Optimal Location and Opening Hours of a Single Facility which Maximally Cover Flows in a Circular City

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This paper proposes a maximum flow-covering location model with a time dimension. A facility which provides service for a given duration in a day is assumed. The number of potential customers is defined as the number of commuters on their way home from work that can fully consumes the service from start to end and can arrive home by a given time. The problem seeks the location and the start time of the service which covers as many commuters as possible. This model can be applied to various situations in which the selection of service providing time greatly influences the number of customers captured.
Key words: Facility Location, Flow-Covering, Circular City, Time-Dependent

1. Introduction

In the areas of operations research and management science, various facility location models have been proposed. Most of these location models only consider the spatial configuration of facilities which optimizes certain objective function (Daskin, 1995; Drezner, 1995; Drezner and Hamacher, 2001).

There are some location problems which explicitly consider the temporal factors into account (Drezner and Wesolowsky, 1991; Current *et al.*, 1998; Snyder, 2006; Farahani *et al.*, 2008). Most of these dynamic models incorporate some aspects of future uncertainty such as changes of demands, market trend over time, and consider the timing of locating facilities in the planning horizon. Although, these models consider the temporal axis over the long term, the model focusing on facility management on a daily basis has not been sufficiently addressed so far.

Tanaka proposed a single facility location model in a linear city considering not only the location of the facility but also the opening hours of the facility within a day (Tanaka, 2006). In this paper, two dimensional extension of Tanaka's model is proposed by considering the problem within a circular city. The model is formulated as a flow-covering location problem in which demand for service occurs not from the demand points but from customers travelling between two points within a city. The model assumes commuters on their way home from work as potential customers for the service. To describe temporal variation of demands, departure time distribution of commuters is introduced. Under these assumptions, the model to find the location of the facility and the start time for service with a given duration that maximizes the number of potential customers is constructed.

In the following section, we will call the proposed problem *the concert problem* to describe the situation intuitively. This name comes from the situation that organizers of a concert are considering the location of a concert hall and start time of a concert which attract as many commuters as possible.

2. The Model and Basic Assumptions

In this section, we consider the concert problem in a circular city of radius R. Let us consider the situation in which the organizer of the concert which lasts c hours is considering where to find a concert hall and when to start the concert. For the organizer, it is desirable to select the location and start time so that lots of people can attend the concert.

As shown in Fig. 1, the location of a concert hall is denoted by (z, θ) using polar coordinates with its origin at the city center. Let τ be the start time of the concert. We consider the problem of deciding the optimal location of the concert hall (z^*, θ^*) and the best start time of the concert τ^* which maximize the number of potential customers. The definition of potential customers is given as follows: commuters that can attend the concert from start to end and can go back home by a given time t_h. If τ is too small, few people can be in time for the start time; on the other hand, if τ is too large, few people can be back home by t_h after attending the concert. Because of this trade-off relationship, there exists the optimal start time of the concert τ^*.

Let us introduce the temporal axis perpendicular to the circular city as shown in Fig. 2. We denote the location of a workplace and a home by (s_1, ϕ_1) and (s_2, ϕ_2) respectively with those origins at the concert hall (z, θ). The duration of concert time c is assumed to be a fixed constant, so that the concert plan can be represented by a point (z, θ, τ) in the space-time region. In Fig. 2, the movement of a potential customer in the space-time region is shown. The following assumptions are made:

(i) The number of commuters having workplaces in unit area at (s_1, ϕ_1) and having homes in unit area at (s_2, ϕ_2) is given by $\rho(s_1, \phi_1, s_2, \phi_2)$;

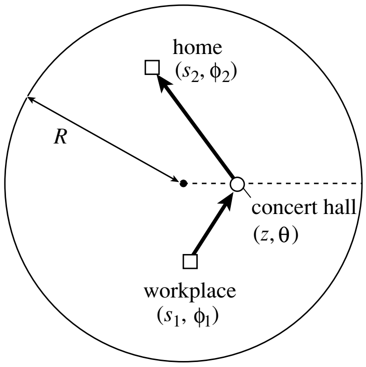

Fig. 1. A circular city of radius R.

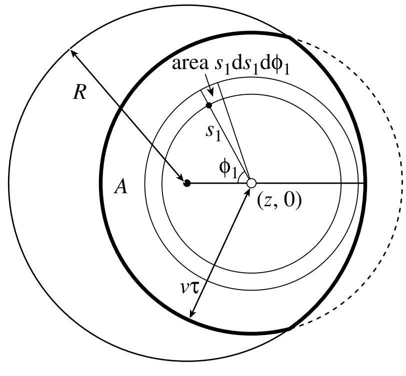

Fig. 3. Small region from which the travel time to the concert hall is s_1/v.

