Nambu-Goldstone Mode and Spatiotemporal Chaos

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It is well known that the Nambu-Goldstone (NG) mode accompanying spontaneous breaking of a continuous symmetry plays important roles in subatomic physics and in condensed-matter physics. Very recently new aspect has been discovered in nonlinear hydrodynamic systems related to NG modes. Here we introduce such a typical but, up to now, unique example which is observed in electroconvection in liquid crystals. The nonlinear interaction between the Nambu-Goldstone mode and the convective one induces a new type of spatiotemporal chaos, the so-called soft-mode turbulence.

Key words: Spontaneous Breaking of Symmetry, Nambu-Goldstone Mode, Spatiotemporal Chaos, Soft-Mode Turbulence

1. Spontaneous Breaking of Symmetry and Phase Transition

The Nobel Prize in Physics 2008 was awarded to Professor Yoichiro Nambu for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics (Nambu and Jona-Lasinio, 1961a, b). The importance of symmetry breaking is not limited in subatomic physics. Symmetry breaking occurring in various scales from microscopic to macroscopic has formed the diversity of the universe. If phase transitions from lower temperature phase to higher temperature one occurs, generally the symmetry of the low temperature phase is broken. Thus symmetry breaking is closely related to various phase transitions.

Let us consider symmetry breaking and phase transition by taking a smectic liquid crystal as an example (de Gennes and Prost, 1993). There are several kinds of liquid crystals that consist of rodlike molecules whose statistical averaged orientation is called the director. The smectic liquid crystal is characterized by the layer structure and many types are classified into depending on molecular orientation and symmetry within the layer (Fig. 1). In those, the smectic A phase has the highest symmetry in which the molecules are perpendicular to the layer (Fig. 1(a)). With decrease of temperature, the molecules tilt to an arbitrary direction with a constant angle below a transition point (Fig. 1(c)). This phase is called the smectic C. The top view of the molecular orientation (the director) during transition from the smectic A to C can be described as follows (see also Fig. 1). The smectic A phase has a continuously rotational symmetry against z-axis, that is the director projection onto x–y plane becomes a point (Fig. 1(b)). In smectic C, it has the tilt director and we define the two-dimensional vector field $\mathbf{C}(\mathbf{r})$-director characterizing the local direction in the smectic C layer. Then the continuously rotational symmetry of $\mathbf{C}(\mathbf{r})$ is broken, because the direction of the $\mathbf{C}(\mathbf{r})$ is not a point but aligns to some directions (Fig. 1(d)). As a result, the symmetry breaking occurs spontaneously. Note here that there is no restriction to determine the direction of the $\mathbf{C}(\mathbf{r})$ because any direction costs equal energy as long as with the same tilt angle.

Let us suppose that perturbation for the direction of the $\mathbf{C}(\mathbf{r})$ is given to the ground state. If the wavenumber k ($\neq 0$) of the perturbation is given, distortion in the $\mathbf{C}(\mathbf{r})$ is induced. Since liquid crystals have elasticity, the relaxation to uniform $\mathbf{C}(\mathbf{r})$ occurs in order to remove the distortion. With making k smaller, the relaxation time τ becomes longer. The perturbation with $k = 0$ does not relax, namely $\tau \to \infty$, since the elastic distortion is not induced. Such a mode with $\tau \to \infty$ for $k \to 0$ is called as the *Nambu-Goldstone* (NG) *mode*.

Generally speaking, in what kind of condition does the NG mode appear? The reason why the perturbation with $k = 0$ does not relax is because the direction of the $\mathbf{C}(\mathbf{r})$ can be freely chosen without any additional cost of energy. Thus this arbitrariness of direction of $\mathbf{C}(\mathbf{r})$ in the smectic C phase results from the continuously rotational symmetry. That is, NG modes appear when a continuous symmetry is spontaneously broken in a short-range interaction system.

Spontaneous breaking of a continuous symmetry also occurs in pattern formation in dissipative systems. Let us take, as an example, the Rayleigh-Bénard convection in fluids heated from below. The spontaneous breaking of the continuously translational symmetry occurs in infinitely large case, when a stripe pattern forms by the periodic arrangement of convective rolls. In this case, the phase of the stripe pattern behaves as a NG mode. The NG mode influences slow motion of defects in the pattern selection process.

