Connecting Pathway between Beginning and Termination —Art Forms of Wheelbase-Based Motion

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(Received December 9, 2011; Accepted February 8, 2012)

The geometric forms of relative motions are the geometric art forms developed by the researcher over the recent years. This study is aimed at probing one type of the relative motion: wheelbase-based motion, developing a systematic model to control its motion parameters, analyzing how the parameters affect its process of motion, and discovering the geometric art forms generated by computing the parameters. Wheelbase-based Motion suggests that the trajectory appears in a straight and direct pattern of motion, moving back and forth in a two-dimensional space from the beginning to the termination. In this case, this study designs and develops the Relative Motion Creator (RMC) as the study tool to test this wheelbase-based motion so that it may compile the model of wheelbase-based motion, complete its motion structure, and operate its motion parameters.
Key words: Pathway, Art Forms, Wheelbase, Relative Motion, Trajectory

1. Introduction

The advance of modern technology and computing instruments has made the plastic art of materials and tools in various dimensions possible. In activities of plastic art, the graphic design is called the two-dimensional art, suggesting the artistic activities happen in a 2D space, while the stereoscopic design implies that the artistic form exists in a 3D space. In this case, if the timeline is added to keep records of the graphic motion, this may be also considered a 4D graphic design. Such an artistic genre is regarded as Kinetic Art/Sculpture as well (Lin, 1999). For example, the prominent Russian sculptor of Constructivism, Gabo (1920), has made use of thin long wires to create his famous kinetic sculpture, *Standing Wave*. This sculpture was vibrating motor-driven by generating a significant oscillation on the strings so that the long still wires can be in the form of curves because of the physical energy.

In the history of Western arts, futurism is seen as people's rebellion against the artistic phenomenon at that time as well as an inspiration of new ideology (Molino, 1992). It rises from literary movement in 1909 when Marinetti advocated the termination of all traditional arts and the establishment of a novel genre integrating arts with the living pace of machine era. As soon as the futurism came into being, it soon spread to influence the creation of all fields, such as arts, music, drama, movie, and photography. Futurists are obsessed with the aesthetics of motion and speed, preferring machines and technology to traditional arts. Therefore, they devote themselves in presenting the motion, speed, and the transformation process of objects. In a futurist painting, spatial concept exists no more and the object is never still. Moreover, the objects are moving and intersecting without a stop.

Futurism is deeply affected by post-impressionism and cubism. Though it has borrowed numerous features of cubism, futurism still preserves its own uniqueness. For instance, cubism is composed of geometric pictures in a still manner. By decomposing and reconstructing, cubism presents the static aesthetics of machinery. However, futurism focuses on motion and change. It decomposes the movement into fragments, blending them with the development of time to demonstrate the beauty of speed (Shlain, 1993). Balla (1912) had created the work of named Dynamism of a Dog on a Leash. The walking of dog and the swinging of leash tied on the dog's neck overlap each other in one single space so that the dog appears to have more than four legs and the leash presents to be more than a curve. Duchamp (1912) had created the work is called Nude Descending a Staircase. This creation successfully reproduces the posture of a nude lady walking downstairs in superimposition. Similarly, Duchamp (1923) had also created the work of named Revolving Glass Plate, it is used out of bicycle wheels to demonstrate how the objects are superimposed and integrated as the time evolves. Both these two works present similar expression and concept of the physical features in space.

To sustain the concept of futurism, any straight lines in a 2D space can be regarded as the collection of a certain vertex moving from the beginning to the termination. Its movement seems to take the geometric forms of relative motion. The geometric forms of relative motions are the geometric art forms developed by the researcher over the recent years. Liao and Sun (2005, 2006) had developed the recursive type of relative motion and tested its parameters of motion. Recursive motion had been regarded as a particular structure of relative motion, and unpredicted on geometric forms of its trajectory. This study is aimed at probing new one type of the relative motion: wheelbase-based motion, develop-

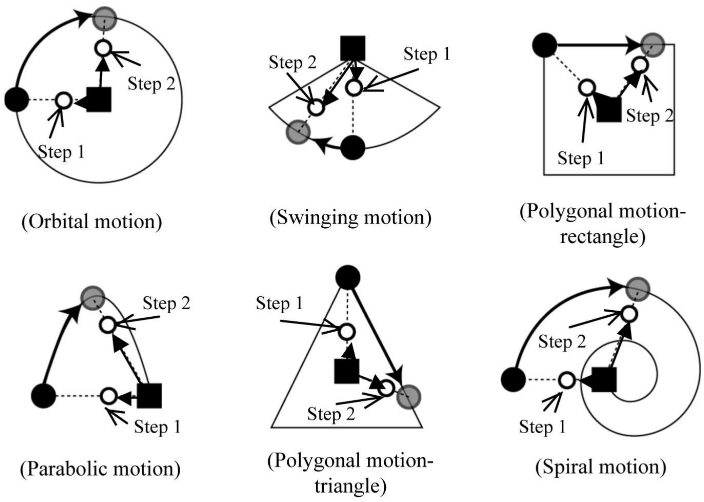


Fig. 1. The demonstrations of wheelbase-based motion (●: subjective system, ■: objective system, ○: wheelbase-based motion).