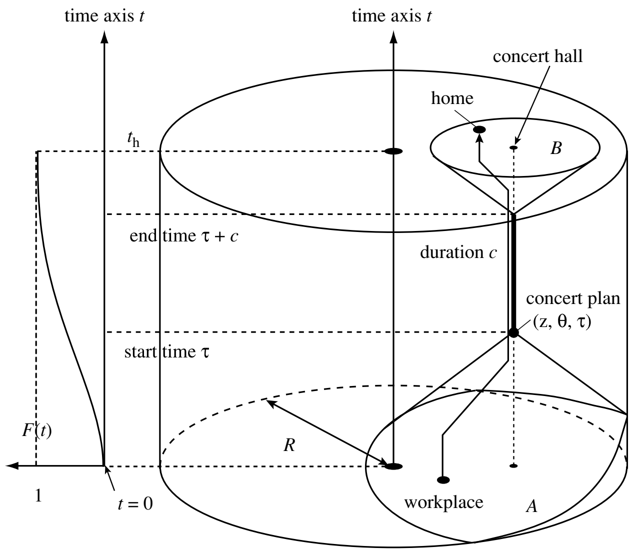

Fig. 2. Movement of a potential customer in the space-time region.

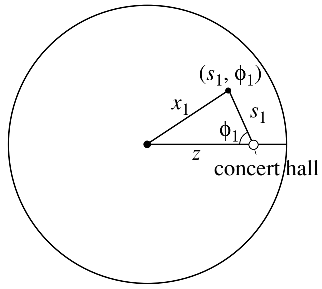

Fig. 4. Transformation of coordinates from (x_1, ψ_1) to (s_1, ϕ_1).

(ii) The cumulative distribution of the departure time t of commuters is given by $F(t)$;
(iii) The distance between any two points is measured by Euclidean distance;
(iv) The traveling speed of commuters is given by a fixed constant v.

The function $\rho(s_1, \phi_1, s_2, \phi_2)$ is called the trip density which is the continuous counterpart of the origin-destination (OD) matrix (Vaughan, 1987). By the definition of trip density, the number of commuters having workplaces within a small region at (s_1, ϕ_1) with area $s_1 ds_1 d\phi_1$ and having homes within a small region at (s_2, ϕ_2) with area $s_2 ds_2 d\phi_2$ is given by

$$\rho(s_1, \phi_1, s_2, \phi_2) s_1 s_2 ds_1 d\phi_1 ds_2 d\phi_2.$$

The departure time distribution of commuters means the proportion of commuters that can leave their workplace by time t. In this paper, we assume that $F(t)$ is not dependent on the location of workplace and home. The time $t = 0$ corresponds to the time of the first commuter leaves one's workplace.

3. Formulation

In this section, we formulate the concert problem. Let us denote the number of potential customers by $n(z, \theta, \tau)$ as a function of the location of a concert hall and the start time. The aim of the concert problem is to find the location and the start time of the concert which maximize this function. If the end time $\tau + c$ exceeds t_h no one can go back home by t_h so $\tau + c$ must be before t_h. This leads that the start time τ must be determined in the range $\tau \in [0, t_h - c]$.

In the following, the objective function $n(z, \theta, \tau)$ is derived. Let us consider the condition that a given commuter be a potential customer. To be a potential customer, the following two conditions have to be met:

(i) Workplace and home of a given commuter must be in region A and region B respectively as shown in Fig. 2.
(ii) This commuter has to leave workplace early enough to be in time for the start time of the concert τ.

The region A is the set of points (workplaces) from which the travel time to the concert hall is within τ, while the region B is the set of points (homes) to which the travel time from the concert hall is within $(t_h - \tau - c)$. By the assumption of constant travel speed, region A represents the intersection of the circle of radius $v\tau$ centered at (z, θ) and the circular city and region B represents the intersection of the circle of radius $v(t_h - \tau - c)$ centered at (z, θ) and the circular city.

Let us consider the condition (i). First, focus on the set of commuters S whose workplaces are within a small region in A with area $s_1 ds_1 d\phi_1$ and whose homes are within a small area in B with area $s_2 ds_2 d\phi_2$. The number of commuters in S is given by $\rho(s_1, \phi_1, s_2, \phi_2) s_1 s_2 ds_1 d\phi_1 ds_2 d\phi_2$ by the

