Model Analysis of Electric Vehicle Charging Infrastructure Development on Highways —An Approximation of the Required Scale of Electric Vehicle Charging Facilities—

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In recent years, electric vehicles (EVs) have gained prominence because of its reduction in CO_2 emissions and its departure from dependence on oil. As an increasing number of EV models are becoming commercially available, their popularity is expected to grow. Ongoing efforts to improve EV performance have not addressed inadequacies in their driving range, which is approximately 160 km. This is an issue particularly on highways in cases of long-distance driving. Hence, this research focuses on the EV support infrastructure of charging facilities on highways and proposes a mathematical model to estimate the number of EVs arriving at the charging facilities. Moreover, the model is applied to Japanese highway networks and its validity is examined. We change the number of power-feed intervals and infrastructure facilities and evaluate the various parameters to estimate the required number of EV charging facilities.

Key words: Electric Vehicle, Electric Vehicle Charging Facility, Japanese Highway Networks, z Transform

1. Introduction

Since the start of the 20th century, there has been an enormous amount of fossil fuel usage. It is not an overstatement to say that our prosperity today has been supported by this huge consumption of energy (MacKenzie, 2000). However, in the past century, this exponential consumption of energy has introduced several problems in exchange for our present prosperity.

Among them, the depletion of fossil fuel is one of the most serious and imminent problems (Roberts, 2004). The depletion of oil, in particular, has been discussed by various organizations and scientists. In the transportation sector, mainly consisting of the automobile industry, which is almost entirely dependent on oil, a reduction in CO_2 emissions and a departure from dependence on oil are seen as acute challenges (International Energy Agency, 2004).

Against this backdrop, electric vehicles (EVs) have gained prominence in recent years. Concept EVs have been developed in the past, but several factors have prevented their practical deployment. Among these factors, a short driving distance limited by battery performance and the time required to recharge the battery are major problems (Larminie and Lowry, 2003). In recent years, owing to the increasing prevalence of hybrid cars, the development of batteries such as lithium-ion batteries has led to significant advances in EV performance. However, even in recent models of EVs, the driving distance covered by a car with a fully charged battery is 100–160 km, at most (Nissan Motor Company, 2012). Alternatively, it will take approximately 30 min to recharge the battery to about 80% capacity using special rapid charging equipment (Husain, 2010). Compared with 400 km (or farther), covered by a gas-engine car with a full tank, this distance is quite unsatisfactory.

Thus, it is inevitable to develop the EV support infrastructure for EV users. In fact, there are continuous efforts to popularize the EV support infrastructures in USA (Morrow *et al.*, 2008) as well as in Japan (Ministry of Land, Infrastructure, Transport and Tourism, 2011). Furthermore, numerous models have been proposed to analyze a system of EV support infrastructure (e.g., Bapna *et al.*, 2002; Kuby *et al.*, 2004, 2009; Kuby and Lim, 2005, 2007; Melaina and Bremson, 2008; Upchurch *et al.*, 2009; Lim and Kuby, 2010; Upchurch and Kuby, 2010) and the references therein). In particular, Kuby and Lim (2005) developed the flow refueling location model to analyze the distribution of EV charging stations. They also applied this model to real-world networks at both the metropolitan and state scales (Kuby *et al.*, 2004, 2009), and have extended it to stations with limited capacities (Upchurch *et al.*, 2009), locations along arcs (Kuby and Lim, 2007), and maximizing trip miles instead of trips (Kuby *et al.*, 2009). Similarly, for Japanese EV infrastructure, Ishigame and Matsuda (2011) also calculated the favorable location of EV charging stations in Osaka Prefecture.

Meanwhile, the problem of EV's limited driving distance is a serious issue particularly on highways for long-distance driving. More precisely, EVs for long-distance driving on highways must recharge their battery on the way to their destination. Furthermore, since the EV's driving distance after quick recharging is approximately 120 km, they have to stop at EV recharging facilities "multiple" times. This

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Table 1. Roles and properties of SAs and PAs.

Item	Purpose	General scale	Number of parking slot	Interval
Service area (SA)	Rest and leisure	Big	100–250	60 km
Parking area (PA)	Only for rest	Small	40–80	20 km

fact indicates that the certain number of “EV charging facilities” must be deployed on highways, and all requests for recharging should be satisfied. Nevertheless, there are few researches that focus on the EV support infrastructure in “highway networks.” If a number of EVs use highway networks, there will be numerous demands to recharge their batteries. This shows that we have to prepare many EV recharging facilities in highway networks. It is essential to approximate the required “number” and “scale” of EV charging facilities in highway networks.

Hence, this research focuses on the EV support infrastructure of charging facilities on highway networks. We propose a mathematical model specialized for highways to estimate the number of EVs arriving at each charging facility. When considering the movements in highways, it is not important to incorporate the “route” of EV movement, because there is only a simple pass from origin to destination in highways. On the other hand, it is very important to consider EV’s “multiple” stops at EV recharging facilities. We will discuss a mathematical model that is simple but quite elegant to analyze the EV support infrastructure in highway networks.

The rest of paper is organized as follows. In Section 2, we summarize the technical conditions of EVs and generally discuss the scale of EV recharging facilities required in Japanese highways. In Section 3, a mathematical model to estimate the number of EVs arriving at each charging facility on highways is formulated on the basis of z transform (Attar, 2006), and basic properties of our model are derived. In Section 4, we apply the model to Japanese highway networks. We approximate the required “number” and “scale” of EV charging facilities and discuss conditions to accomplish the total conversion from gasoline vehicles to EVs. We finally conclude our study and present ideas for future studies in Section 5.