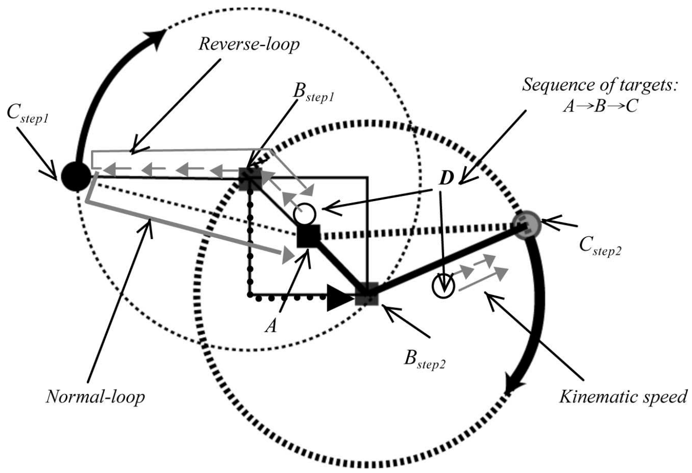


Fig. 2. Manipulate parameters of wheelbase-based motion.

ing a systematic model to control its motion parameters, analyzing how the parameters affect its process of motion, and discovering the geometric aesthetic form generated by computing the parameters. In this case, this study designs and develops the Relative Motion Creator as the study tool to test this wheelbase-based motion so that it may compile the model of wheelbase-based motion, complete its motion structure, and operate its motion parameters.

2. Concept of Wheelbase-Based Motion

Wheelbase-based Motion suggests that the trajectory appears in a straight and direct pattern of motion, moving back and forth in a two-dimensional space from the beginning to the termination. If the beginning and the termination is static, the trajectory of Wheelbase will appear simply in a straight line and the two points of beginning and termination in the space will be the vertexes of this line. As the computing progresses, the trajectory will move in a fixed span on the wheelbase. No matter what motion structure or speed the beginning or termination has, the trajectory of wheelbase shall be moving close to the termination with the development of computing.

This study defines the beginning as the absolute static center in this two-dimensional space of motion meaning the ‘objective system’ of motion structure, while the termination is regarded as its ‘subjective system.’ If there is a certain motion of the subjective system going against the objective one, when the objective system is in a static manner, the subjective system will dominate the moving of Wheelbase-based Motion, including its direction and speed. In this case, the motion speed has little effect on the original speed of wheelbase. As it is seen in Fig. 1, the objective system is the static center in this 2D space. When the subjective system is moving against the objective system, for instance, in Orbital motion, Swinging motion, Rectangle motion, Parabolic motion, Triangle motion, and Spiral motion respectively, wheelbase-based motion will be affected as well. This study adopts ‘step 1’ and ‘step 2’ in the figure to demonstrate how the trajectory is moving from the beginning (namely, the objective system) towards the termination (that is, the subjective system) and defines it as the fundamental structure of wheelbase-based motion.

Therefore, if the motion structure of subjective system is modified, the trajectory of wheelbase will be different likewise. Meanwhile, if the objective system shares the characteristic of subjective system (that is, the former is not

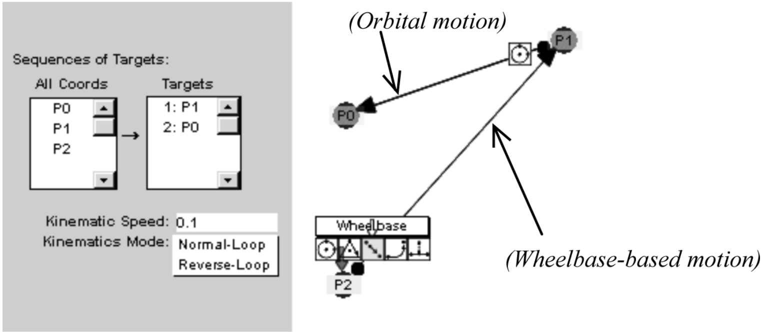


Fig. 3. Relative Motion Creator (http://can.elt.nhcue.edu.tw/cgflab/download.html).

static but with a unique motion structure), the trajectory of wheelbase will be more complicated to predict. A set of wheelbase-based motion can be so complicated to include several objective and subjective systems. In this case, the following things can be concluded from all factors affecting the trajectory of wheelbase: the speed status of objective and subjective systems in their motion structures (i.e. the angular variation), the span between objective and subjective systems, and the kinematic speed of wheelbase (which determines its distance of moving). On the other hand, the 'Sequences' defined by the moving sequence of wheelbase will affect the motion of trajectory directly. Take Fig. 2 for example, if the orbital motion is integrated with the rectangle motion (C $\rightarrow$ B: orbital motion, B $\rightarrow$ A: rectangle motion), $C_{step1} \rightarrow C_{step2}$ will be likely to revolve in a clockwise direction, while $B_{step1} \rightarrow B_{step2}$ moves in a counterclockwise direction. D suggests the wheelbase-based motion. The moving sequence of wheelbase can be in the sequences of targets of A $\rightarrow$ B $\rightarrow$ C. In other words, when the trajectory moves to C, it will be back to A and continue moving in this circulation. The whole process is called 'Normal-loop.' Otherwise, if the trajectory moves back to its previous target after C, the whole process is called 'Reverse-loop.' Consequently, the kinematic speed of wheelbase will present a corresponding moving distance. The faster it is, the farther the distance will be.

It can be concluded that when the subjective system is in a relative motion against the objective system and the objective system is in a static manner, wheelbase-based motion will be determined by the direction of subjective system. Providing that the subjective system is in an orbital motion, the trajectory of wheelbase will be the pathway generated with the process of computing. Moreover, the relationship between kinematic speed and orbital speed will make the pathway appear in various curved spirals. If the objective system shares the characteristic of subjective system, the beginning and termination of wheelbase will be able to change their directions and posts actively, generating dynamic and unpredictable trajectory in a geometric manner. On the other hand, the span between subjective and objective systems suggests the number of sequences needed for the completion of wheelbase-based motion. In this case, if the subjective system itself has the model of dynamic radius variation, wheelbase-based motion will change its corresponding number of sequences.