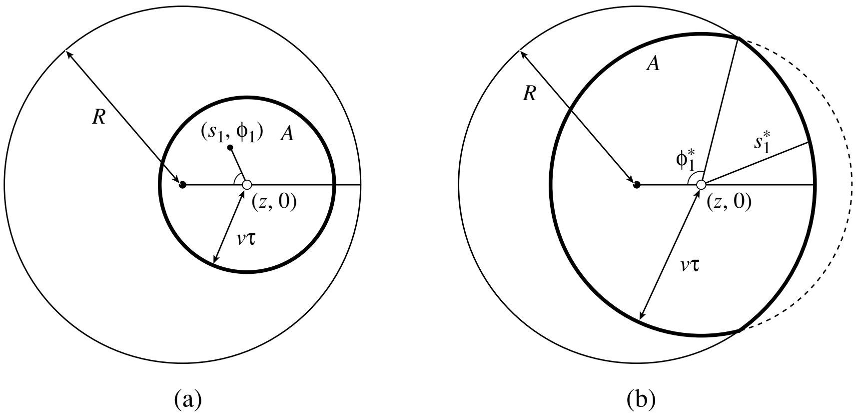

Fig. 5. The set of workplaces from which the access time to the concert hall is within τ as represented by region A. (a) $0 \leq \tau \leq \frac{R-z}{v}$, (b) $\frac{R-z}{v} \leq \tau \leq \frac{R+z}{v}$.

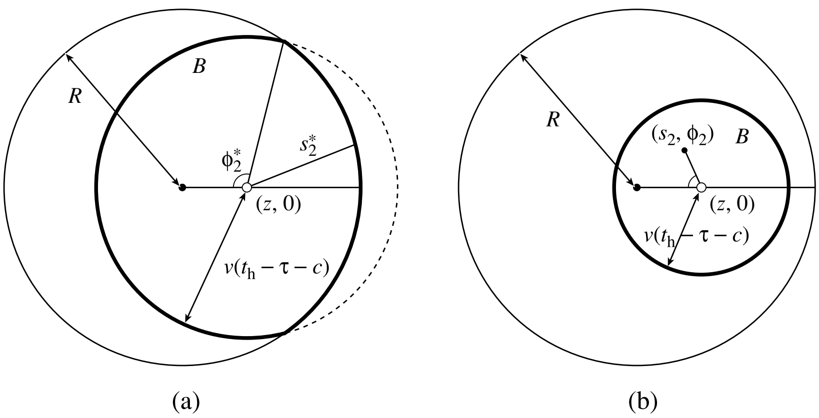

Fig. 6. The set of homes to which the access time from the concert hall is within $t_h - \tau - c$ as represented by region B. (a) $t_h - c - \frac{R-z}{v} \leq \tau \leq t_h - c$, (b) $0 \leq \tau \leq t_h - c - \frac{R-z}{v}$.

definition of trip density.

Next, we focus on the number of potential customers in S by considering the condition (ii). For a given commuter in S to be a potential customer, this commuter must be in time for the start time of the concert τ. This condition can be restated that the commuter can leave their workplaces by

$$t = \tau - s_1/v,$$

considering that the travel time from workplace to the concert hall is given by s_1/v as shown in Fig. 3. Because the proportion of commuters that can leave their workplace by $t = \tau - s_1/v$ is given by $F(\tau - s_1/v)$, the number of potential customers in S is given as follows:

$$F\,(\tau - s_1/v) \cdot \rho(s_1, \phi_1, s_2, \phi_2)\, s_1 s_2 \mathrm{d}s_1 \mathrm{d}\phi_1 \mathrm{d}s_2 \mathrm{d}\phi_2. \quad (1)$$

The total number of potential customers in the whole city $n(z, \theta, \tau)$ can be obtained by integrating Eq. (1) within region A and region B:

$$n(z, \theta, \tau) = \iint_A \iint_B F\left(\tau - \tfrac{s_1}{v}\right) \cdot \rho(s_1, \phi_1, s_2, \phi_2) \cdot s_1 s_2 \mathrm{d}s_1 \mathrm{d}\phi_1 \mathrm{d}s_2 \mathrm{d}\phi_2. \quad (2)$$

From the above discussions, the concert problem in a circular city can be formulated as follows:

$$\max_{z,\theta,\tau}.\ n(z, \theta, \tau) \quad (3)$$
$$\text{s. t.}\quad 0 \leq z \leq R, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \tau \leq t_h - c.$$

Given $F(t)$ and $\rho(s_1, \phi_1, s_2, \phi_2)$, Eq. (2) is calculable in princple. The analytical calculation of the objective function, however, is complicated even under simple cases. In addition, since the domain of integration changes depending upon the set of parameter and variable values, determining the domain of integration for each possible set of parameter and variable values is almost intractable. By expressing Eq. (2) more explicitly, however, the value of $n(z, \theta, \tau)$ can be calculated numerically. In the following, the expression of the objective function more suitable for numerical integration will be presented.