2. Nambu-Goldstone Mode in a Homeotropic Nematic System

Nematic liquid crystals (NLCs) are characterized by orientational order without layer structures. In NLCs, applying an electric field beyond a threshold voltage V_c, convection starts. This is called the electrohydrodynamic instability

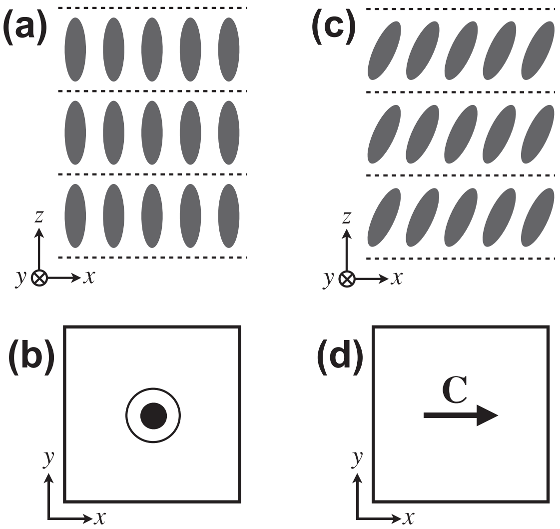

Fig. 1. Smectic liquid crystal. The broken lines show layer structures. (a), (b) Smectic A phase. (c), (d) Smectic C phase.

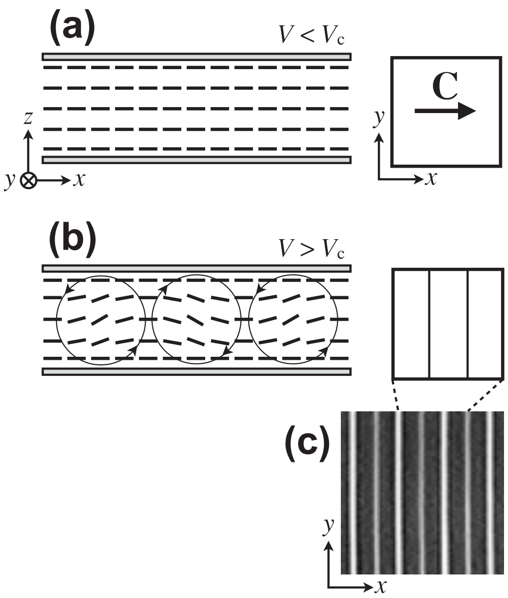

Fig. 2. Planar alignment system of a nematic liquid crystal. (a) Initial state. (b) Electroconvective state. (c) Convective pattern observed under a polarizing microscope.

and explained by the Carr-Helfrich effect (de Gennes and Prost, 1993). The electroconvection in nematic liquid crystals has been supplied rich variety of subjects of nonlinear dynamics because it has many advantages for experimental and theoretical researches comparing to conventional fluid systems, e.g., Rayleigh-Bénard convection systems. Now, let's describe briefly how to set our sample. A nematic liquid crystal is sandwiched between two electrodes and the direction of the molecules (the director) is aligned parallel to the electrodes. By a surface treatment of the electrodes, e.g., polymer coating and rubbing procedure along the x-direction, the directors can be aligned to one direction. This surface treatment artificially breaks the continuous rotational symmetry in the electrode plane, such as the x–y direction, that is, the x and y directions have not

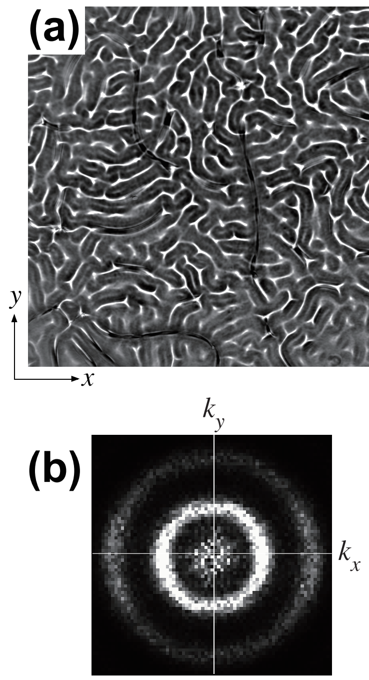

Fig. 3. (a) Pattern of the soft-mode turbulence. (b) Two-dimensional spectrum of (a).

equal property each other. This system is called the planar alignment. The direction of the convective rolls due to the electrohydrodynamic instability here is perpendicular to the initial alignment of the directors (Fig. 2). Therefore, the wavevector $\mathbf{q}$ of the stripe pattern becomes uniformly parallel to the x-direction. In electroconvection of nematic liquid crystals, generally convective flow leads an anisotropic torque on the $\mathbf{C}$ because of the viscous anisotropy. In the planar system, since the rotation of the $\mathbf{C}$ by the viscous torque is suppressed by the initial breaking of the rotational symmetry due to the rubbing, the $\mathbf{C}(\mathbf{r})$ and the convective stripe pattern can be stable at primary state above V_c.

