Patterns of Water Waves

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1. Introduction

Wave motion of a free surface of water, such as waves generated by the oscillation of a cup partially filled with water and ocean waves observed on beaches, is called water waves, and is one of the most familiar motions of fluids for us. Water waves in a container can be generally expressed as a superposition of many wave components each of which has its own spatial pattern determined by the shape of container, similarly to the expression of oscillation of drumhead. In each of these wave components called an eigenmode of standing water waves, the free surface oscillates with a specified frequency called eigenfrequency. Since the eigenfrequencies of eigenmodes are usually different from each other except for the case of degenerate eigenmodes, only one eigenmode or a few degenerate eigenmodes of water waves can be generated by a periodic resonant forcing with an appropriate frequency. The superposition of these generated eigenmodes yields a variety of regular patterns of water waves.

2. Experiments by Faraday

Faraday waves, widely known as an example of regular pattern of water waves, are generated by the vertical oscillation of a container partially filled with water with an appropriate frequency, and were named after the experiments on this generation of waves by Michael Faraday, famous for his remarkable contributions to electromagnetics, performed in the 19th century. In Faraday waves, eigenmodes with eigenfrequencies about half the forcing frequency (the frequency of vertical oscillation) are generated, similarly to the excitation of motion of a pendulum by the vertical oscillation of its supporting point. Vertical oscillation of a container is equivalent to the oscillation of gravitational acceleration experienced by water around its average value. The resonance caused by the periodic variation in a parameter of a system around its average value with an appropriate frequency is called parametric resonance. Therefore, Faraday waves are a typical example of waves excited by parametric resonance. Here, since the wavelength of an eigenmode of water waves of larger frequency is known to be shorter, the wavelength of Faraday waves can be of the same order as the usual size of a container if forcing frequency is small. In this case, only one eigenmode or a few degenerate eigenmodes of water waves are excited strongly as Faraday waves. The pattern of water waves in this case is simple and regular, but the amplitude of each generated eigenmode can be time-dependent and the variation of wave pattern with time is not necessarily simple.

3. Various Patterns of Faraday Waves

Contrary to the above case, if forcing frequency is sufficiently large, we observe a generation of Faraday waves whose wavelength is much smaller than the size of container. In this case, the effect of side walls of container can be neglected in the central part of container. Therefore, it is reasonable to consider water waves in an infinite region to describe the local waves in this part. Such water waves are expressed as a superposition of one-dimensional waves with parallel straight contour lines of free-surface displacement at each time, with various directions of these contour lines. Therefore, the pattern of Faraday waves is expressed as a superposition of one or more one-dimensional waves of the same wavelength determined by the forcing frequency and of different directions of contour lines. A variety of wave patterns can be formed by changing the number of superposed waves.

In the images of Faraday waves obtained in experiments, we observe a few patterns that are recognized as the superposition of two or more one-dimensional waves, in addition to a simple stripe pattern composed of a one-dimensional wave. The directions of n one-dimensional waves in the former pattern are determined so that their contour lines intersect with the angle of $2\pi/n$. Square patterns composed of two one-dimensional waves (Fig. 1(a)) as well as hexagonal patterns (Fig. 1(b)) and triangular patterns (Fig. 1(c)) composed of three one-dimensional waves are observed in experiments. Wave patterns composed of four or more one-dimensional waves, called quasi-pattern, are also observed. As found from the example of quasi-pattern composed of four one-dimensional waves shown in Fig. 1(d), it is not easy to determine only from an image of wave pattern whether it is a quasi-pattern composed of several one-dimensional waves or not. However, we can determine it by