4. Calculation of the Objective Function

In this section, we assume that workplaces and homes are independently distributed. Let us denote the location of a workplace and a home by (x_1, ψ_1) and (x_2, ψ_2) respectively by using polar coordinates with its origin at the city center. From the independence assumption, trip density can be described as follows (Vaughan, 1987):

$$\rho(x_1, \psi_1, x_2, \psi_2) = N \cdot \lambda(x_1, \psi_1)\mu(x_2, \psi_2), \quad (4)$$

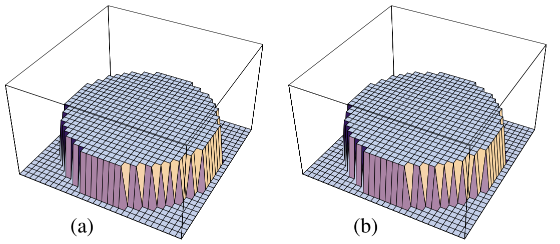

Fig. 7. Distribution of workplaces (a) and homes (b) for case 1.

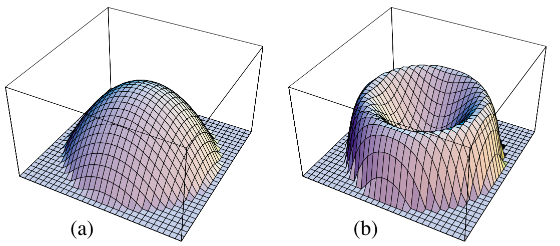

Fig. 8. Distribution of workplaces (a) and homes (b) for case 2.

where N represents the total number of commuters, $\lambda(x_1, \psi_1)$ and $\mu(x_2, \psi_2)$ denote the number of workplaces and homes per unit area at (x_1, ψ_1) and (x_2, ψ_2), respectively. It should be noted that the origin of polar coordinates in Eq. (4) is different from that of Eq. (2); the origin of the former is the city center while that of the latter is the concert hall. In the following analysis, we assume that workplaces and homes are radially sysmetric; densities of workplaces and homes are expressed as a function of the distance from the city center only:

$$\lambda(x_1, \psi_1) = \lambda(x_1), \quad \mu(x_2, \psi_2) = \mu(x_2). \tag{5}$$

To carry out the integration in Eq. (2), trip density must be expressed as a function of $(s_1, \phi_1, s_2, \phi_2)$ instead of $(x_1, \psi_1, x_2, \psi_2)$. As illustrated in Fig. 4, the transformation of coordinates of workplaces from (x_1, ψ_1) to (s_1, ϕ_1) is explained. Using the law of cosines, x_1 can be related to s_1 and ϕ_1 as follows:

$$x_1^2 = s_1^2 + z^2 - 2s_1 z \cos\phi_1. \tag{6}$$

Similarly for homes, x_2 can be related to s_2 and ϕ_2:

$$x_2^2 = s_2^2 + z^2 - 2s_2 z \cos\phi_2. \tag{7}$$

Using these relationship, the densities of workplaces and homes can be written as follows:

$$\lambda(x_1) = \lambda\left(\sqrt{s_1^2 + z^2 - 2s_1 z \cos\phi_1}\right), \tag{8}$$

$$\mu(x_2) = \mu\left(\sqrt{s_2^2 + z^2 - 2s_2 z \cos\phi_2}\right). \tag{9}$$

From the above discussions,

$$\rho(s_1, \phi_1, s_2, \phi_2) = N \cdot \lambda\left(\sqrt{s_1^2 + z^2 - 2s_1 z \cos\phi_1}\right) \cdot \mu\left(\sqrt{s_2^2 + z^2 - 2s_2 z \cos\phi_2}\right). \tag{10}$$

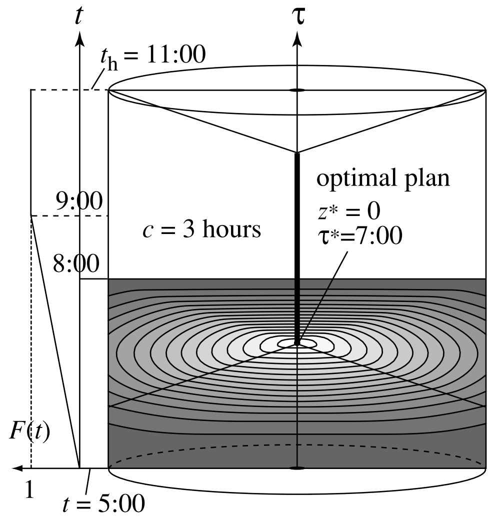

Fig. 9. Contour plot of $n(z, \theta, \tau)$ for case 1.

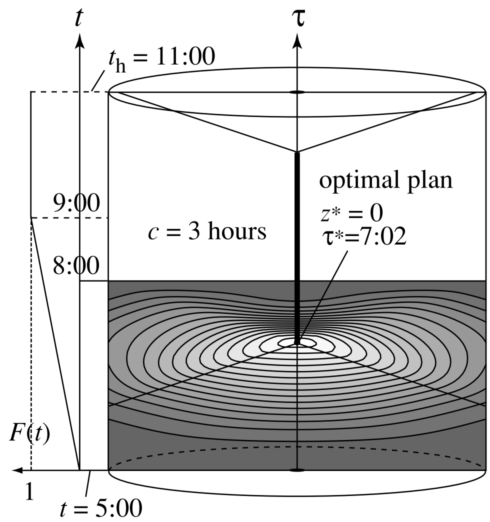

Fig. 10. Contour plot of $n(z, \theta, \tau)$ for case 2.

