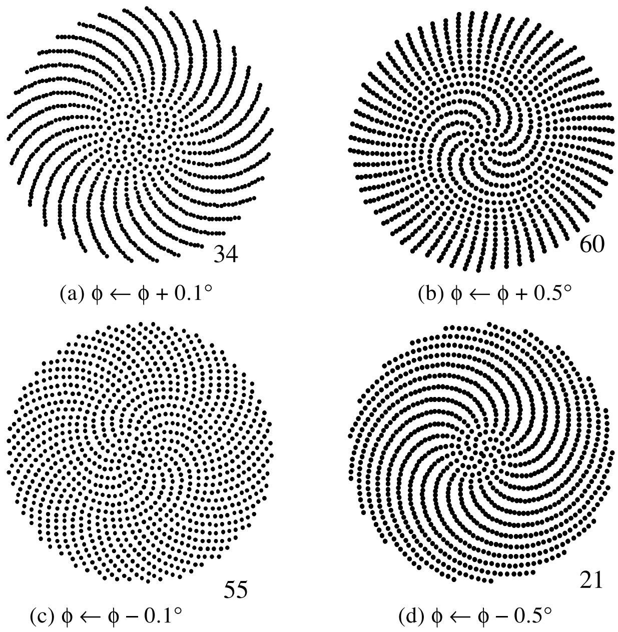


Fig. 4. Pixel distribution and spiral number by different ϕ. Each number of lower right are most probable spiral one of out side in case of n = 1000.

points are determined as (l, θ) (Fig. 2(b)). In this case, the drawings of the cylinder base are ignored until it grows a certain size.

3. Simulation of the Pixel Drawings

3.1. Sunflower model

Positions of the each point are drawn by the method mentioned in Fig. 2(a) after computing $r = n^{1/2}$ and $\theta = n\phi$ using n. The pixels are dispersed in the constant plane, and the distribution of pixels coincide with the well known sunflower seeds spirals. For instance, when n = 1000, the distribution of pixels result as shown in Fig. 3(a) whose two spirals agree with Fibonacci number 55/89. As shown in Fig. 3(b), which is the pixel distribution of 65536 utilizing τ of significant figure 15, the pixels are homogeneously dispersed. Fibonacci number for this distribution is 377/610. However, only slight change in ϕ can cause different form of pixel distribution; so that, the number of spirals at the certain distance outside of a center will not be Fibonacci number (Fig. 4(b)).

3.2. Pineapple model

Like the sunflower model, each drawing point is, first, calculated from $n = l$ and $\theta = n\phi$. Each point is, then, drawn one by one over the rectangle that is made by cutting through the cylinder surface of pineapple model (Fig. 5(a)). The pixels are homogeneously

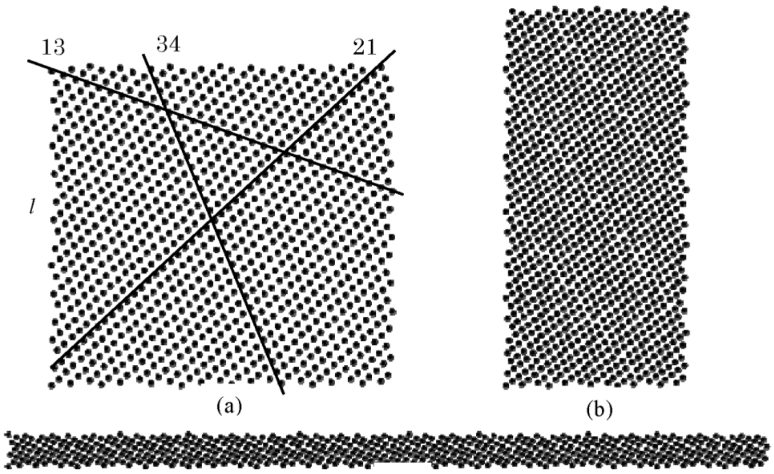


Fig. 5. Pixel distribution by pineapple model in case $n = 1000$. The number of upper side in (a) is one of parallel lines at illustrated line in lap 2π.

distributed within the certain size rectangle. It is important to notice that the distribution of pixels is not affected by changes in aspects of rectangular ratio (Figs. 5(b) and (c)). The numbers in (a) are shown the number of lines that are parallel to the illustrated lines with lap 2π. Just like the sunflower model, slight changes in ϕ causes different form of the pixel distribution (Fig. 6).