3. Manipulation of Motion Parameters on Art Forms

Strictly speaking, wheelbase-based motion is a concept rather than a type of motion. It is unlikely to be in motion automatically, but determines its position in accordance with the expected objective. Therefore, the motion type of subjective system can determine the trajectory variation of wheelbase straight away. In order to understand how the manipulation of motion parameters of wheelbase-based motion affects its trajectory, this study designs and develops an edit tool, Relative Motion Creator (RMC), to control the motion parameters systematically. As it is seen on the right side of Fig. 3, which demonstrates the fundamental structure of wheelbase-based motion shown in Fig. 2, whose subjective system appears in the orbital motion, each motion structure is presented in the manner of motion icons to show its specific motion type. $P_1 \rightarrow P_0$ means the orbital motion, while $P_2 \rightarrow P_1$ suggests the wheelbase-based motion. Moreover, relevant motion parameters are shown on the left side: sequence of targets, kinematic speed, kinematics mode (normal-loop/reverse-loop). The geometric graphic test is simplified initially as the wheelbase-based motion of 2-RL (Relative levels) to observe its motion and tendency so as to further analyze the form developed by its trajectory for the reference of studying the transformation of trajectory in higher motion structure of RL.

3.1 Two relative levels of wheelbase-based motion

The wheelbase-based motion of 2-RL is, in fact, quite similar to the trajectory generated by the revolving motion mentioned earlier. As the structure of 2-RL is seen in Case 1 (as Table A1), its subjective system (P_1) adopts the motion type of revolving. When the orbital speed of P_1 and the wheelbase speed of P_2 are manipulated respectively at the same time and the radius of P_1 is defined as the fixed and dynamic variations, the transformation of trajectory is shown exactly as Table A1. It is discovered that when the radius is fixed, if the orbital speed of P_1 is the same as the wheelbase speed of P_2, they may produce the same pattern of trajectory. The only difference lies in the boldness of lines, suggesting that the two trajectories are moving in a slightly different pathway to generate such various bold lines. If the wheelbase speed of P_2 is fixed and is made to increase with the orbital speed of P_1, when comparing the trajectory of P_1 (0.05, 0.15) and that of P_1 (0.1, 0.2), it is discovered that the latter pattern of each group has higher density of intersection than the former one. Since the test selects the trajectory generated by 100,000 iterations, if the computing continues unlimitedly, the trajectory will eventually form a filled up circle. Likewise, if the orbital speed of P_1 is fixed and the wheelbase speed of P_2 varies, the transformation of trajectory will be likely the same as the analysis mentioned previously. Generally speaking, when the orbital speed is the same, the higher the kinematic speed is, the more angles the trajectory will produce. Nevertheless, such a rule does not seem to apply to this empirical study. For example, in Case 1, comparing the transformation of trajectory of P_2 (0.05, 0.1, 0.15, 0.2) when P_1 (0.05) is fixed, the visual variation of angles is 9, 9, 27, and 18 respectively. However, if viewed from the perspective of wheelbase-based motion to observe the drawing of trajectory, the angles shall be 9, 18, 27, and 36 respectively. In other words, P_2 (0.1) and P_2 (0.2) have two sequences of targets overlapped each other twice on the same place of trajectory (or, it may be only a very slight difference between the two). That's why their trajectories produce only half of the real angles visually. On the other hand, the trajectory generated by various orbital speed of P_1 when P_2 (0.15) is fixed, some curves are presented in a rather dark and bold trajectory. This may be the consequence when the deviation of trajectory is getting smaller when the sequence of targets has been repeated twice. Observing the trajectory produced when P_1 is in the model of dynamic radius, the transformation of trajectory appears to be more complicated and strange. Since the beginning of wheelbase-based motion is P_1, when the wheelbase speed picks up, it will reach the second sequence of targets (P_0) sooner to complete the first sequence of targets of wheelbase-based motion and to proceed to P_1 again. Meanwhile, under the double effects of P_1 dynamic radius, the trajectory of P_1 is, actually, closer to that of P_2, making P_2 reach the position of P_1 soon, and the pattern repeats itself constantly.

In order to detect how the manipulation of wheelbase speed and orbital speed affect the transformation of trajectory in an easier way, this study sets up a much slower orbital speed to collaborate with a smaller slight variation of radius. For instance, in Case 2 (seen as Table 1), the rather bold trajectory is the orbital trajectory of P_1, while the rather thin one is the wheelbase trajectory of P_2. The faster the Kinematic speed is, the sooner it will reach the sequence of targets. However, if the object moves at a rather slow speed, the trajectory of wheelbase will appear to be extremely sharp. Therefore, as the wheelbase speed of P_2 increases, the trajectory nearly fills up the spiral scope formed by the revolving of P_1. On the other hand, under the impact of low orbital speed, the trajectory of wheelbase is more likely to present its spiral structure in bigger curves. If a pair of spirals is regarded as the shape of a pedal, the faster the wheelbase speed is, the smaller the width of pedal will be; meanwhile, the number of pedals may increase and the gap between pedals is smaller. On the other hand, concerning the transformation of P_2 (0.005$\sim$0.08), since P_1 revolves to develop a 3-layered spiral, the trajectory of wheelbase will change the length of pedals with the shrinking spirals. However, when the density of pedals increases, it is unlikely to detect the slight change happening inside the spirals since it is nearly filled up with darker lines. Therefore, if the Kinematic speed is faster than the orbital speed, the trajectory will be more likely to have sharp angles. Besides, as the difference of the two gets bigger, the number of pedals will increase in a denser manner. Similarly, if the two are almost equivalent at speed, the width of pedal will be larger and the number of pedals will decrease. On the other hand, if the Kinematic speed is slower than the orbital speed, the pedal can be close to the proportion of a square. In addition, the more difference the two have, the more likely the pedals will be big in scale.