By expressing trip density as a function of $(s_1, \phi_1, s_2, \phi_2)$ using the relationship shown in Eq. (10), $n(z, \theta, \tau)$ can be calculated by carrying out the integration in Eq. (2) in principle. The analytical calculation of the integral, however, is very complicated. Depending on the set of parameter and variable values, the shape of region A and region B varies as illustrated in Figs. 5 and 6. The domain of integration of Eq. (2) is given by the direct product of region A and B. Therefore, there are four shapes of the domain of integration: A-(a) and B-(a), A-(a) and B-(b), A-(b) and B-(a) and A-(b) and B-(b). In Eq. (11), Eq. (12), Eq. (13) and Eq. (14), the expressions of the objective function corresponding to the above four cases are presented. Although it is very difficult to specify which of the four cases occurs for each possible parameter and variable set, the domain of integration can be easily specified given a parameter and variable set. Consequently, the objective function can be calculated by numerical integration.

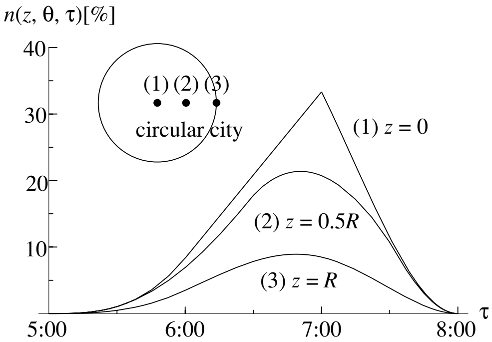

Fig. 11. Plot of $n(z, \theta, \tau)$ as a function of start time τ at three different points in the city for case 1.

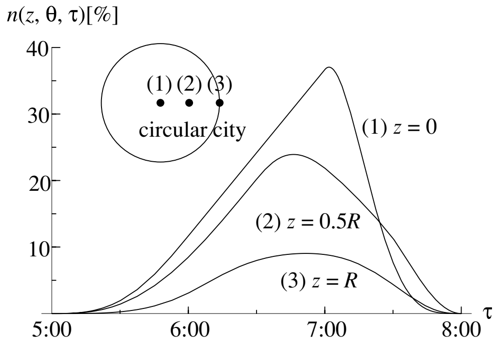

Fig. 12. Plot of $n(z, \theta, \tau)$ as a function of start time τ at three different points in the city for case 2.

A-(a) and B-(a)

$$\begin{aligned} n(z,\theta,\tau) = 4 &\int_{\phi_1=0}^{\pi}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\phi_2^*}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \\ &+4\int_{\phi_1=0}^{\pi}\int_{s_1=0}^{v\tau}\int_{\phi_2=\phi_2^*}^{\pi}\int_{s_2=0}^{s_2^*} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \end{aligned} \tag{11}$$

A-(a) and B-(b)

$$\begin{aligned} n(z,\theta,\tau) = 4 &\int_{\phi_1=0}^{\pi}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\pi}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \end{aligned} \tag{12}$$

A-(b) and B-(a)

$$\begin{aligned} n(z,\theta,\tau) = 4 &\int_{\phi_1=0}^{\phi_1^*}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\phi_2^*}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \\ &+4\int_{\phi_1=0}^{\phi_1^*}\int_{s_1=0}^{v\tau}\int_{\phi_2=\phi_2^*}^{\pi}\int_{s_2=0}^{s_2^*} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \\ &+4\int_{\phi_1=\phi_1^*}^{\pi}\int_{s_1=0}^{s_1^*}\int_{\phi_2=0}^{\phi_2^*}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \\ &+4\int_{\phi_1=\phi_1^*}^{\pi}\int_{s_1=0}^{s_1^*}\int_{\phi_2=\phi_2^*}^{\pi}\int_{s_2=0}^{s_2^*} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \end{aligned} \tag{13}$$