2. Need for EV Support Infrastructure Development on Highways

In this section, we first describe the importance of EV support infrastructure development on highways, which is the focus of this research.

As previously stated, the most significant challenge to the popularization of EVs is the performance limit of the battery (Husain, 2010). The continuous driving range of the latest commercial EV is approximately 160 km, and a 30-min quick charge can replenish up to approximately 80% battery life (Husain, 2010). Therefore, when driving continuously for a long distance, it is necessary to recharge the vehicle for 30 min approximately once every 120 km.

On highways, as is apparent from the purpose of this development, many long-distance trips are expected. Moreover, there are almost only route for EV drivers when they travels highways. More specifically, when attempting to make a trip by an EV on the highway, a driver must complete the trip by repeatedly recharging the vehicle at the charging facilities provided on the highway.

In Japan, there are two types of rest areas on the highway networks: “service areas” (SAs) and “parking areas” (PAs). The roles and properties of SA and PA are summarized in Table 1. The reality is that EV charging facilities must be installed at existing SAs and PAs. As indicated in Table 1, SAs and PAs are located at every 20 km, so the “interval” is sufficiently short. However, owing to the constraint of parking slots, there are only approximately 100–200 EV chargers. Is this scale of charging facilities sufficient to support EV infrastructure?

To consider this question, we applied a simple calculation using the Tomei Expressway as an example. Tomei Expressway is the most crowded Highway in Japan. According to Japanese road traffic census 2010 (Ministry of Land, Infrastructure, Transport and Tourism, 2010), approximately 30,000 vehicles enter from the Tokyo Interchange (IC) for 12 h on a weekday. In our scenario, all vehicles are replaced with EVs, and two-thirds of them use the highway up to the Fuji IC, which is more than 120 km away. It is then expected that these 20,000 EVs need recharging once before reaching the Fuji IC because of the upper limit of the driving range.

A total of two SAs and five PAs are located between the Tokyo IC and Fuji IC. Therefore, if charging facilities are developed at all SAs and PAs, the following number of vehicles will arrive at one charging facility:

$$\frac{20000}{12 \times 7} \approx 240 \quad \text{[vehicles per hour]}. \tag{1}$$

To closely discuss, we now introduce the queueing theory (Haviv, 2013). An EV charging facility can follow the typical M/M/s queueing model. Therefore, $\lambda / s\mu < 1$ must be satisfied; here λ is the arrival rate of EV per hour, μ is the service rate of EV quick charger per hour, and s is the number of EV quick chargers in an EV facility. This equation means that the number of arrivals to the queueing system cannot exceed the total service potential of the system. Therefore, this equation must be satisfied in any queueing model. Hence, if all EVs occupy the charger for a 30-min quick charge on average, 120 quick chargers are required. Similarly, if we set the average waiting time to less than 5 min, 124 quick chargers will be needed from the M/M/s formula. This fact indicates the need for a large-scale support infrastructure.

EVs enter the highway from other ICs, and some EVs recharge before the Fuji IC; therefore, the previous calculation is rather a rough estimation. In practical applications, the charging patterns vary according to the number of ve-

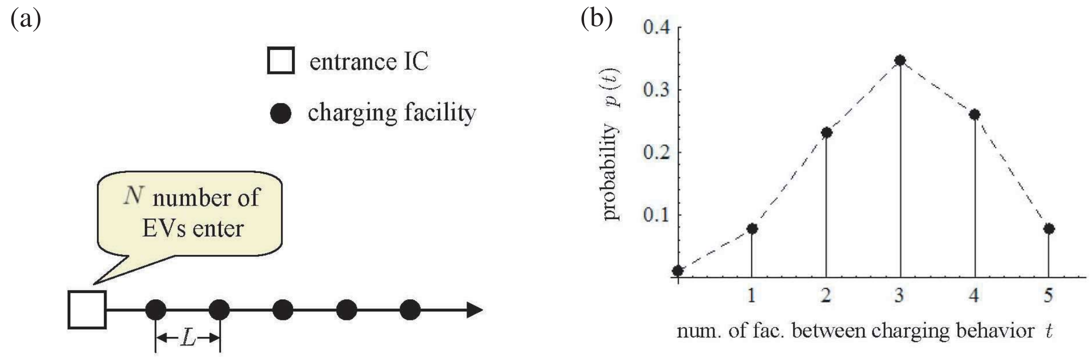

Fig. 1. Formulation in this study. (a) Highway model. (b) Example of $p(t)$.

hicles arriving at each SA and PA at a given time. Hence, the number of EVs arriving at the charging facilities, which is an important criterion for judging infrastructure development, depends on the following variables:

- The pattern of EV origin destination (OD) on the highway.
- The EV power-feed pattern.
- The placement pattern of charging facilities.

In this research, a mathematical model is proposed to estimate the number of EVs arriving at the charging facilities upon determination of these three elements.

3. Formulation of the Mathematical Model

In this section, we formulate a mathematical model based on z transform to analyze the EV charging infrastructure development on highways. In Subsection 3.1, we propose the simplest model of vehicle movement used in this research, a model with one start point and no end point. Although this assumption may seem odd, in Subsection 3.2, we expand the model to one start point and multiple end points, and further expand it to multiple start and end points. We show that these models are sufficiently descriptive. In Subsection 3.3, we use z-transform to derive a concrete analytical solution for the mathematical model in Subsection 3.1. Then, we examine concrete numerical examples and analyze the results shown in our model in Subsection 3.4. In Subsection 3.5, we discuss generalizations that consider the case of initial power-feed pattern.

