# Non-equilibrium in Foreign Currency Exchange Determination

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(Received December 27, 2013; Accepted December 31, 2013)

It is humanly possible to accomplish even what economic prediction proves wrong. We carried out empirical studies employing fractal analysis and statistical tests to address the problem of finding the number of variables describing the monthly JPY/USD exchange rate. Principal component analysis extracts essential variables in economic fundamentals. We herein examine whether the foreign currency exchange determination can be described by a non-equilibrium system relating to the complex economic structure of both countries, and to its change.

**Key words:** Exchange Rate, Correlation Dimension, Multiple Regression, Principal Component (PC), Non-equilibrium

## 1. Introduction

We understand from the principles of economics that the Japanese yen/US dollar (JPY/USD) exchange rate is decided by demand and supply mechanics in the foreign exchange market over a day (Stiglitz, 1993). Hence, several discussions have arisen, in the finance and economics literature, on the factors influencing changes in the JPY/USD exchange rate. To explain this we may rely on balance of trade and interest rate differences.

The exchange rate is the relative price of two currencies, determined by the interaction of supply and demand factors for the two currencies in the market they are traded in. This has been studied extensively in the literature, and has been widely discussed among investors, government officials, academicians, traders, etc. However, there are still no definitive explanations on its determination (Stiglitz, 1993; Rosenberg, 1996).

According to Cross (1998), there are several approaches in determining the exchange rate. The first is the *purchasing power parity* approach. This theory claims that in the long run, exchange rates will adjust to equalize the relative purchasing power of currencies. The second is the Balance of Payments and Internal-External Balance approach, postulating that exchange rate movements are determined by an international difference in prices, or changes in prices and tradable items. The third is the Monetary approach, based on the proposition that exchange rates are established through the process of balancing total supply and total demand of the national currency, in each economy. The fourth is the Portfolio Balance approach, which takes a shorter-term view of exchange rate into account, and broadens its scope from the supply conditions for money to the demand-supply conditions for other financial assets as well.



This paper is based on a part of my special lecture in the 76th Symposium of Society for Science on Form.

Although the approaches noted above are some of the most general and familiar ones, they have not proved adequate in providing a reliable explanation on exchange rate mechanisms, even in past studies (EPA, 1995; Hamada, 1996; Moriyama, 1999). We discuss the factors that may describe the process of the JPY/USD exchange rate.

## 2. Fractal Dimension Analysis of JPY/USD Exchange Rate

The delay coordinate is defined in an $m$-dimensional embedding space as follows:

$$\mathbf{X}_t = (x_t \quad x_{t+\tau} \quad \mathrm{L} \quad x_{t+(m-1)\tau}), \tag{1}$$

where $\tau$ expresses a sampling time step, or delay time in case we take notice of nonlinearity in the mathematical generator. According to Takens' theorem (Takens, 1981), a transformation from the time series $\{x_t\}$ to the attractor of $\mathbf{X}_t$ is embedding on the condition of $m \geq 2n + 1$. In this study, $\tau$ was set to be a month.

We calculated the correlation dimension of the exchange rate according to the idea that the time series of the exchange rate is generated by chaos. The correlation dimension of the attractor with delay coordinates is defined in an $m$-dimensional embedding space as follows:

$$D_2(m) = \lim_{\epsilon \to 0} \frac{\log C(\epsilon)}{\log \epsilon}, \tag{2}$$

$$s.t \quad C(\epsilon) = \frac{1}{N^2} \sum_{i,j=1}^{N} \Theta\left(\epsilon - |\mathbf{X}_i - \mathbf{X}_j|\right), \tag{3}$$

where $\Theta$ and $N$ express Heviside's function and the number of delay coordinates measured in this study, respectively. In general, $C(\epsilon)$ is called the correlation integral. The correlation dimension converged to 1.39 for $m > 3$, which is also estimated by the calculation of the Hurst exponent, $\mathcal{H}$ (Takada, 2013). That is, two variables at most are enough to describe the time series (Yoshimori *et al.*, 1999a). We

Table 1. Coefficients of principal components (4). The data for the economic variables were from monthly reports, financial statistics, business statistics, and surveys of current businesses.