A-(b) and B-(b)

$$\begin{aligned} n(z,\theta,\tau) = 4 &\int_{\phi_1=0}^{\phi_1^*}\int_{s_1=0}^{v\tau}\int_{\phi_2=0}^{\pi}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \\ &+4\int_{\phi_1=\phi_1^*}^{\pi}\int_{s_1=0}^{s_1^*}\int_{\phi_2=0}^{\pi}\int_{s_2=0}^{v(t_h-\tau-c)} F\left(\tau-\tfrac{s_1}{v}\right) \\ &\cdot\rho(s_1,\phi_1,s_2,\phi_2)s_1 s_2 \mathrm{d}s_2\mathrm{d}\phi_2\mathrm{d}s_1\mathrm{d}\phi_1 \end{aligned} \tag{14}$$

where

$$\begin{aligned} s_1^* &= z\cos\phi_1 + \sqrt{z^2\cos^2\phi_1 + R^2 - z^2}, \\ \phi_1^* &= \arccos\left(\frac{z^2 + v^2\tau^2 - R^2}{2v\tau z}\right), \\ s_2^* &= z\cos\phi_2 + \sqrt{z^2\cos^2\phi_2 + R^2 - z^2}, \\ \phi_2^* &= \arccos\left(\frac{z^2 + v^2(t_h-\tau-c)^2 - R^2}{2v(t_h-\tau-c)z}\right). \end{aligned} \tag{15}$$

5. Numerical Examples

In this section, we first consider the following two radially symmetric models for densities of workplaces and homes.

case 1

$$\lambda(x_1) = \frac{1}{\pi R^2}, \qquad \mu(x_2) = \frac{1}{\pi R^2}, \tag{16}$$

case 2

$$\lambda(x_1) = \frac{2}{\pi R^2}\left(1 - \frac{1}{R^2}x_1^2\right), \quad \mu(x_2) = \frac{6}{\pi R^4}x_2^2\left(1 - \frac{1}{R^2}x_2^2\right). \tag{17}$$

The case 1 is the simplest model assuming that workplaces and homes are uniformly distributed within a city as shown in Fig. 7. The case 2 assumes that workplaces are densely distributed at the center and homes are densely distributed at some distance from the city center.

In the example below, the departure time distribution is given by a linearly increasing function from 5:00 p.m. to 9:00 p.m. as shown in Figs. 9 and 10. We assume the following parameter values: $c = 3$ hours, $2R/v = 2$ hours, $t_h = 11:00$ p.m. Under these assumptions, the objective function $n(z, \theta, \tau)$ is calculated by numerical integration for various values of (z, θ, τ).

Figures 9 and 10 show the contour plot of $n(z, \theta, \tau)$. From these figures, the unique optimal solution can be found. In both cases, the optimal location of the concert hall is at the city center: $z^* = 0$. In the case of Fig. 9, the optimal start time satisfies the distinctive property: the best plan is to start concert as late as possible while satisfying that every commuters attendable to the concert can also go back home by t_h.

Figures 11 and 12 show $n(z, \theta, \tau)$ as a function of τ at three different points in the city. The interesting point to note is that the optimal start time depends upon the location

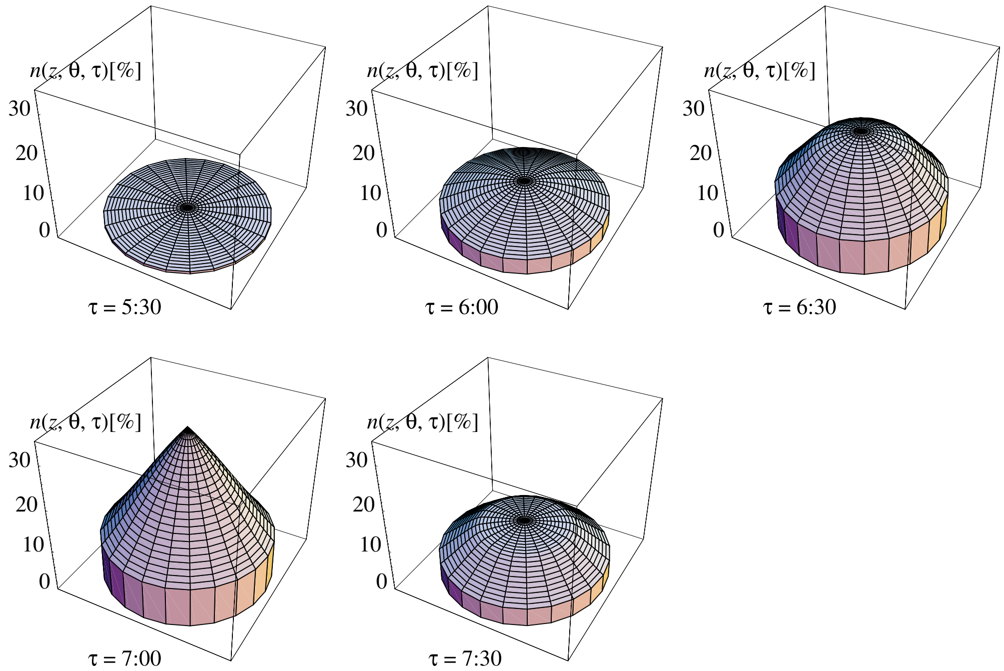

Fig. 13. 3D plot of $n(z, \theta, \tau)$ as a function of the location of the concert hall at various start time for case 1.