3.1 Assumptions

Consider an urban model of a highway with charging facilities at regular intervals L with N number of EVs entering from an IC. In this case, assume that these EVs do not exit the highway but continue along the route and are repeatedly charged. As shown in Fig. 1-(a), this can be interpreted as an infinitely long highway with only one IC at one end and charging facilities located at regular intervals (as stated above, a generalization to multiple start and end points will be discussed in Subsection 3.2).

For the EV power-feed pattern, as shown in Fig. 1-(b), we assume

$$p(t) = \text{[probability of recharging at } t+1\text{th facility after the last charging facility (or the IC entry)]}. \quad (2)$$

To simplify the discussion, this section assumes that the vehicle is charged just before entering the highway (the relaxation for this assumption is discussed in Subsection 3.5). In this case, the following equation is calculated for each charging facility:

$$r(t) = \text{[ratio of EVs charged at the } t\text{th facility from the IC]}. \quad (3)$$

3.2 Extension of the problem

The previous formulation considers only EVs that drive from one start point to an undetermined end point and does not consider multiple OD patterns. Prior to explaining the calculation in detail, an extended formulation that introduces multiple OD patterns is described.

First, we examined the case of one start point and multiple end points. Using the highway illustrated in Fig. 1 as an example, EVs can exit at any of the multiple ICs. Thus,

$$s(t) = \text{[ratio of EVs exiting from ICs that are located beyond the } t\text{th facility from the entry point]}. \quad (4)$$

Then the above indicator can be calculated. If both a charging facility and an exit IC are present, it should be assumed that an IC is located just before the facility because smart exit IC for only ETC is located just before a SA or PA. In this case, it is possible that $s(t)$ or this OD pattern is determined independently of the charging behavior. Therefore, the number of EVs charged at the tth facility can be calculated using $N \cdot r(t) \cdot s(t)$. As shown in Fig. 2-(a), this generalization reflects the case of “one” start point and “multiple” end points.

Next we examined the case of multiple start points and multiple end points. Because an increase in the number of EV arrivals or crowded charging facilities does not affect the EV power-feed pattern even if the number of entry ICs or start points increases, it is not necessary to calculate each as an individual case. Therefore, the generalization applied to “multiple” start points can be calculated by solving $N \cdot r(t) \cdot s(t)$ for each entry IC and summing them. Figure 2-(b) shows calculations for an example with three start points at $t = 0$, $t = 2$, and $t = 4$, where N_1, N_2 and N_3 be the number of EVs entering from each start point. For EVs that entered at $t = 0$, the distribution of the number of vehicles arriving at a facility is calculated by $N_1 \cdot r(t) \cdot s_1(t)$. For

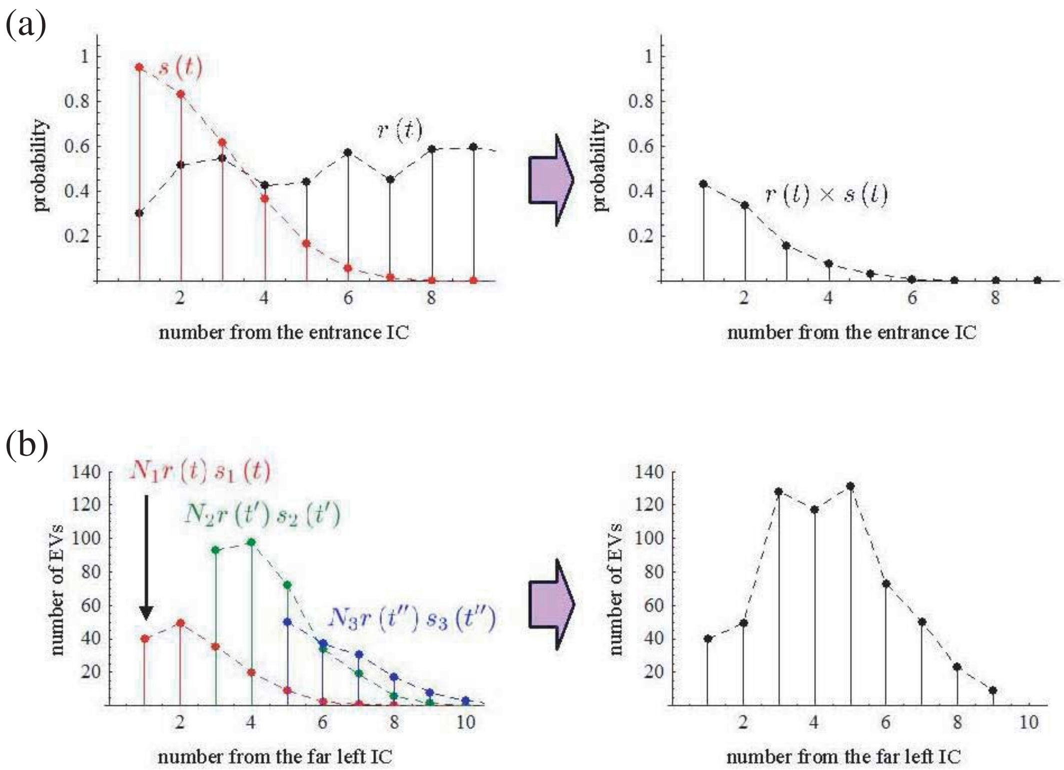

Fig. 2. Generalization of the model. (a) Generalization to one start point and multiple end points. (b) Generalization to multiple start points and multiple end points.

EVs that entered at $t = 2$, the distribution of the number of vehicles arriving at a facility is calculated by $N_2 \cdot r(t') \cdot s_2(t')$. Likewise, for EV vehicles that entered at $t = 4$, the distribution of the number of vehicles arriving at a facility is calculated by $N_3 \cdot r(t'') \cdot s_3(t'')$. These numbers can then be summed. However, note that $t \neq t' \neq t''$. The point $t = 2$ corresponds to $t' = 0$; similarly, the point $t = 4$ corresponds to $t'' = 0$. Therefore, for the sixth charging facility from the far left, the number of EVs stopping to recharge would be calculated by $N_1 \cdot r(6) \cdot s_1(6) + N_2 \cdot r(4) \cdot s_2(4) + N_3 \cdot r(2) \cdot s_3(2)$.