4. Tendency of the Pixel Distribution

In each model, a quantity s can be rewritten as $s = c_k n^x$ (c_k is constant) if s represents growth quantity of both r (sunflower model) and l (pineapple model). In sunflower model, the pixels are uniformly dispersed and held constant when $x = 0.5$ (Fig. 7(b)). In other cases, not only are the pixels uniformly distributed but also the number of spiral lines (Fibonacci number) observed outside of the distribution is different from on another (Figs. 7(a), (c) and (d)).

In pineapple model, the pixels are uniformly dispersed over the surface when $x = 1$ (Fig. 7(g)) while the pixel distributions can not be uniformly dispersed when $x \neq 1$. Therefore, when $\phi = 2\pi(1 - 1/\tau)$, the dispersion of pixels is determined by x.

In pineapple model, the pixel dispersion tendency is analyzed. The number of pixels is counted for 100, 1000, 10000 points within 360°, and the pixels are counted for every 2° (Fig. 8). The scale at the left side, A, shows the number of pixels when 10000 point, and the scale at right side, B/C, shows the number of pixels when 1000 and 100 points.

When 10000 points were examined, 54–57 pixels were observed every 2°. Likewise, 5–7 pixels were observed when 1000 points were examined, and 0–1 pixels were observed when 100 points were observed. Each pixel was homogeneously dispersed in each degree.

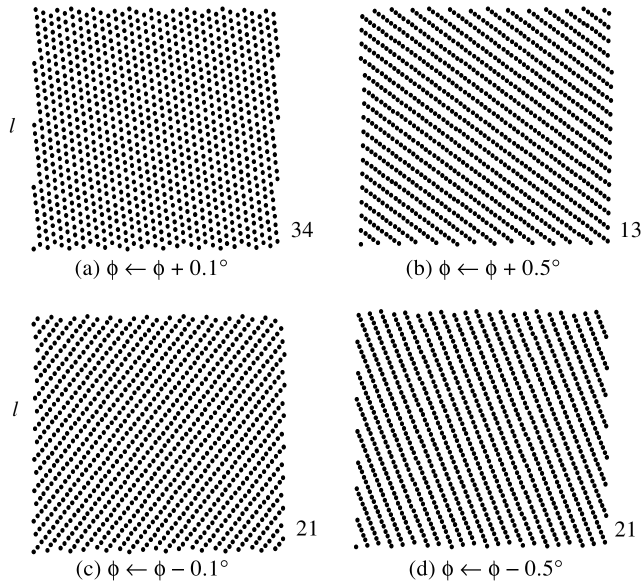


Fig. 6. Appearance of changing distribution when different ϕ. The number of lower side in each figure is one of parallel lines at illustrated line in lap 2π. These are Fibonacci number all.

As a result, since l is constant, the pixel dispersion over the surface is not affected by changes in ratio aspect. Moreover, intervals between pixels are the non-regular intervals.

5. Application to the Image Processing

In order to improve the information transmission speed for image expression and distinction, we are required to randomly sample pixels from an object image. The pixel-filling method by the sunflower and pineapple model enables us to sample pixels with the arbitrary pixel number from an arbitrary area. An image shown in Fig. 9(a) is the original image drawn by conventional pixels with regular intervals, and images (b) through (g) are drawn by used pixel-filling method. The pineapple model is used in the image (b) through (e), and the sunflower model is used in image (f) and (g). (b), (c) and (d) are images drawn by sampling 30000, 25637 and 2557 points from the original (a), respectively. Images in (e) are drawn by sampling 9973 points from arbitrary. Images in (f) and (g) are drawn by sampling 10000 and 9973 points with sunflower model method from an arbitrary area.