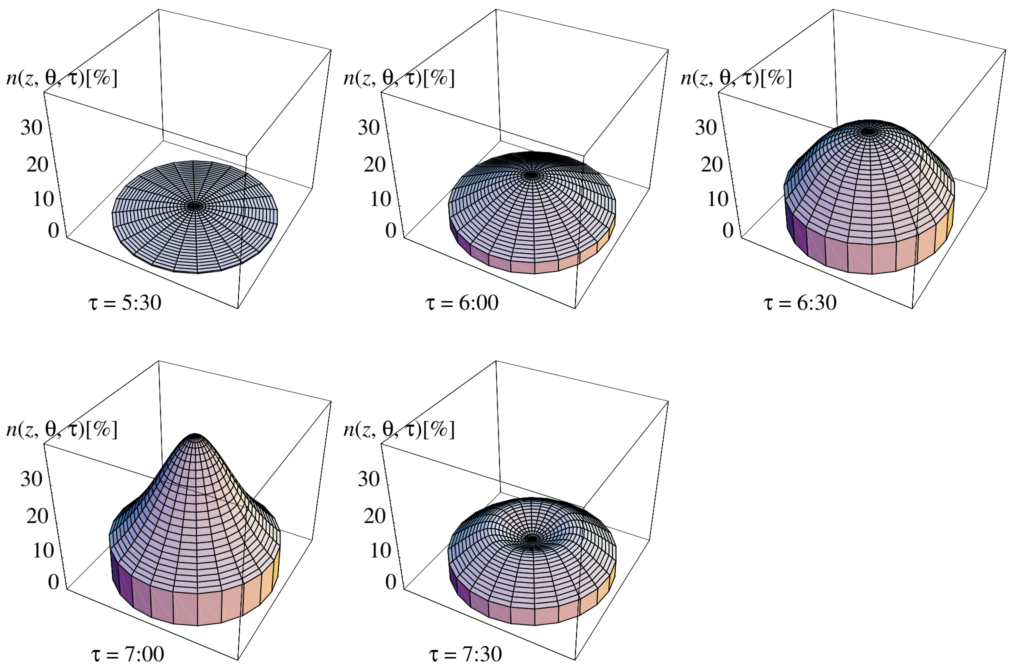

Fig. 14. 3D plot of $n(z, \theta, \tau)$ as a function of the location of the concert hall at various start time for case 2.

of the concert hall. As can be seen from Fig. 12, the city center is not a good place for the concert when τ is large. This is because that density of homes near the city center is low; the number of commuters that can go back home by t_{h} after the end of the concert $\tau + c$ is small.

In Figs. 13 and 14, $n(z, \theta, \tau)$ is shown as a function of locations at various start time of the concert for case 1 and case 2.

As the next example, we consider the case in which the optimal location is not at the city center but the point some distance from the center. Figure 15 shows the contour of $n(z, \theta, \tau)$ and Fig. 16 shows the plot of $n(z, \theta, \tau)$ at three different points in the city when $c = 5$ for case 2. Other parameter values are the same as those of Fig. 9 to Fig. 12. Interesting point to note is that the city center is not the best place to find a concert hall. There exist points around the densely populated areas that more people can go back home by t_{h} after the end of the concert in comparison with the city center.

Next, we investigate the effect of concentration of trip origins and destinations at the city center on $n(z, \theta, \tau)$. In Figs. 17 and 18, we compare the value of $n(z, \theta, \tau)$ at

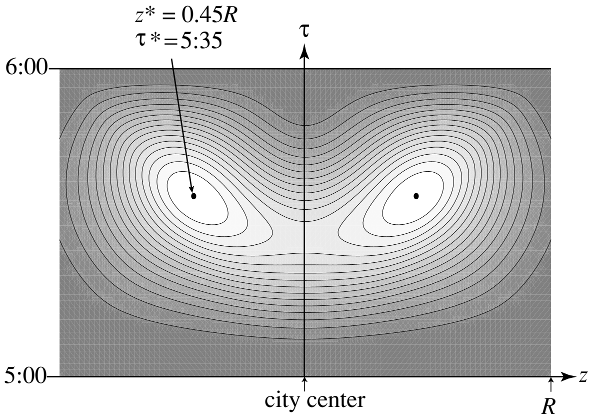

Fig. 15. Contour of $n(z, \theta, \tau)$ in which the optimal location is not at the city center.