3.2 Three relative levels of wheelbase-based motion

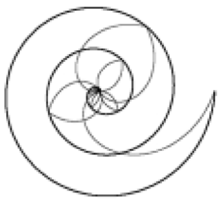
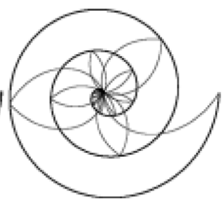
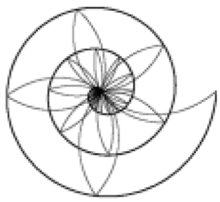
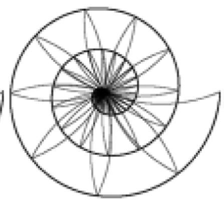
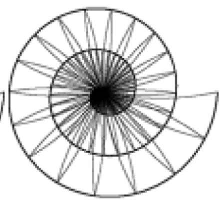
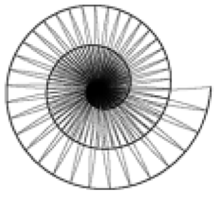
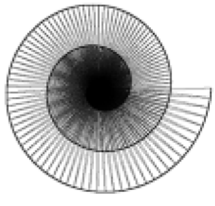
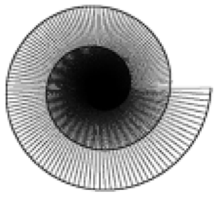
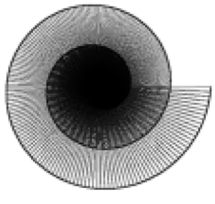
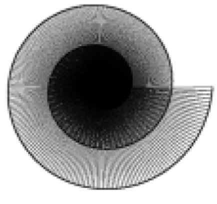
The wheelbase trajectory can be considered the results highly dynamic in accordance with the complexity of motion structure. When the sequence of targets scatters in all levels of motion structure, the wheelbase will change its position dynamically to produce various trajectories. Case 3 (seen as Table A2) is the test conducted to understand how different sequence of targets and various kinematics modes affect the wheelbase trajectory (P_3). This test focuses on manipulating the orbital speed of P_1 against P_2 in 3-RL, Normal-loop and Reverse-loop, and the two sequence arrangement as ($\rightarrow P_0 \rightarrow P_1 \rightarrow P_2$) and ($\rightarrow P_0 \rightarrow P_2 \rightarrow P_1$). The generated trajectory is shown as Appendix.

When observing the transformation of trajectory, it is discovered that different orbital angles of the objective system (P_2, P_1) will affect the wheelbase trajectory directly. Basically, the appearance of wheelbase trajectory has the same number of corner-places as the orbital trajectory of P_2. P_2 (0.45) can be regarded as the average of P_2 (0.4) and P_2 (0.5). Therefore, it is predictable that the trajectories of P_2 (0.25), P_2 (0.35), and P_2 (0.45) will be exactly with 7, 9, and 11 angles. No matter how the sequence of targets changes and what kinematics mode it is, as long as the objective system proceeds its orbital motion based on similar angular relationship and P_2 is included in the sequence of targets, the wheelbase trajectory will possess the aforementioned rules.

Comparing the trajectory of Normal-loop and that of Reverse-loop, it is discovered that Normal-loop is more likely to produce delicate and smooth trajectory visually, connecting the corner-places scattering around the pattern to the central point (P_0). This shall be the consequence when the wheelbase reaches its final sequence of targets (P_2 or P_1), it will proceed to the next sequence arrangement (P_0) again. Besides, the relationship between the orbital speed of P_2 and P_1 has an impact on the complexity of this phenomenon. On the other hand, concerning the trajectory of Reverse-loop, the sequence of targets changes its arrangement back and forth. That is, when the loop reaches its final sequences (P_2 and P_1), the original sequence arrangement will reverse. In this case, its wheelbase trajectory tends to have less external links to the pattern center visually. Because of this, the setting of Reverse-loop requires relatively more times of computing to produce a nearly complete appearance of corner-places. Moreover, the trajectory close to P_0 tends to have a more complicated structure, yet its transformation is quite clear. The motion structure of Case 3 is ($P_2 \rightarrow P_1 \rightarrow P_0$). When the arrangement of sequence of targets is similar to its original structure, the trajectory may be more complicated and different from the original structure; For instance, the sequence of ($\rightarrow P_0 \rightarrow P_1 \rightarrow P_2$), which starts from P_0 and ends in P_2, may generate a trajectory of

Table 1. Case 2: the test of 2-RL wheelbase-based motion (transformation under different wheelbase speed).

The Trajectory of P_2			
Structure	Manipulate Parameters		
	Motion Relation: $P_1 \rightarrow P_0$ Motion Type: #orbital Initial Angle: 0	Angular Increment: 0.005 Ceiling-limitation: 0 Floor-limitation: 0 Incremental Variation: 0	X-Radius: 200 Y-Radius: 200 Radius X-variation: 0.001 Radius Y-variation: 0.001
	Motion Relation: $P_2 \rightarrow P_1$ Motion Type: #wheelbase	Sequences of Targets: ($\rightarrow P_1 \rightarrow P_0$)	Kinematic Speed: 0.005~1 (dynamic) Kinematics Mode: Normal-loop