If the directors are aligned perpendicular to the electrodes, what happens now? This state traditionally called the homeotropic alignment (HA) in the liquid crystal fields. HA becomes unstable for an electric field beyond a threshold voltage V_F, the Fréedericksz transition point (de Gennes and Prost, 1993), at which the directors tilt against the electrode. In some sense, this aspect has a good analogy to the transition from smectic A to C phase though their intrinsic physics is different. Then the continuous rotational symmetry in the the x–y plane is spontaneously broken, since the tilt direction is arbitrary as previously described. Consequently, the fluctuation of the azimuthal angle ϕ of the $\mathbf{C}$ can be regarded as a NG mode similar to that in the smectic C phase (note, we are talking about only one layer of smectic C as an anology). Further increasing the control parameter, electroconvection occurs at a secondary threshold voltage V_c. Now note also that this state is dissipative while Fréedericksz state is not. In this system, two NG modes which originate respectively from the rotational and the translational symmetries coexist. In other words, a short-wavelength mode of electroconvection coexists with the NG mode of ϕ. In the homeotropic system, the rotation of the **C** behaves as a NG mode and the **C** can rotate by the viscous torque. The **q** also has to rotate to follow the **C** so that the electroconvection can be maintained. This means that the nonlinear interaction between the short-wavelength mode and the NG one make the convective pattern unstable. As a result, the convective pattern become disordered as shown in Fig. 3(a). However, the regular convection is locally kept and azumuzal angle becomes random in time, namely $|\mathbf{q}(\mathbf{r})| = \text{const}$. Such motions are shown as a ring in the two-dimensional spectrum (Fig. 3(b)), and indicate spatiotemporal chaos related to the direction of **q**. This type of spatiotemporal chaos was named as *soft-mode turbulence* (SMT), since the softening of the fluctuation can be observed approaching to the bifurcation point (Kai *et al*., 1996; Hidaka *et al*., 2006). Both patterns shown in Figs. 2 and 3 are obtained using the same liquid crystal and same field conditions. Nevertheless, the convective pattern and its dynamics in the homeotropic system is drastically different from those in the planar system. This means that symmetry plays quite important roles in emergence in dissipative structures as well as in phase transition in condensed matters and subatomic systems. In well known route of pattern developments, an ordered state appears at a primary threshold, complex and chaotic states appears after successive bifurcations. The homeotropic electroconvection is not the case and therefore quite nontrivial. This is clearly related to the NG mode via the symmetry problem of the system.

3. Soft-Mode Turbulence in Other Systems

Angular momentum of **C** is a conserved quantity in the Fréedericksz state. On the other hand, the Navier-Stokes equation is invariant under the Galilean transformation. Therefore, in a layer of normal fluid, long-wavelength fluid flow in the horizontal plane has long relaxation time due to the momentum conservation. The role of this flow called as mean flow effect was investigated in the occurrence of secondary instability in convective systems. If the horizontal size of the system is infinite and the friction at the top and bottom boundaries is negligible, the fluid velocity parallel to the horizontal plane behaves as a NG mode. Though actually it seems very difficult to realize such an experimental system, the research using numerical simulation for the Rayleigh-Bénard convection in a large system with free boundary conditions was done (Xi *et al*., 1997). The results are very similar to ones of the SMT, such as observed patterns and bifurcation to spatiotemporal chaos. Furthermore, the so-called Nikolaevskii equation which was originally derived against the propagation of longitudinal seismic waves in viscoelastic media have been intensively investigated (Tribelsky and Tsuboi, 1996). The equation which includes nonlinear coupling between a short-wavelength mode and a NG mode exhibits spatiotemporal chaos similar to the SMT. Most important result for all these is that spatiotemporal chaos appears via a single supercritical bifurcation. This is only related to the symmetry problem. Thus, it can be concluded that SMT is quite universal and independent of specific properties of individual systems except symmetry. So far, however, actually observed unique system is only homeotropical electrohydrodynamics of liquid crystals. We therefore wish to discovery of further examples for more advanced development of science.

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