As a consequence, it is clear from these discussions that the generalization applied to multiple start points and multiple end points can be obtained through simple addition and multiplication even if only $r(t)$ is identified.

3.3 Derivation of $r(t)$

On the basis of the previous subsection, here we derive $r(t)$ from $p(t)$. First, we examine the relationship between $p(t)$ and $r(t)$. Initially, the derivation of $r(1)$, which is the probability that the EV will be charged at the first charging facility after entering the highway from the entry IC, is as follows:

$$\begin{aligned} r(1) &= p(0) \\ &= r(0) \cdot p(0). \end{aligned} \tag{5}$$

Here $r(0) \stackrel{\text{def.}}{=} 1$, which corresponds to the probability of EVs entering from the entry IC. Next, $r(2)$ is derived by adding the probability that the EV entering from the entry IC will be charged at the second charging facility and the probability that the EV charged at the first charging facility will be recharged at the nearest facility. Arranged as an equation,

$$r(2) = r(0) \cdot p(1) + r(1) \cdot p(0). \tag{6}$$

Similarly, to derive $r(3)$, the probability that the EV entering from the entry IC will be charged at the third charging facility is added to the probability that the EV charged at the first charging facility will be recharged at a facility that is two facilities after the first and the probability that the EV charged at the second charging facility will be recharged at the nearest facility, as shown in Eq. (7):

$$r(3) = r(0) \cdot p(2) + r(1) \cdot p(1) + r(2) \cdot p(0). \tag{7}$$

From $r(4)$ onward, the same calculation can be used. Hence, a recurrence equation can be applied to the relationship between $p(t)$ and $r(t)$:

$$\begin{cases} r(0) = 1 \\ r(t+1) = \displaystyle\sum_{\tau=0}^{t} r(\tau) \cdot p(t-\tau). \end{cases} \tag{8}$$

This equation is simply a convolution of $p(t)$ and $r(t)$.

A z transform (Attar, 2006) is valid for such a discrete convolution algorithm. Here if Eq. (8) is calculated using the z transform, the following equation is obtained:

$$z^{-1}\{R(z) - r(0)\} = R(z)P(z), \tag{9}$$

where $P(z)$ and $R(z)$ are z-transformed functions of $p(t)$ and $r(t)$, respectively. Moreover, if Eq. (9) is rearranged,

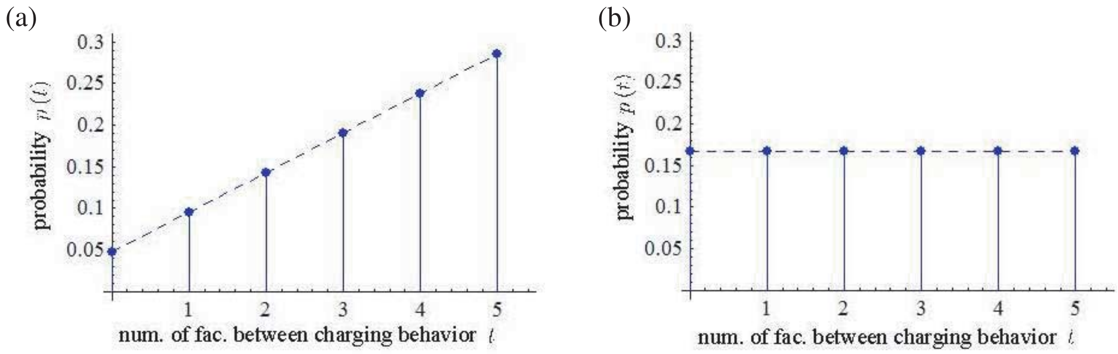

Fig. 3. Examples of $p(t)(T = 6)$. (a) Linear function. (b) Uniform function.

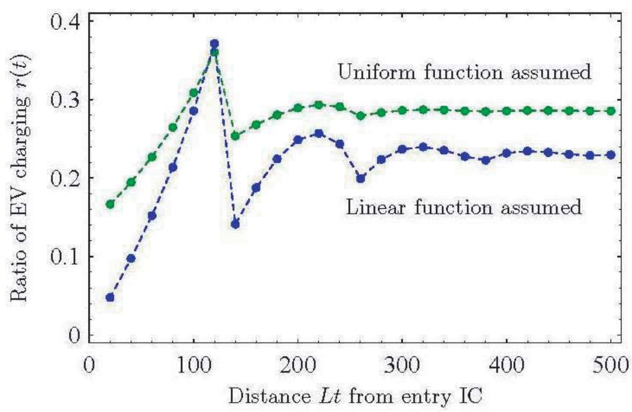

Fig. 4. $r(t)$ according to the various power-feed patterns $p(t)$.