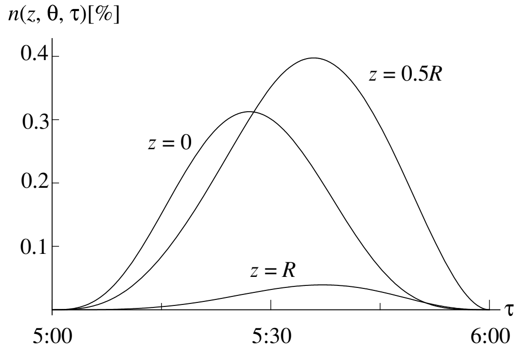

Fig. 16. Plot of $n(z, \theta, \tau)$ in which the optimal location is not at the city center.

$z = 0$ and $z = 0.5R$ respectively between case 1 and case 3 in which both workplaces and homes are more densely distributed at the city center:

case 3

$$\lambda(x_1) = \frac{2}{\pi R^2}\left(1 - \frac{1}{R^2}x_1^2\right), \quad \mu(x_2) = \frac{2}{\pi R^2}\left(1 - \frac{1}{R^2}x_2^2\right). \tag{18}$$

As can be seen from these figures, the concentration of origins and destinations at the city center makes the value of $n(z, \theta, \tau)$ larger. This result indicates that concentration of trip origins and destinations at the city center is advantageous in terms of accessibility.

6. Future Works

In this paper, the continuous model of the maximum flow covering location problem with temporal axis is proposed. There are various future directions of this work.

Multi-facility location problems should be considered. In this case, an algorithm to find optimal solutions should also be developed.

The proposed model can be formulated as an integer programming problem by constructing the discrete version of this model. This model can be considered as an extended version of the maximal flow covering problem by (Berman, 1997).

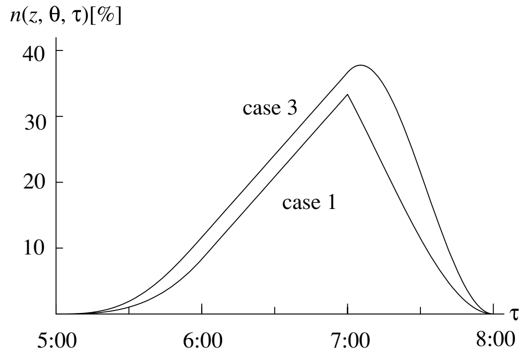

Fig. 17. Comparison of $n(z, \theta, \tau)$ at $z = 0$ between two different trip densities.

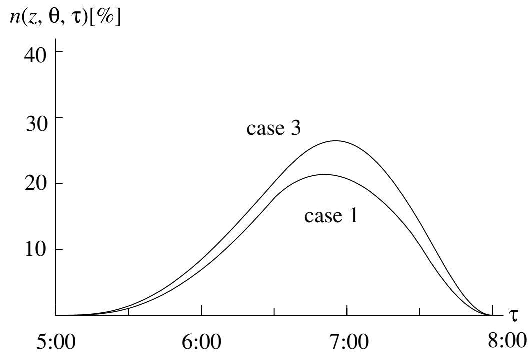

Fig. 18. Comparison of $n(z, \theta, \tau)$ at $z = 0.5$ between two different trip densities.

Another possible variation of this model is the cost minimization problem. By introducing the space-time cost for accessing the service, the problem seeking the location and the start time which minimizes the total space-time cost can be constructed.

Examining the impact that a change of the departure time distribution will have on the number of potential customers is also important. This approach can be used to evaluate the effects of flexible working hours on the space-time accessibility of commuters to various services.

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References

Berman, O. (1997) Deterministic flow-demand location problems, *Journal of Operational Reserach Society*, **48**, 75–81.

Current, J., Ratick, S. and ReVelle, C. (1998) Dynamic facility location when the total number of facilities is uncertain: A decision analysis approach, *European Journal of Operational Research*, **110**, 597–609.

Daskin, M. S. (1995) *Network and Discrete Location: Models, Algorithms, and Applications*, Wiley-Interscience.

Drezner, Z. (ed.) (1995) *Facility Location: A Survey of Applications and Methods*, John Wiley & Sons, New York.

Drezner, Z. and Hamacher, H. W. (eds.) (2001) *Facility Location: Applications and Theory*, Springer, New York.

Drezner, Z. and Wesolowsky, G. O. (1991) Facility location when demand is time dependent, *Naval Research Logistics*, **38**, 763–777.

Farahani, R. Z., Drezner, Z. and Asgari, N. (2008) Single facility location and relocation problem with time dependent weights and discrete planning horizon, *Annals of Operations Research*.

Snyder, L. (2006) Facility location under uncertainty: a review, *IIE Transactions*, **38**, 547–564.

Tanaka, K. (2006) A maximum flow-covering location model considering the time dimension, *Papers of the Annual Conference of the City Planning Institute of Japan*, **41**, 265-270 (in Japanese).

Vaughan, R. J. (1987) *Urban Spatial Traffic Patterns*, Pion Limited.