(P_2) Kinematic Speed (dynamic)				
0.005	0.01	0.02	0.04	0.08
				
0.16	0.4	0.6	0.8	1
				

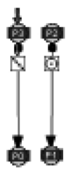
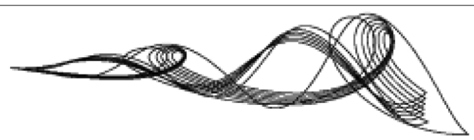
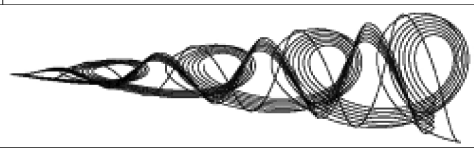
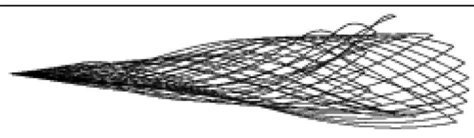
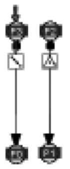
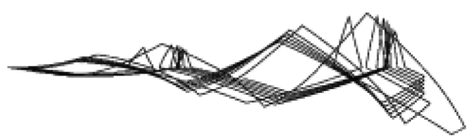
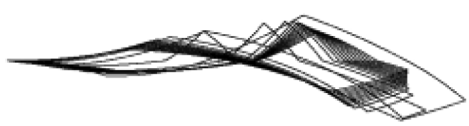
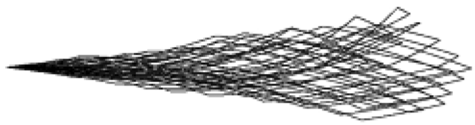
($\rightarrow P_0 \rightarrow P_2 \rightarrow P_1$). When the sequence arrangement is different from the original structure, the wheelbase trajectory will enter the stage of repeating the position of previous trajectories with less number of computing. Therefore, the trajectory appears to be simpler in terms of abstractive levels. Both Normal-loop and Reverse-loop will develop similar transformation of trajectory.

3.3 Static/dynamic subjective motion in the sequence of targets

If the sequence of targets of Wheelbase-based motion includes static motion system of different spatial positions along with dynamic subjective system, the trajectory it produces can be diverse and interesting. For example, if a static object is placed alone outside most of the wheelbase-based motion, whenever the wheelbase moves towards this static object, sharp corner-places are likely to appear. The manipulation of the other dynamic objects' motion parameters will make the trajectory developed by the wheelbase running back and forth between the static and dynamic objects look like many spirals tangling and intersecting with one another. As it is seen in Case 4 (seen as Table 2), the transformation of wheelbase trajectory is generated from the test of 3-RL, which integrates orbital motion and polygonal motion. Since P_0 is a static system on the left side of pattern and all sequence of targets starts from P_0, the trajectories forming around it are sharp. On the other hand, the orbital motion of P_2 has its target (P_1) on the right side of pattern. Therefore, it can be concluded from the transformation of trajectory that the relationship between the wheelbase speed and the orbital speed of object has an impact on the development of trajectory.

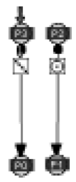
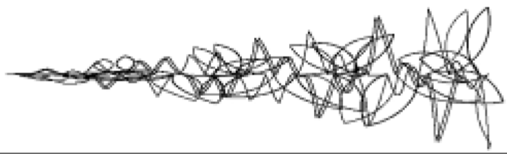
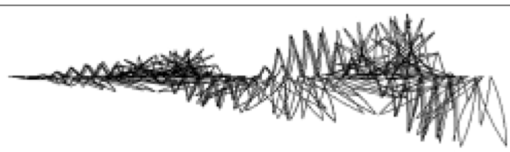
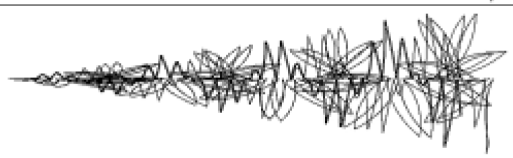
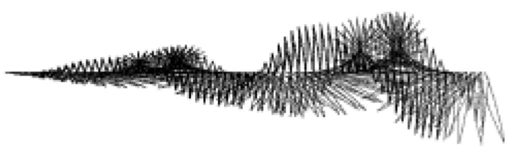
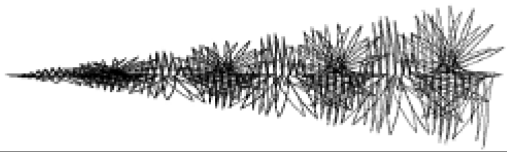
Analyzing the orbital motion of P_2, generally speaking, it is discovered that when the kinematic speed (0.1) is slower than the orbital speed, the trajectory appears to be constructed by many spirals to present a rhythm and a gradual change. However, if the wheelbase speed is increased to be (0.2), this phenomenon tends to slowly fade away. Up until the speed is increased to be (0.3), faster than the orbital speed, with the curvature of trajectory decreases, the rhythm exists no longer. On the other hand, though P_2 (0.24) is lower than P_3 (0.3), their relative difference will not be able to affect the curvature of trajectory. When analyzed the structure formed in the center of the pattern, it is noticed that the wheelbase trajectory from the right to the left turns from big into small. This may be the result of the orbital motion P_2 demonstrating its dynamic radius variation. Moreover, the relationship between the slight variation of radius of P_2 and its orbital angle will definitely affect the appearance and structure of the central trajectory. When the slight variation of radius has a higher variation

Table 2. Case 4: the test of wheelbase-based motion of various spatial positions (the manipulation of orbital and polygonal motions).