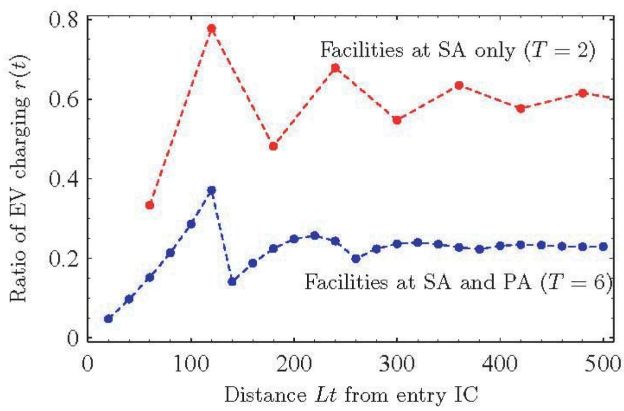

Fig. 5. $r(t)$ at various maximum power-feed intervals T.

the following equation is obtained:

$$R(z) = \frac{1}{1 - zP(z)}. \tag{10}$$

Therefore, $r(t)$ to be solved can be derived by calculating $R(z)$ using an inverse z transform:

$$r(t) = Z^{-1}\left[\frac{1}{1 - zP(z)}\right]. \tag{11}$$

3.4 Numerical examples

Using the example in Fig. 3, a numerical example is given with $p(t)$ as a linear function,

$$p_L(t) = \begin{cases} \alpha(t+1) & t = 0, \ldots, T-1 \\ 0 & t \geq T, \end{cases} \tag{12}$$

$$\alpha = \left[\sum_{t=0}^{T-1} t + 1\right]^{-1}, \tag{13}$$

and a uniform function,

$$p_U(t) = \begin{cases} \frac{1}{T} & t = 0, \ldots, T-1 \\ 0 & t \geq T, \end{cases} \tag{14}$$

where $T - 1$ is the maximum number of facilities that can be skipped with one charge.

If each case is calculated using the z transform, the following equations are obtained:

$$p_L(z) = \frac{\alpha(1 - (T+1)z^T + Tz^{T+1})}{(1-z)^2}, \tag{15}$$

$$p_U(z) = \frac{1 - z^T}{T(1-z)}. \tag{16}$$

Therefore, $r(t)$ can be calculated by substituting these parameters into Eq. (11). However, in such generalization functions, it is difficult to derive the inverse z transform using simple calculations. Therefore, in this research, we used the differential theorem

$$\lim_{z \to 0} \frac{1}{t!} \frac{d^t}{dz^t} F(z) = f(t) \tag{17}$$

for the z transform, and derived the inverse z transform by an analytical calculation using Mathematica (Wolfram, 2013).

Assuming that SAs and PAs are located at intervals of approximately 20 km and the continuous driving range after quick charging is 120 km, $r(t)$ for the linear and uniform functions when $L = 20$ and $T = 6$ are shown in Fig. 4. As is clear from this figure, the first power-feed peak occurs at the continuous driving range limit of 120 km ($T = 6$); at the second and third power-feed peaks, the probability ratio becomes constant. This result indicates that the EV power-feed patterns become more widespread when the facility is

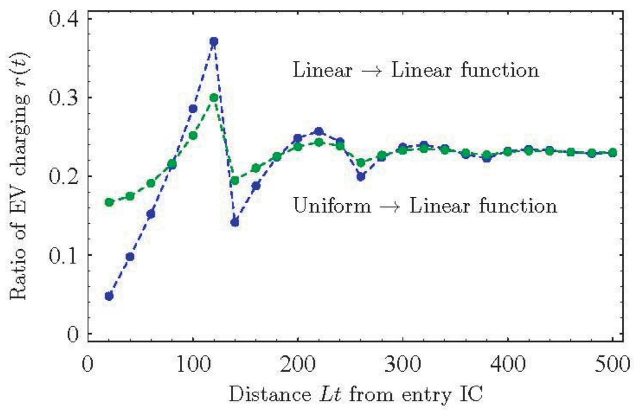

Fig. 6. $r(t)$ according to different initial power-feed patterns $q(t)$.

further away from the entry IC. By using the following limit theorem for the z transform,

$$\frac{1-z}{z}F(z) = \lim_{t\to\infty} f(t), \tag{18}$$

a convergence value for $r(t)$ for both linear and uniform functions can be calculated as

$$\lim_{t\to\infty} r_L(t) = \frac{3}{2T+1}, \tag{19}$$

$$\lim_{t\to\infty} r_U(t) = \frac{2}{T+1}. \tag{20}$$

Next, we examined the impact of intervals between power-feed facilities. Comparisons when $L = 60$ and $T = 2$ are shown in Fig. 5. A linear function was assumed for the power-feed patterns, and the charging facilities in this case were only installed at SAs located at approximately 60 km intervals. As is clear from this figure, the number of power-feed facilities is reduced to one-third, and the ratio of EVs charging at each facility increases. Moreover, as a result of the limited number of power-feed facilities, it can be confirmed that variation at each facility does not become uniform even if the distance from the entry IC is longer.

3.5 Considerations for initial power-feed pattern

As the last point regarding formulation, we discuss the generalization applied to EVs that are "not" charged immediately before entering the highway. The generalization can be applied easily to this assumption by adding

$$q(t) \overset{\text{def.}}{=} [\text{probability of recharging at the } t+1\text{th facility from the entry IC}] \tag{21}$$

and $p(t)$. In this case, the following recurrence equation is formed from $r(t)$, $p(t)$, and $q(t)$:

$$\begin{cases} r(0) = 1 \\ r(t+1) = q(t) - p(t) + \displaystyle\sum_{\tau=0}^{t} r(\tau)\cdot p(t-\tau). \end{cases} \tag{22}$$

Therefore, from

$$z^{-1}\{R(z) - r(0)\} = Q(z) - P(z) + R(z)P(z), \tag{23}$$

$r(t)$ can be derived from

$$r(t) = Z^{-1}\left[\frac{zQ(z) - P(z) + 1}{1 - zP(z)}\right], \tag{24}$$

and by using inverse z transform of $R(z)$. The result to consider initial power-feed pattern $q(t)$ is shown in Fig. 6. Assuming $q(t)$ = Uniform function, there are a large number of EVs that are not fully charged when entering the IC. We see that the ratio of EVs charging at facilities 1–3 increases. On the other hand, the ratio of EVs charging at facilities 4–6 decreases. As a result, the initial rise in the first charging peak (or "power-feed peak") is not as severe. Because the first recharge (or "initial power-feed") takes place by the sixth facility, from the seventh facility onward, there is no significant change. Eventually, we see that the numbers converge to similar values. The results show that the consideration of initial charge patterns (or "initial power-feed pattern") largely influences $r(t)$ through the initial charging peak (or "initial power-feed peak").