| # | Index | $PC1$ | $PC2$ | $PC3$ |
|---|---|---|---|---|
| $X_1$ | Business conditions in Japan | 0.045 | 0.416 | 0.653 |
| $X_2$ | Money supply in Japan | −0.346 | −0.128 | 0.033 |
| $X_3$ | Trade balance in Japan | −0.192 | 0.137 | 0.030 |
| $X_4$ | Index of wholesale price in Japan | 0.286 | −0.256 | −0.287 |
| $X_5$ | Foreign reserves in Japan | −0.316 | 0.226 | −0.112 |
| $X_6$ | Industrial production in Japan | −0.274 | −0.262 | −0.235 |
| $X_7$ | Business conditions in the USA | 0.317 | −0.011 | −0.244 |
| $X_8$ | Money supply in the USA | −0.346 | 0.035 | −0.054 |
| $X_9$ | Trade balance in the USA | −0.171 | 0.398 | −0.485 |
| $X_{10}$ | Index of wholesale price in the USA | −0.345 | −0.155 | 0.085 |
| $X_{11}$ | Foreign reserves in the USA | −0.275 | −0.332 | 0.281 |
| $X_{12}$ | Industrial production in the USA | −0.347 | −0.065 | −0.087 |
| $X_{13}$ | Difference in interest rates | 0.131 | −0.552 | 0.167 |

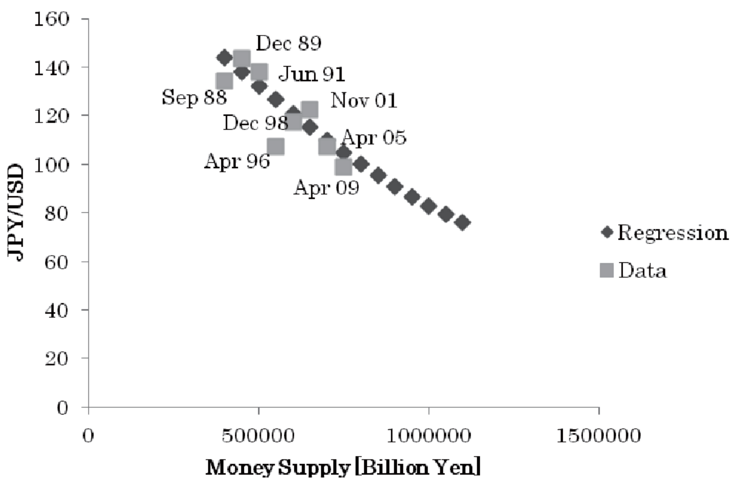


Fig. 1. A partial regression plot with standardized partial regression coefficients of money supply in Japan. The exchange rates can be estimated by the inverse standardization of regression values.

later identify two variables based on principal component analysis.

## 3. Problem and Solution in Mathematical Analysis of JPY/USD Exchange Rate

In this section, we discuss what the essential variables are, mentioned above, based on economic fundamentals.

We seek to look for correlations between interest rate and exchange rate. Many economists have studied the interdependence between interest rate and exchange rate. In our previous research (Matsugi *et al.*, 2001; Yoshimori *et al.*, 2001), it was concluded that the interest rate, especially the difference between Japanese and American long-term interest rates, was not as significant as certain other economic variables describing the exchange rate. In some quarterly models (Fukao, 2000), interest rates are estimated to be significant as a determinant of exchange rate. We now examine whether the interest rates of both countries are effective for the analysis of the exchange rate. In our monthly model, we analyze relations between the exchange rate and economic variable with backward and forward time lags. The term “lag” is defined to show a correlation between interest rate at a time, and exchange rate prior. We found that the Japanese short-term interest rate has the highest correlation coefficient with the exchange rate. The next significant correlation is with the US long-term interest rate.

We investigated for economic variables that seemed to depend on exchange rate, exploring several papers and books (Amano, 1978; EPA, 1995; Rosenberg, 1996; Cross, 1998; Ronald and Oono, 1998; Moriyama, 1999) for variables such as money supply, trade balance, index of wholesale prices, foreign reserves, industrial production, interest rates, current account, direct investment, portfolio investment, and unemployment rates. We also considered the variables such as business conditions, the macro index, and share prices (Yoshimori *et al.*, 2003).