<table>
<tr><td colspan="4">The Trajectory of P_3 (80,000 iterations)</td></tr>
<tr><td>Structure</td><td colspan="3">Manipulate Parameters</td></tr>
<tr><td rowspan="3"></td><td colspan="3">Motion Relation: P_0 (static)</td></tr>
<tr><td>Motion Relation: $P_2 \rightarrow P_1$
Motion Type: #orbital
Initial Angle: 0</td><td>Angular Increment: 0.12 、0.24 (dynamic)
Ceiling-limitation: 0
Floor-limitation: 0
Incremental Variation: 0</td><td>X-Radius: 100
Y-Radius: 100
Radius X-variation: 0.0005
Radius Y-variation: 0.0005</td></tr>
<tr><td>Motion Relation: $P_3 \rightarrow P_0$
Motion Type: #wheelbase</td><td>Sequences of Targets: ($\rightarrow P_0 \rightarrow P_2$)</td><td>Kinematic Speed: 0.1~0.3 (dynamic)
Kinematics Mode: Normal-loop</td></tr>
<tr><td rowspan="2">Kinematic Speed (P_3)</td><td colspan="3">(P_2) Angular Increment (dynamic)</td></tr>
<tr><td colspan="2">0.12</td><td>0.24</td></tr>
<tr><td>0.1</td><td colspan="2"></td><td></td></tr>
<tr><td>0.2</td><td colspan="2"></td><td></td></tr>
<tr><td>0.3</td><td colspan="2"></td><td></td></tr>
<tr><td>Structure</td><td colspan="3">Manipulate Parameters</td></tr>
<tr><td rowspan="3"></td><td colspan="3">Motion Relation: P_0 (static)</td></tr>
<tr><td>Motion Relation: $P_2 \rightarrow P_1$
Motion Type: #polygonal
Nums of Side: 4</td><td>Speed of Movement: 0.12 、0.24 (dynamic)
X-Radius: 100
Y-Radius: 100</td><td>Radius X-variation: 0.0005
Radius Y-variation: 0.0005</td></tr>
<tr><td>Motion Relation: $P_3 \rightarrow P_0$
Motion Type: #wheelbase</td><td>Sequences of Targets: ($\rightarrow P_0 \rightarrow P_2$)</td><td>Kinematic Speed: 0.1~0.3 (dynamic)
Kinematics Mode: Normal-loop</td></tr>
<tr><td rowspan="2">Kinematic Speed (P_3)</td><td colspan="3">(P_2) Speed of Movement (dynamic)</td></tr>
<tr><td colspan="2">0.12</td><td>0.24</td></tr>
<tr><td>0.1</td><td colspan="2"></td><td></td></tr>
<tr><td>0.2</td><td colspan="2"></td><td></td></tr>
<tr><td>0.3</td><td colspan="2"></td><td></td></tr>
</table>

than the orbital speed, the wheelbase trajectory tends to be more stable. Therefore, it is concluded that the manipulation of smaller slight variation of radius helps P_2 develop smooth spirals and makes the trajectory appear in a form of rhythm and gradual change. On the contrary, larger slight variation of radius is likely to cause P_2 to produce dynamic spirals, leading the variation of wheelbase trajectory to be bigger. Case 5 (seen as Table 3) is a modification of Case 4, focusing on the test of orbital motion. When the slight variation of radius of P_2 ranges as (0.5∼2), apparently, its number of sequence of targets decreases. Although the spiral trajectory does not exist anymore, a radial trajectory of pedals appears instead, scattering and intersecting around. With the increase of slight variation of radius, the pedal appears to be thinner and longer and a different presentation of wheelbase trajectory then develops.

Table 3. Case 5: the test of wheelbase-based motion of various spatial positions (the manipulation of orbital motion and different slight variations of radius).

The Trajectory of P_3 (80,000 iterations)			
Structure	Manipulate Parameters		
	Motion Relation: P_0 (static)		
	Motion Relation: $P_2 \to P_1$ Motion Type: #orbital Initial Angle: 0	Angular Increment: 0.12 、0.24 (dynamic) Ceiling-limitation: 0 Floor-limitation: 0 Incremental Variation: 0	X-Radius: 100 Y-Radius: 100 Radius X-variation: 0.5~2 (dynamic) Radius Y-variation: 0.5~2 (dynamic)
	Motion Relation: $P_3 \to P_0$ Motion Type: #wheelbase	Sequences of Targets: ($\to P_0 \to P_2$)	Kinematic Speed: 0.1 Kinematics Mode: Normal-loop
(P_2) X-Y variations	(P_2) Angular Increment (dynamic)		
	0.12	0.24	
0.5			
1			
2			

Similarly, if P_2 is a polygonal motion with dynamic radius variation, its appearance and structure of trajectory will be like what is mentioned previously. When analyzing the central section of wheelbase trajectory, it is discovered that many lines are bent with right angles by the motion type of P_2. Moreover, comparing the wheelbase trajectory of P_2 in orbital motion and that in polygonal motion, it is noticed that their turning, intersecting, and twining are quite identical. The only difference may lie in that the trajectory appears to be smooth spirals or clear broken lines.

4. Summary and Discussion

By means of manipulating the motion parameters systematically, this study is able to discover how the motion features and structure affect the transformation of trajectory. Running back and forth from the beginning and the termination, if the sequence of targets of wheelbase-based motion is defined as a corresponding structure for subjective-objective systems, the trajectory will be rather complicated. That's because when the wheelbase is increased with the number of computing and moves in the whole process of subjective system, more sequences of targets will generate more intensive trajectories. On the other hand, when the sequence arrangement is different from its original structure, since the sequence of targets is relatively low and the trajectory repeats its position constantly, the appearance of trajectory seems simple. Therefore, if the sequence arrangement is set up as a motion structure corresponding to subjective-objective system, the geometric form will be rather simple and sparse and the overall result hardly distributes over the 2D space covered with the motion structure.

When the structure of subjective-objective motion system is rather complicated (i.e. ≧3-RL), the kinematics mode (Normal-loop/Reverse-loop) turns out to be obvious. From the previous tests, it is discovered that Normal-loop often produces delicate and smooth trajectories visually, connecting the corner-places scattering around the pattern to the center of motion structure (i.e. the static center of 2D space). The main reason may be that when a sequence of targets is completed, the trajectory enters directly the first sequence of the original sequence of targets (e.g. P_0). On the other hand, under the impact of Reverse-loop, the sequence of targets' arrangement varies back and forth. When the last sequence is completed, the trajectory will reverse its original sequence arrangement again. Therefore, the wheelbase trajectory seldom appears to connect from external corner-places to the pattern center. In this case, it can be concluded that Normal-loop is relatively easier to produce a rather simple and center-focused geometric shape than Reverse-loop. Nevertheless, Reverse-loop requires relatively more times of computing to accomplish it. Basically, manipulating the range of kinematic speed makes the sharpness of corner-places of trajectory easy to control and it helps to determine if the area constructed by the entire motion structure will be fully covered. When the Kinematic speed is fast, the trajectory moves like a line connecting the subjective-objective systems directly. Moreover, the lines will intersect each other with rhythms. On the other hand, when Kinematic speed is slow, the trajectory is seriously affected by the motion direction of subjective system. Besides, the level of influence depends on how close the trajectory is to the subjective system to transfer gradually (in other words, how far the trajectory away from the objective system).