Fig. 7. Target highway network.

4. Application to the Japanese Highway Network

In this section, we conduct a calculation for the Japanese Highway Network using the model described in Section 3.

4.1 Target network and preparation of OD data

In this section, the model is applied to a Japanese highway network for examining the need for infrastructure development with the popularization of EVs. The target network is described in Fig. 7, which consists of 1,030 ICs (nodes). There are also 112 SAs and 260 PAs, which are potential locations of EV charging facilities. We incorporated all Japanese highways besides those in Hokkaido and

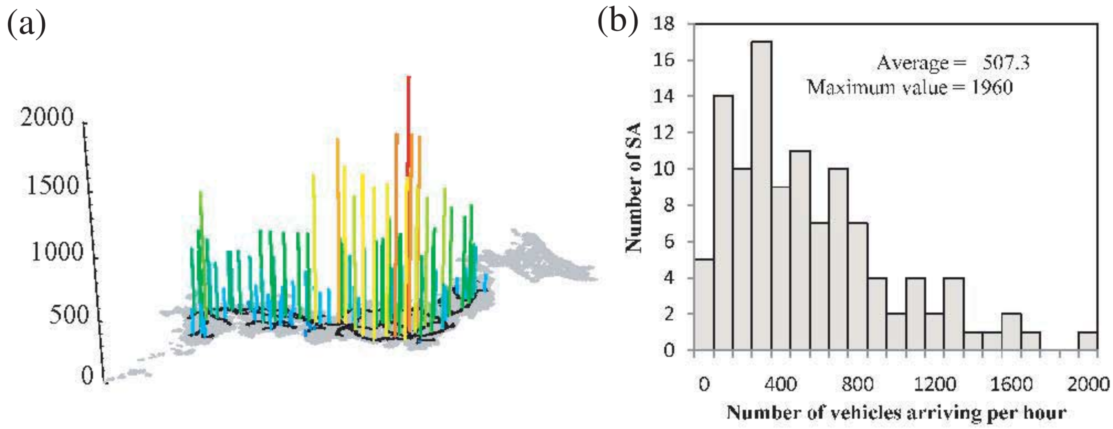

Fig. 8. Estimated number of vehicles arriving at SAs per hour (cont. driving range = 120 km). (a) Distribution. (b) Histogram.

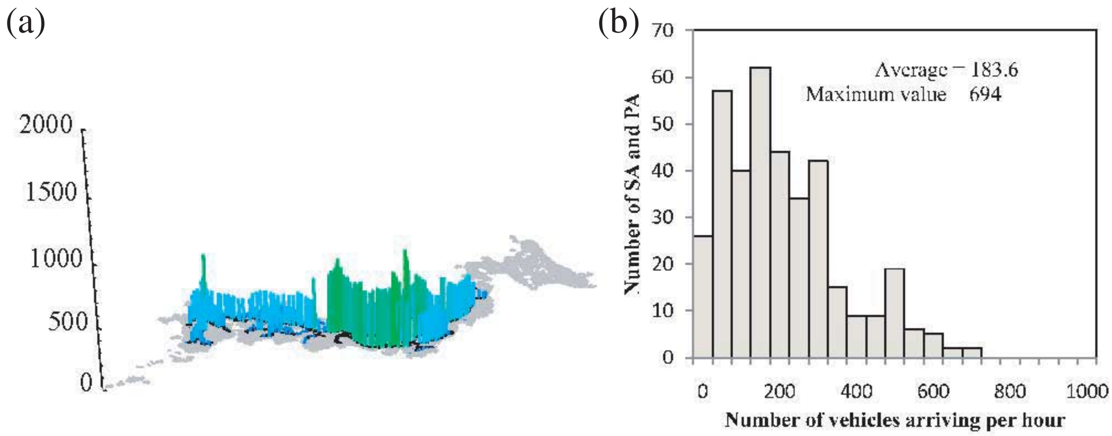

Fig. 9. Estimated number of vehicles arriving at SAs and PAs per hour (cont. driving range = 120 km). (a) Distribution. (b) Histogram.

Okinawa Prefectures (these are separated islands and not connected by strait bridge).

In this research, we calculated the number of charging facilities and number of vehicles arriving at these facilities. We assume that the vehicles driving in the target network were all replaced with EVs. Japanese highway networks are very busy currently; therefore, a calculation that replaces currently used vehicles with EVs is necessary. We prepared the real OD traffic volume between the ICs provided by Nippon Expressway Company Limited (Central Nippon Expressway Company Limited, 2012; East Nippon Expressway Company Limited, 2012; West Nippon Expressway Company Limited, 2012), which was 12 h of traffic in a weekday. The total number of vehicles in the OD matrix was 965,338,579.

4.2 Estimation result

We first analyzed the most probable estimation result based on our mathematical model. For the initial power-feed pattern, we assumed $q(t)$ to be a uniform function, because the remaining battery amount of EVs on highways would be varying. Meanwhile, for the power-feed pattern after recharging, we assumed $p(t)$ to be a linear function, because recharged EVs would be driven till their capacity. These assumptions of $q(t)$ and $p(t)$ are stable in the rest of this section.