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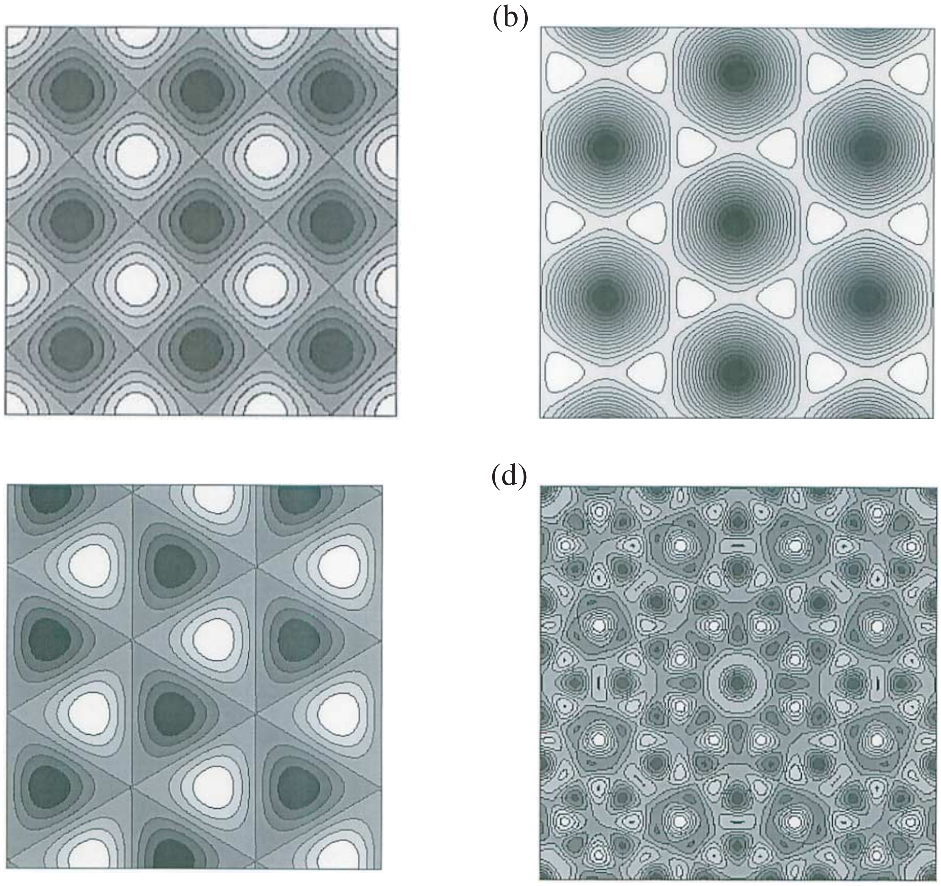

Fig. 1. Patterns of Faraday waves. Free-surface displacements are expressed by gray scale. (a) square pattern, (b) hexagonal pattern, (c) triangular pattern, (d) quasi-pattern.

the two-dimensional Fourier transform of this image.

In experiments, quasi-patterns composed of 4, 5 and 6 one-dimensional waves were found through this method. Detailed discussion on patterns of Faraday waves was made in Kudrolli and Gollub (1996) and Muller *et al.* (1998).

4. Waves Generated by the Motion of a Wavemaker

Water waves are generated also by the motion of a solid body partially submerged in water. A typical example is the waves generated by a periodic motion of a wavemaker installed at the end of a long wave tank with a constant frequency, as shown in Fig. 2. A one-dimensional wave whose contour lines are parallel to the front face of wavemaker is always generated by this motion of wavemaker, and propagates along the wave tank.

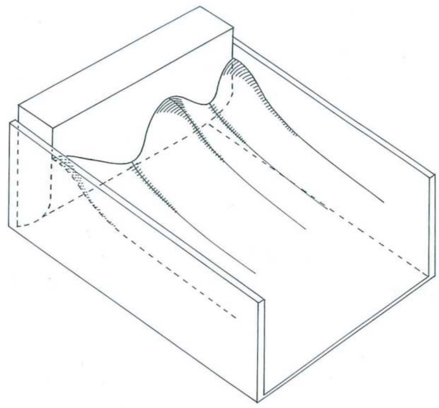

Fig. 2. Cross waves generated by the motion of a wavemaker.

However, if the frequency of wavemaker is close to twice the eigenfrequency of a transverse one-dimensional mode of standing water waves whose contour lines are parallel to two side walls of wave tank (shown in Fig. 2), another kind of waves is generated by parametric resonance, in addition to the propagating one-dimensional waves described above. It is a transverse one-dimensional standing wave of much larger amplitude, called a cross wave.

It is interesting that these waves varying along the transverse direction of wave tank are generated by the transversely uniform motion of wavemaker. In the experiments on cross waves, if amplitude a_f of the motion of wavemaker is relatively small, cross waves are localized near the wavemaker as shown in Fig. 2, and their amplitudes are time-independent at each location. However, if a_f is large, the amplitude of cross waves near the wavemaker increases with time, and then cross waves of sufficiently large amplitude are shed from the wavemaker. This process is observed repeatedly. We can find also an irregular behavior of cross waves for still larger a_f. Examples of these interesting behaviors of cross waves were reported by Shemer and Kit (1989).

5. Petal-shaped Waves around an Oscillating Sphere

Generation of waves similar to the above-mentioned cross waves is observed also when a sphere is half submerged in water and forced to oscillate vertically with frequency f_s. If amplitude a_s of this oscillation is sufficiently small, only concentric waves of frequency f_s are generated around the sphere and then propagate outward. However, for large a_s, a regular pattern of petal-shaped waves is observed around the sphere for appropriate value of f_s. Since the frequency of petal-shaped wave is half of f_s, it is suggested that this wave is excited by parametric resonance, similarly to the case of cross waves. Moreover, the pattern of petal-shaped waves can rotate around the sphere, unlike the cross waves generated by a wavemaker. The observation of petal-shaped waves in experiments was reported by Taneda (1986).

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