Relative motion has its unique motion parameters. There can be various motion types inside one single motion structure. Even if it is the most simplified motion structure, it may possess a combination of multiple dimensions. Therefore, this study can only observe the motion tendency to some extent and analyze its construction. Yet, it is unable to present all sorts of variance combination of motion parameters and to learn what the trajectory may appear visually. In the future, this study may try the motion structure of Chaotic Term to turn the stabilized motion phenomenon into an unpredictable and dynamic scenario. In this case, the motion tendency will be difficult to observe and its trajectory may change as it wishes. However, by means of the RMC tool, which helps to manipulate and experiment the parameter combination and motion structure, this study may be able to observe the transformation of trajectory and to understand the relationship among corresponding motion parameters.

Acknowledgments. The author is very grateful to National Science Council (Taiwan, R.O.C.) for providing budget to accomplish this research project (NSC 98-2410-H-134-024-).

Appendix A.

Appendix includes Table A1 and Table A2, and see on p. 17–18.

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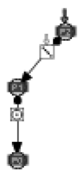
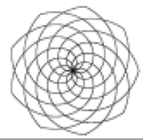
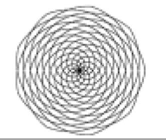
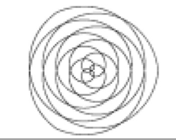
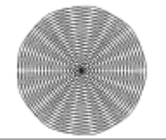
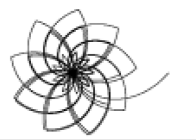
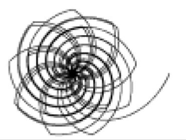
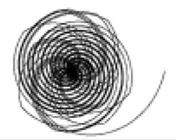
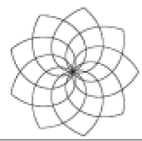
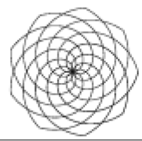
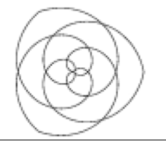
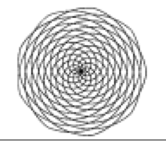
Table A1. Case 1: the test of 2-RL wheelbase-based motion (the variation combination of wheelbase speed and orbital speed).

The Trajectory of P_2 (100,000 iterations)			
Structure	Manipulate Parameters		
	Motion Relation: $P_1 \to P_0$ Motion Type: #orbital Initial Angle: 0	Angular Increment: 0.05~0.2 (dynamic) Ceiling-limitation: 0 Floor-limitation: 0 Incremental Variation: 0	X-Radius: 200 Y-Radius: 200 Radius X-variation: 0 、0.1 (dynamic) Radius Y-variation: 0 、0.1 (dynamic)
	Motion Relation: $P_2 \to P_1$ Motion Type: #wheelbase	Sequences of Targets: ($\to P_1 \to P_0$)	Kinematic Speed: 0.05~0.2 (dynamic) Kinematics Mode: Normal-loop

(P_2) Speed	(P_1) Angular Increment (various change)			
	0.05	0.1	0.15	0.2
0.05	Radius X-Y variation: 0			
	 1			
	Radius X-Y variation: 0.1			
				
0.1	Radius X-Y variation: 0			
	 5	 2		
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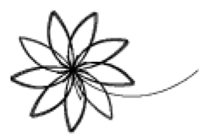
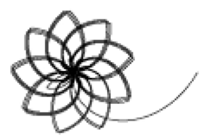
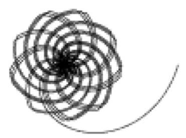
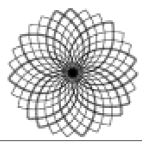
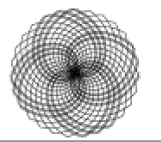
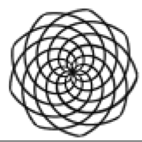
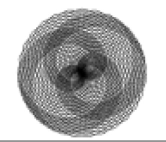
				
0.15	Radius X-Y variation: 0			
	 6		 3	
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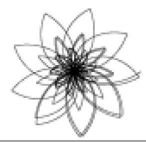
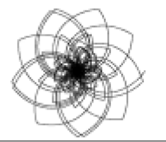
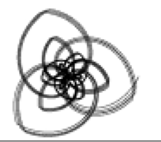
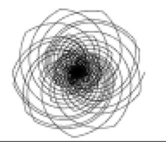
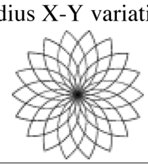
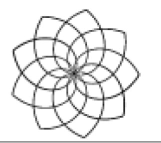
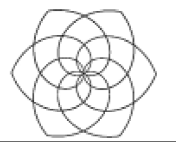
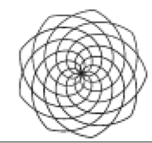
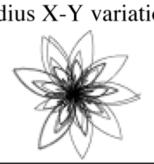
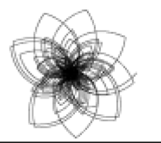
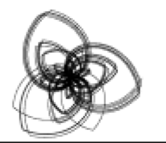
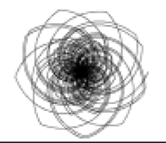
				
0.2	Radius X-Y variation: 0			
	 7			 4
	Radius X-Y variation: 0.1			
				

Note 1 2 3 4: when the angular variation equals the wheelbase speed, it may produce trajectories with the same number of corner-places and appearance.