It is very likely for EV charging facilities to be installed at SAs only in early stage of EV diffusion. This means that EVs can skip one SA at a maximum ($T = 2$) because SAs are located at intervals of approximately 60 km and the continuous driving range after quick charging is 120 km. The calculation results are shown in Fig. 8. As shown in the figure, when charging facilities are installed only at SAs, the number of vehicles arriving at the facilities is high because of the few facilities, and the number of vehicles arriving per hour is approximately 500–1,960. Now, we assume that each EV occupies the charging station for approximately 35 min on average (we considered storing, charging, and retrieving) and the number of SA parking slots is 250, and the actual diffusion rate of EVs is α. Here, similar to the discussion in Section 2, EV charging facilities should be considered to follow the queueing model, and $\lambda/s\mu < 1$ must be satisfied. That is,

$$\frac{1{,}960 \times \alpha}{250 \times \dfrac{60}{35}} < 1$$

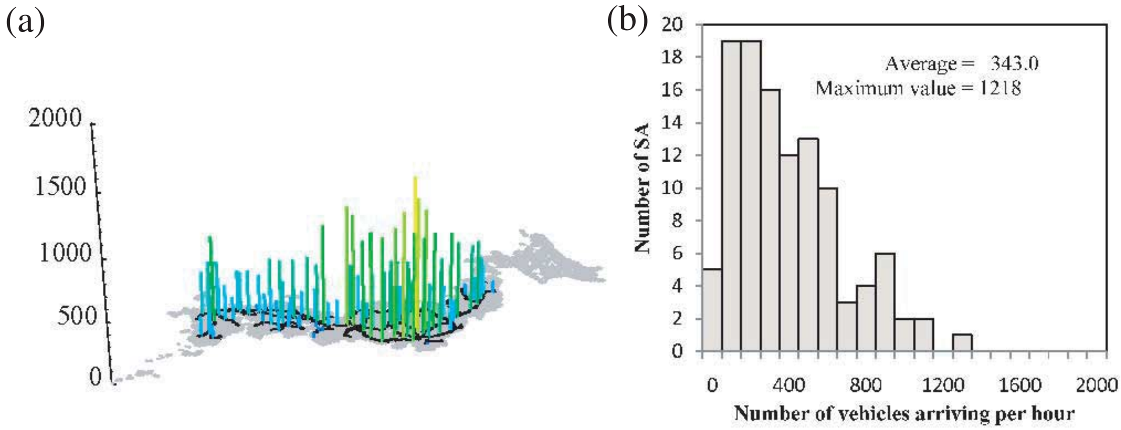

Fig. 10. Estimated number of vehicles arriving at SAs per hour (cont. driving range = 180 km). (a) Distribution. (b) Histogram.

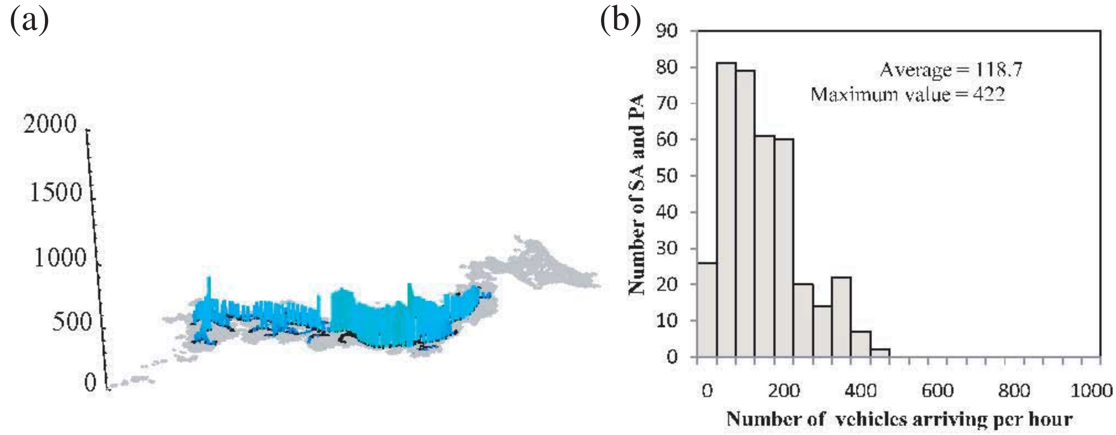

Fig. 11. Estimated number of vehicles arriving at SAs and PAs per hour (cont. driving range = 180 km). (a) Distribution. (b) Histogram.

$$\alpha < \frac{250 \times \frac{60}{35}}{1{,}960} \approx 22\%. \tag{25}$$

Hence, when charging stations are installed in car parks at all SAs, we can manage if the actual diffusion rate of EVs is less than 22%. Please recall that we could introduce the formula of M/M/s queueing model, if we have to calculate the waiting time at EV facilities. For example, if we would like to set the average waiting time to less than 5 min, $\alpha <$ 21.4% should be satisfied from M/M/s formula. In any case, the most important point is to calculate the arrival rate of EVs, which is shown in Fig. 8.

Upon viewing Fig. 8, it can be seen that the number of EVs arriving at SAs near metropolitan areas, such as Tokyo, Nagoya, Osaka, and Fukuoka, evidently increases. In addition, the regions connecting Tokyo, Nagoya, and Osaka, known as the Higashi-Meihan regions, have been confirmed to have a significant number of arriving EVs. Since the number of arriving EVs must be proportional, to some extent, to the passing traffic at that spot from a general perspective, the aforementioned result can be properly understood. Even though rural areas such as Tohoku, Chugoku, and Sanyo do not have as many EVs as those in metropolitan areas, these rural areas have a number of SAs, where approximately 1,000 EVs are estimated to arrive at. From this viewpoint, it is suggested that infrastructure improvements be important on a nationwide scale.