We calculated correlation functions between the exchange rate and each economic variable listed in Table 1;

1) We employed the moving averages method so that the time series smoothened out, and then calculated correlation functions between the exchange rate and each economic variable with seven-month moving averages. The time in-

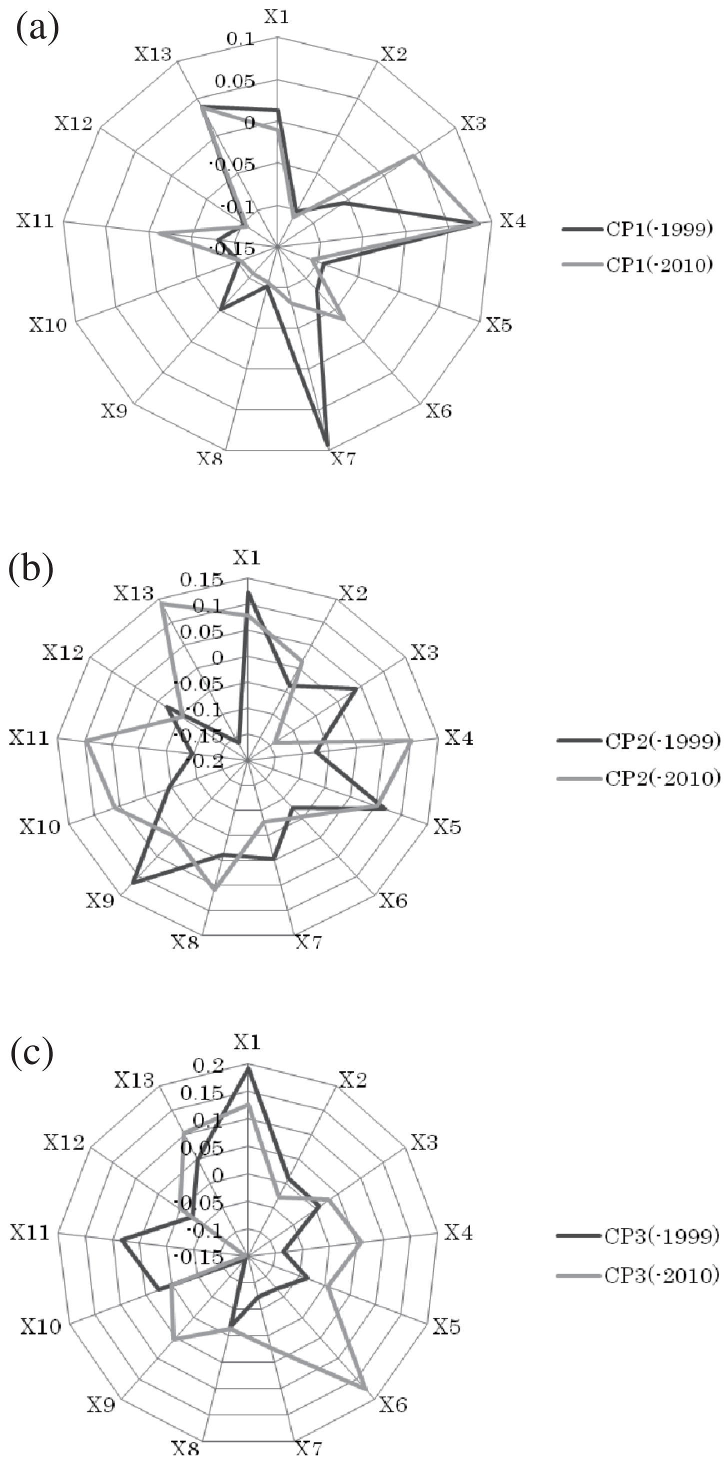


Fig. 2. Temporal changes in the principal components: $PC1$ (a), $PC2$ (b), and $PC3$ (c).

tervals with maximum correlation are given in Yoshimori *et al.* (1999b).