Note 1 5 6 7: when the multiple relationships between wheelbase speed and angular variation is changed, the trajectory's corner-places may have multiples of direct proportion. But 5 7 belong to the second round sequence of targets.

Table A2. Case 3: the test of 3-RL wheelbase-based motion (the manipulation of different combination of sequence of targets, Kinematics Mode, and orbital angle).

<table>
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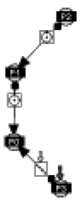
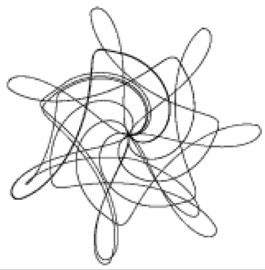
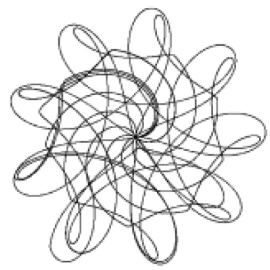
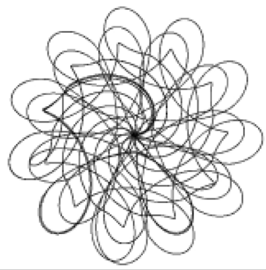
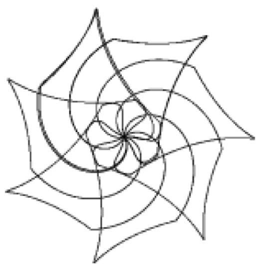
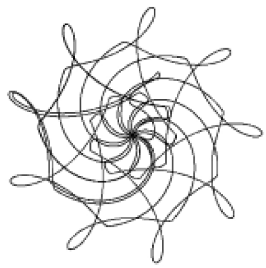
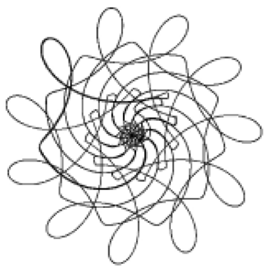
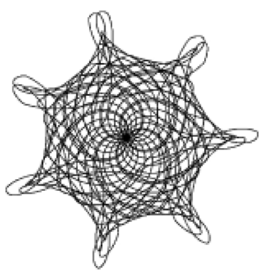
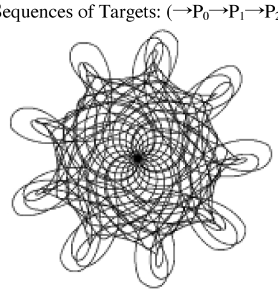
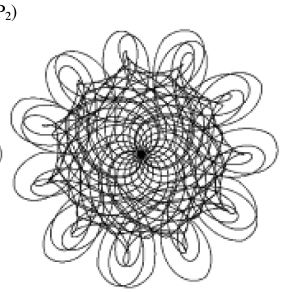
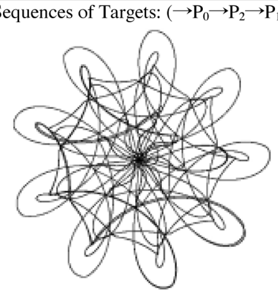
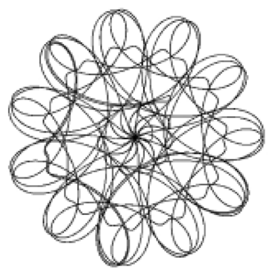
<tr><td rowspan="3"></td><td>Motion Relation: $P_1 \rightarrow P_0$
Motion Type: #orbital
Initial Angle: 0</td><td>Angular Increment: -0.1
Ceiling-limitation: 0
Floor-limitation: 0
Incremental Variation: 0</td><td>X-Radius: 150
Y-Radius: 150
Radius X-variation: 0
Radius Y-variation: 0</td></tr>
<tr><td>Motion Relation: $P_2 \rightarrow P_1$
Motion Type: #orbital
Initial Angle: 0</td><td>Angular Increment: 0.25~0.45 (dynamic)
Ceiling-limitation: 0
Floor-limitation: 0
Incremental Variation: 0</td><td>X-Radius: 100
Y-Radius: 100
Radius X-variation: 0
Radius Y-variation: 0</td></tr>
<tr><td>Motion Relation: $P_3 \rightarrow P_0$
Motion Type: #wheelbase</td><td>Sequences of Targets: ($\rightarrow P_0 \rightarrow P_1 \rightarrow P_2$) 、($\rightarrow P_0 \rightarrow P_2 \rightarrow P_1$) (dynamic)</td><td>Kinematic Speed: 0.1
Kinematics Mode: (dynamic)</td></tr>
<tr><td rowspan="2">Kinematics Mode (P_3)</td><td colspan="3">(P_2) Angular Increment (dynamic)</td></tr>
<tr><td>0.25</td><td>0.35</td><td>0.45</td></tr>
<tr><td rowspan="4">1

Normal-Loop

(50,000) iterations</td><td colspan="3">Sequences of Targets: ($\rightarrow P_0 \rightarrow P_1 \rightarrow P_2$)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="3">Sequences of Targets: ($\rightarrow P_0 \rightarrow P_2 \rightarrow P_1$)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td rowspan="4">2

Reverse-Loop

(100,000) iterations</td><td colspan="3">Sequences of Targets: ($\rightarrow P_0 \rightarrow P_1 \rightarrow P_2$)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="3">Sequences of Targets: ($\rightarrow P_0 \rightarrow P_2 \rightarrow P_1$)</td></tr>
<tr><td></td><td></td><td></td></tr>
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Note1 2: In comparison, Reverse-loop tends to have a rather complicated structure of trajectories. Moreover, if the sequence of targets and the motion structure it depends on are the same, the trajectories can be more complicated.