4.3 When installed at not only SAs but also PAs

We next analyzed the situation when EV charging facilities are installed at not only SAs but also PAs. This scenario is likely in the middle ages of EV diffusion and more charging facilities are required. In this case, since intervals of EV charging facilities are approximately 20 km, EVs can skip five PAs (or SAs) at a maximum ($T = 6$). We also assumed that the continuous driving range after quick charging is 120 km. The calculation results are shown in Fig. 9. As shown in the figure, when charging facilities are installed at both SAs and PAs, it is apparent that the number of vehicles arriving at the facilities is significantly reduced, and is approximately 180–694 vehicles per hour. Therefore, assuming the number of SA and PA parking slots are both 250 (we need to expand PAs), the actual diffusion rate α should satisfy the following:

$$\alpha < \frac{250 \times \frac{60}{35}}{694} \approx 62\%. \tag{26}$$

Hence, when charging stations are installed in car parks at all SAs and PAs, we can manage if the actual diffusion rate of EVs is less than 62%. (To set the average waiting time to less than 5 min, $\alpha < 60.5\%$ should be satisfied.)

Similar to Fig. 8, Fig. 9 shows that a large number of EVs increasingly arrive at SAs and PAs in metropolitan areas such as Tokyo, Nagoya, Osaka, Fukuoka, and the Higashi-Meihan regions. Figure 9 also indicates smooth distribution as a whole, as a result of the narrowed installation intervals of power supply facilities according to a scenario where EVs can also be charged at PAs. This can be easily inferred on the basis of the model characteristics in Fig. 5. In other words, it becomes clear that the equalization of the number of EVs arriving at the power supply facilities can be achieved by installing the facilities at SAs and PAs, thereby narrowing the installation intervals of the facilities.

4.4 Difference based on continuous driving range

Finally, we considered the difference based on a continuous driving range. We assumed the continuous driving range to be extended from 120 km to 180 km. Then the case with installation at only SAs calculated as $T = 2$ was compared with that calculated as $T = 3$ (Fig. 10), and the case with installation at both SAs and PAs calculated as $T = 6$ was compared with that calculated as $T = 9$ (Fig. 11). It is apparent from these figures that the number of vehicles arriving at each charging facility is significantly reduced when the continuous driving range is extended from 120 km to 180 km.

Moreover, as a result of varying the power-feed cycle, the magnitude relation for the number of vehicle arrivals varies. When charging facilities are installed at only SAs, the number of vehicles arriving at each charging facility is 340–1218 per hour. It means that the following actual diffusion rate of EVs should be satisfied:

$$\alpha < \frac{250 \times \frac{60}{35}}{1{,}218} \approx 35\%. \tag{27}$$

Similarly, to set the average waiting time to less than 5 min, $\alpha < 34.5\%$ should be satisfied.

Finally, when charging facilities are installed at both SAs and PAs, the number of vehicles arriving at each charging facility is 118–432 per hour. It means that the following actual diffusion rate of EVs should be satisfied:

$$\alpha < \frac{250 \times \frac{60}{35}}{432} \approx 99\%. \tag{28}$$

where the number of chargers at PA is assumed to be 250. Similarly, to set the average waiting time to less than 5 min, $\alpha < 97.2\%$ should be satisfied. Hence, it is apparent that by preparing parking spaces for 250 vehicles and installing charging stations at each SA/PA, the demand necessary for a total conversion to EV can be satisfied.

In addition, from Figs. 10 and 11, it is commonly recognized that an increasingly large number of EVs arrive at SAs and PAs in metropolitan areas such as Tokyo, Nagoya, Osaka, Fukuoka, and the Higashi-Meihan regions. The histograms of these figures should be noted here, which show that the facilities deviating from the average value have clearly decreased when compared with the histograms of Figs. 8 and 9. This means that the equalization of the number of EVs arriving at facilities can be expected when the driving ranges of EVs are extended. Figure 11 is the best example and indicates that if the spreading of facilities and the performance improvements of EVs are sufficiently carried out, the number of arriving EVs must be well distributed. This seems to be a good example to show that the innovation of technical performance may lead to a prospective large-scale infrastructure investment.

5. Conclusion

This research focused on the EV support infrastructure of charging facilities on highways and proposes a mathematical model to estimate the number of EVs arriving at each charging facility. In particular, we formulated a mathematical model to estimate the number of EVs arriving at each charging facility on highways based on z transforms and discussed basic properties of the above problem. Furthermore, we applied the model to Japanese highway networks and approximated the required "number" and "scale" of EV charging facilities. These applications indicated that (i) the installation of EV charging facilities to all SAs and PAs, (ii) about 250 chargers for each EV charging facility, and (iii) extension of the EV's continuous driving range to 180 km, are required to accomplish the total conversion to EVs. Future work should include analyzing more practical situations with several collaborated facilities as well as social optimal system, which maximizes the user benefits.

Appendix A. Formulas for z Transform

z transform (Attar, 2006) is defined by the following expression:

$$F(z) = Z[f(t)] = \sum_{t=0}^{\infty} z^t f(t). \tag{A.1}$$

When this transform is applied, the following equation relative to function convolution is formed:

$$Z\left[\sum_{\tau=0}^{t} f(\tau)g(t-\tau)\right] = F(z)G(z). \tag{A.2}$$

Moreover, a parallel translation of the function is

$$g(t) = \begin{cases} f(t+1) & t \geq 0 \\ 0 & t < 0, \end{cases} \tag{A.3}$$

$$G(z) = z^{-1}\{F(z) - f(0)\}. \tag{A.4}$$

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