6. Summary

The characteristics and applications of the pixel are discussed by using Fibonacci spiral which is represented by sunflower seeds. The results show that in the sunflower model, pixels are dispersed in plane all over. On the other hand, in the pineapple model, random pixels are extracted not depending on the aspect ratio. That suggests the possibility

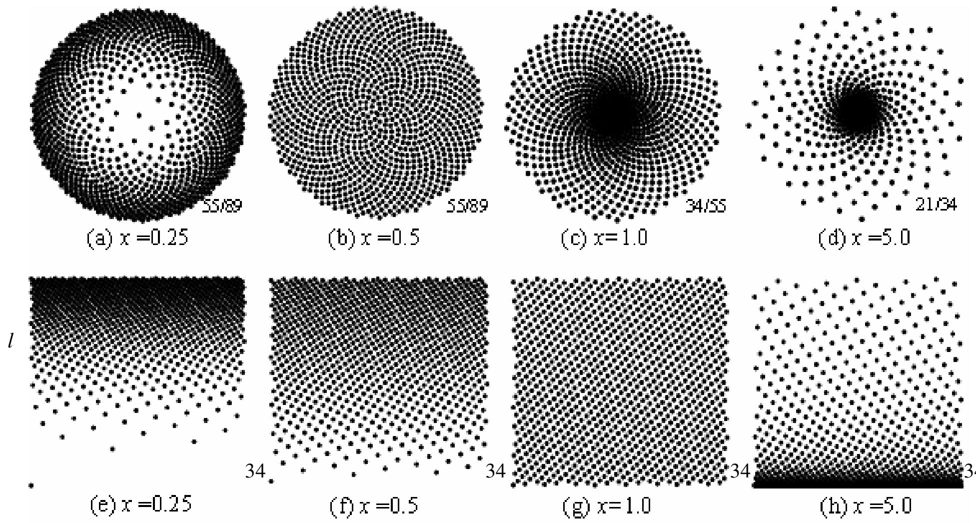


Fig. 7. Variety of the pixel distribution by x in n = 1000. (a)–(d) are by sunflower model, (e) and (f) pineapple model. Each number of lower rights is spiral one in each figure, and those are Fibonacci number all.

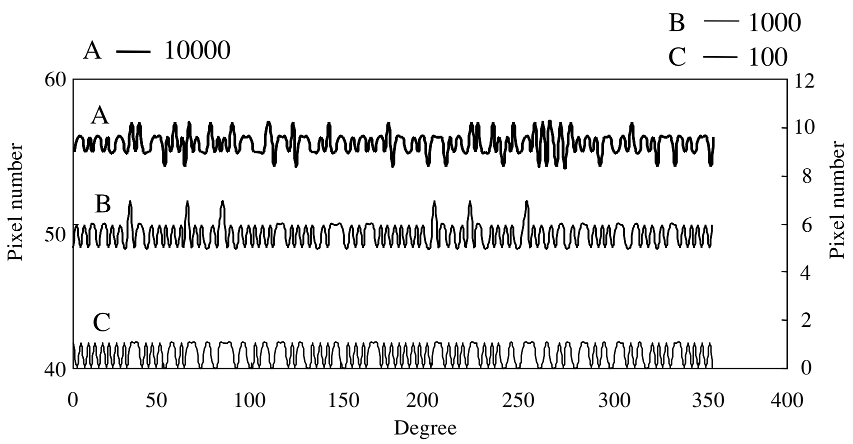


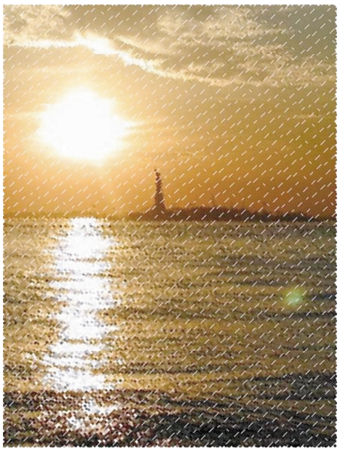
Fig. 8. Pixel distribution with each sampling number. A is for 10000 points, B 1000, C 100, respectively.

of expressing and recognizing information of an image at higher speed. Also, it is possible that moiré fringe not to occur on printed images because of the pixels with non-regular intervals.

Therefore, the results above are believed to be useful to built better printing devices since expressing and recognizing speeds could be higher and moiré fringe free images could be made if Fibonacci spiral is applied.

(a) Original image (256376 pixels)

(b) 30000 points



(c) 25637 points

(d) 2557 points

(e) 9973 points

(f) 10000 points

(g) 9973 points

Fig. 9. Example images of pixel samplings from an arbitrary area. (a) is the original image. “Pineapple model” was applied as to (b), (c), (d) and (e), and “Sunflower model” was applied as to (f) and (g).

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