2) We employ multiple regression analysis (Green, 2000) for exchange rate with economic variables listed in Table 1. The exchange rate and fundamentals were standardized in this analysis. First, we try to get the coefficient of determination for $M_0$ variables. Secondly, we eliminate the next economic variable that had the smallest $t$-value. Thirdly, the coefficient of determination is derived for $M_0 - 1$variables. $M_0$ is initially set to be 13.

3) The procedure stated in 2) is iterated, until $M_0$ goes to 3.

As a result of the variable selection method stated above, the exchange rate is expressed by the money supply in Japan and the wholesale price in the US. Furthermore, we observed the multicollinearity that often occurred in the multiple regression; it was necessary to orthogonize those variables through principal component analysis. We then employed principal component analysis as the method of parsimony.

We first calculated elements of the simple correlation matrix using the same 13 economic variables. Based on this matrix, we calculated 13 eigen-values as well as 13 eigen-vectors. The first principal component $PC1$ corresponds to the eigen-vector with the largest eigen-value; the second, $PC2$, corresponds to that with the second largest eigen-value, and so on. Moreover, we employed multiple regression analysis of the exchange rate by the use of $PC1$ $\cdots$ $PC13$. $PC$ defines the principle component, i.e., a set of variables defined as a projection which encapsulates the maximum amount of variation in a dataset and is orthogonal to the previous principle component of the same dataset. We understand that $PC1$ is composed of those factors affecting economic activities in the United States and their financial policies, because increasing $X_7$, $X_{10}$ and $X_{12}$ indicates a US economic boom, while $X_2$, $X_5$, and $X_8$ are highly correlated to $PC1$ as shown in Table 1. Correlation coefficients between $PC1$ and the economic variables can be estimated by the product of square root of the 1st eigen-value and the component of eigen-vector (the coefficient of the principal component). $PC2$ and $PC3$ are mainly influenced by the difference of interest rates and Japanese boom, respectively. In addition, these economic variables also represent the Japanese recession. This implies that $PC3$ would have negative effects on the exchange rate $S_t$ through increasing exports from Japan to the United States.

## 4. Future Research Directions

In previous research, economic variables were added in mathematical models to enhance their explanatory power. Dummy variables were also seen in EPA (1995). In contrast, the number of essential variables was estimated by fractal dimension analysis for the attractor reconstructed from the time series. This estimation restricts the number of explanatory variables. The principal component analysis can statistically indicate economic meanings of the essential variables; this is similar to analyses by foreign exchange dealers as they also include economic fundamentals in their decision-making. This kind of analogy may find future research directions in mathematical science.

It was found that the regression in 12 steps gave an estimated coefficient of determination of 0.76, with the two principal components $PC1$ and $PC3$ as

$$S_t = 0.731PC1(t) - 0.463PC3(t), \qquad (4)$$

for $t = 1985.10$ to 1999.9. This regression model can be applied to the prediction of future periods, for $t = 1999.10$ to 2010.12. We have specially focused on money supply in Japan, because (1) the exchange rate is expressed using this factor in our multi-regression model demonstrated in the last section, and (2) $PC1$ is highly correlated with this factor (the correlation coefficient was more than 0.96). Using a standardized partial regression coefficient, the money supply in Japan was regressed to the exchange rate as shown in Fig. 1. Although the prediction complements this partial regression, the exchange rate is reduced with an increase in the money supply in Japan, which is against the *purchasing power parity* theory (Cassel, 1918). The foreign currency exchange determination may depend upon the complex economic structure of both countries, and the transformations herein. We should also accommodate the size of the markets in both countries.

Similarly, as stated above, a more recent regression model can be obtained, composed of the principal components in those macroeconomic factors for $t = 1985.10$ to 2010.12. (Fig. 2). We compared the coefficients of principal components $PC1$, $PC2$, and $PC3$ in this recent regression model with those in the previous one (4). In particular, $PC2$ and $PC3$ showed remarkable changes, as shown in Fig. 2; therefore, it can be proposed that foreign currency exchange determination might be described by a non-equilibrium system.

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